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Aryabhata dalit, his philosophy of *ganita*, and its contemporary applications[1]

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Introduction: the bhata

Aryabhata (आर्यभट) was a dalit as his name *bhata* (भट) shows.[2] This is not a retrospective superimposition of a modern category, dalit: his contemporary critic Brahmagupta[3] (himself a *vaisya* as his name *gupta* shows) criticizes him in the most contemptuous terms, and even Aryabhata's admirer Bhaskar-I refers to the "bhata and his disciples".[4]

However, persistent attempts have been made in recent times to appropriate Aryabhata's legacy by Brahamanising him. One trick is to change the spelling of his name, to the homonymous Aryabhatta (आर्यभट्ट).[5] State authority was misused to propagate that, until I publicly pointed it out.[6]

Another trick is to use images showing him as a Brahmin (with *shikha*, *janeyu* etc.), as in Aryabhata's objectionable statute in IUCAA, which still stands. Since there is much confusion about *upanayana* on Google, let us recall what Ambedkar said about it

> What is the technique which the Brahmins employed to bring about the degradation of the Shudras...? My answer to the question is that the technique employed by the Brahmins for this purpose was to refuse to perform the Upanayana of the Shudras. I have no doubt that it is by this technique that the Brahmins

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1 This paper was first presented at the national seminar on "Dalit narratives and Indian philosophy" at the A. N. Sinha Institute of Social Science, Patna University, 27-29 March 2016 and is due to appear in the Proceedings.

2 According to the Monier Sanskrit English dictionary the word *bhata* (भट) means mercenary, servant, slave, in *Mahabharata*; it is the name of a demon or a mixed-caste person. It is from the root भट् which means to hire (as in Hindi भाडा). Similar meanings of *bhata* (भट:) are also found in the Apte Sanskrit Hindi dictionary, which include भडैत सैनिक, भाड़े का टट्टू, जातिबाहिष्कृत, वर्णसंकर, पिशाच.

3 Brahmagupta, *Brahmasphutasiddhanta*, chapter 11, Tantraparikshadhyaya, criticizes Aryabhata incessantly, and says his errors are two numerous to recount. The commentary repeatedly puns about आचार्य भट:, alluding to all the negative meanings of the word.

4 E.g., Bhaskar 1, महाभास्करीय, 2.5, speaks about the "disciples of the bhata" (भटस्य शिष्या:).

5 The homonymous word *bhatta* (भट्ट) means master (स्वामी) and is also commonly the title of a learned Brahmin. Hence, this wrong spelling turns a dalit into a Brahmin. There is not the slightest doubt that the correct spelling is आर्यभट, and is the spelling used in all manuscripts of the *Aryabhatiya,* and its commentaries. See, e.g., *Aryabhatiya*, Eng. Trans. K. S. Shukla and K. V. Sharma, or Hindi trans. Ram Nivas Rai, both INSA, New Delhi, 1983. It is also the spelling used by his critics, as noted above.

6 "इतिहास के विचलन" *Jansatta*, 24 Jan 2008, http://ckraju.net/papers/Jansatta-Euclid.jpg. Also, "Teaching Racist History", *Indian Journal of Secularism*, **11**(4) (2008) 25–28, http://ckraju.net/papers/Teaching-racist-history.pdf.

> accomplished their end and thereby wreaked their vengeance upon the Shudras. [7]

The *upanayana* involves the chanting of the *gayatri mantra* which is whispered in the ear of the initiate. (Recall the restriction that the Veda must not be recited before shudras.) As Ambedkar further points out,[8]

> Under the Maratha rule any one other than a Brahmin uttering a Veda Mantra was liable to have his tongue cut off and as a matter of fact the tongues of several Sonars (goldsmiths) were actually cut off by the order of the Peshwa for their daring to utter the Vedas contrary to law.

Due to these social legacies, at the present time, the *janeyu* remains an unmistakeable symbol of casteist restrictions, which is the stock identifying mark of a *dvija.* As such the statue is misleading and communicates the wrong message that Aryabhata was non-dalit.

A statue depicting Aryabhata at IUCAA showing him with a janeyu. However, instead of the shikha which ought to accompany it, he is shown with flowing locks to emphasize the caucasian stereotype.

An "eminent astrophysicist" continued with the statue in full knowledge[9] that it paints a false image. Because of such authoritative backing, this image has been given wide currency by Wikipedia and our own media to the detriment of dalits. The mainstream media refuses to carry the counter story. These attempts to Brahamanize Aryabhata need to be resisted by a #StopBrahminisationOfAryabhata campaign along the lines of #RhodesMustFall campaign.

A dalit icon in science

Why is it important that Aryabhata was a dalit? First, while there are numerous respected dalit *religious* figures,[10] there is as yet no known dalit icon in *science.* Aryabhata made an invaluable contribution to

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7 B. R. Ambedkar, Who were the shudras? Chp. X "Degradation of the shudras". Quoted from the online version at http://www.ambedkar.org/ambcd/38C2.%20Who%20were%20the%20Shudras%20PART%20II.htm.

8 B. R. Ambedkar, http://www.ambedkar.org/ambcd/57.%20Manu%20and%20the%20Shudras.htm.

9 Personal communication with PRO, IUCAA, email dated 19 June 2006 et seq. The PRO had asked me for the authentic reference to the verse in which Aryabhata compares the spherical earth to a *kadamba* flower (Gola, 6-7). In my reply I added that the spelling of Aryabhata he was using was wrong, like the spelling in the IX th standard NCERT texts in mathematics authored by his Director, Narlikar, which changes Aryabhata's caste. He replied stating that Narlikar knew about it. He certainly did, since the spelling in the school text he authored was subsequently changed, but the statue still stands. So, it is a "mistake" made in full knowledge of the disservice it does to dalits.

10 Sanjay Paswan, *Cultural Nationalism and Dalit*, Samvad Media, 2014.

science by inventing the calculus[11] used to formulate current science. Second, Aryabhata and his followers such as Lalla[12] and Vateshwara[13] persistently resisted for centuries many common superstitions such as the belief that Rahu and Ketu are demons or that the earth is supported by शेष नाग etc. as "false knowledge" (मिथ्या ज्ञान) . This fact[14] of a long tradition of *resistance* to superstition in India, has been regrettably suppressed, with typical missionary fervour, by Western and colonised writers. It is very important, for it proves absence of hegemony, and serves as a concrete historical precedent to Ambedkar's navayana which emphasizes resistance to superstition.

Third, the fact is that the followers of the dalit Aryabhata from Patna included even the highest-caste Namboodiri Brahmins from Kerala,[15] thus transcending both regional and caste divide. Aryabhata was no singularity, since there was a second Aryabhata, author of *Mahasiddhanta,* who comes some 5 centuries later. These facts call for a re-examination of facile theories of caste, floated by Western (trained) sociologists.[16] A proper understanding of the caste system is essential to the goal of annihilating caste. It is commonsense that casteism could not have been as oppressive when Buddhism was widespread, or under Muslim rule, since people could easily convert.

Colonialism itself may have played a significant role in furthering the casteist oppression so manifest in the time of Ambedkar.[17] Thus, for example, irrespective of any pre-colonial religious animosity, Hindu-Muslim *riots* were certainly a colonial creation which helped bolster insecure colonial rule. Likewise, the colonised often saw caste through the blinkers of race, a retrograde association still being non-verbally propagated by the statue of Aryabhata at IUCAA, which has caucasian features emphasized by a muscular physique, and flowing locks (a *janeyu*, but no *shikha*)! Recall further that a legion of racist Western philosophers like Hume, Kant etc., justified racism using a false church history of science to deny the existence of any black achievers, by the trick of appropriating the knowledge of black Egypt through early Greeks, real or concocted, but declared White.[18] I will not go further into these matters, since my objective here is only to point out the importance of dalit scientific figures in understanding caste and refuting stereotypes.

The existence of such dalit scientific figures also refutes the stereotype that education was confined to Brahmins in pre-colonial India. While pre-colonial (non-Vedic) education did *not* exclude dalits, as we first learnt through Dharampal, post-independence mathematics education in India did: it has remained within the tight grip of Brahmins in the TIFR math school, exempt from reservation. After Kosambi (and because of the attempt to oust him) that math school has functioned on marked casteist and regional lines[19] on the one hand, and has acted as a stooge of West, on the other.[20] Not only have they propagated the Western formalist philosophy of mathematics, they have actively suppressed any

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11 C. K. Raju, *Cultural Foundations of Mathematics: the nature of mathematical proof, and the transmission of the calculus from India to Europe in the 16th c.*, Pearson Longman, 2007.

12 लल्ल, शिष्यधीव्रद्धिद chp. 20 मिथ्याज्ञाननिराकरणम्

13 वटेश्वर, गोल, भूगोलः, 5.5.

14 *Cultural Foundations of Mathematics*, cited above, chp. 4, Time, latitude, longitude and the globe.

15 Such as नीलकंठ सोमसूत्वन, author of आर्यभटीयभाष्य

16 e.g. Ghurye, Dumont etc. Thus, Ghurye cites Ambedkar just 4 times. G. S. Ghurye, *Caste and race in India*, Popular Prakashan, Mumbai, 5th edition, 1969.

17 However, the last word on the colonial contribution to caste has certainly not been said by Dirks, Cohn or the subaltern historians all from Princeton.

18 For a detailed discussion of racism and false history, see C. K. Raju, *Euclid and Jesus*, Multiversity, 2012.

19 C. K. Raju, "Kosambi the Mathematician", Special article, *Economic and Political Weekly* **44**(20) May16–22 (2009) 33–45. http://ckraju.net/papers/Kosambi-EPW.pdf.

20 In particular, the TIFR math school, despite being richly state-funded, has not contributed an iota of practical value to the Indian people in over 50 years. All they have to show for themselves are certificates of Western approval.

dissent, and provided no room for Aryabhata's philosophy, going back to the *sulba sutra* (a manual for masons) that *ganita* is primarily concerned with practically useful but approximate calculations.

Contrasting philosophies of ganita and mathematics

This practical philosophy is of central importance to Aryabhata who solved differential equations numerically to calculate his 24 precise sine values.[21] It is also of central importance to contemporary applications, since most engineering applications (including "rocket science") are still done in almost the same way[22] by numerical calculations performed on computers today, using floating point numbers. In contrast, in my experience, people indoctrinated into formal mathematics are rendered unfit to work on those practical applications.

Present-day formalist mathematics conflicts with *ganita* in another fundamental way: Aryabhata advocated empirical proofs[23] ridiculed as "inferior" by church/racist/colonial historians,[24] and prohibited in formal mathematics. Formal mathematics permits only deductive proofs based on axioms. 2+2=4 may *not* be proved by pointing to chairs, or anything we can *see*, but only by reference to Peano's axioms or formal set theory, which few understand.

The belief that deductive proofs are "superior" to fallible empirical proofs, though a key aspect of Western philosophy, is founded only on some silly myths and superstitions[25] such as those about Pythagoras, "Euclid" and their purported pure deductive proofs. To expose that "Euclid" is pure myth, I have offered a prize of Rs 2 lakhs for serious evidence of "Euclid".[26] Further, again contrary to long-standing Western myths, the fact is that the book *Elements* he supposedly authored does NOT have pure deductive proofs of either the "Pythagorean" or any other proposition: the proofs in it are just as empirical as the proofs of the "Pythagorean theorem" found in Indian tradition, only more prolix.[27]

In Indian philosophy only empirical proof (प्रत्यक्ष प्रमाण) was *universally* accepted, by *all* schools of philosophy. Further, the Lokayata accepted *only* empirical proofs; specifically they rejected deductive proof (अनुमान) as inferior. As the Lokayata critique of deductive proofs shows, and as even formal mathematicians today admit, deductively proven theorems are, at best, true *relative* to postulates. Hence, mere deductive proof does NOT lead to valid knowledge (the goal of Indian philosophy) until the postulates are empirically validated, as in science.

As a concrete example (द्रष्टान्त) of the Lokayata critique of deductive proof I point out the historical problems Europeans had for centuries with navigation which was their major scientific challenge from the 15th until the 18th c. During this period, they used "dead reckoning" (involving a version of the "Pythagorean theorem") to determine longitude at sea.[28] This problem of European navigation arose

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21 For details, see *Cultural Foundations of Mathematics*, chp. 3, Infinite series and π.

22 While Aryabhata used linear interpolation or the wrongly-named "Euler's" method to numerically solve differential equations, Brahmagupta used quadratic interpolation, and Aryabhata's Kerala disciples used $11^{th}/12^{th}$ order polynomials to calculate sine values. In contrast, the thermal design of our INSAT 2D satellite used only a $4^{th}/5^{th}$ order Runge-Kutta (Fehlberg) method.

23 For a detailed discussion and concrete examples, see C. K. Raju, "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhâsâ", *Philosophy East and West*, **51**:3 (2001) 325–362.

24 e.g. W. W. Rouse Ball, *A Short Account of the History of Mathematics*, Dover, New York, 1960, pp. 1–2,

25 C. K. Raju, "Ganita vs mathematics: ten myths of formal math and the need to refute them", paper presented at International Conference on Plurality in Math, Kolkata, Dec 2015. http://ckraju.net/blog/?p=111.

26 C. K. Raju, *Euclid and Jesus: how and why the church changed mathematics and Christianity across two religious wars*, Multiversity, Penang, 2012.

27 e.g., *Euclid and Jesus*, cited above. In fact the book is NOT about deductive proof as misinterpreted by the church.

28 For a detailed account of the European problems with latitude, longitude and loxodromes, see *Cultural Foundations of*

just because the "Pythagorean theorem" (even if deductively proved) is *not* valid knowledge for triangles drawn on the curved surface of the earth. That requires empirical inputs. Aryabhata's commentator Bhaskar I had explicitly[29] noted this point a thousand years earlier, as did Aryabhata's critic Brahmagupta[30] who observed that precise knowledge of the radius of the earth was needed to determine longitude correctly. (Europeans were ignorant also of the correct radius of the earth, until late 17th c.; Columbus and Newton had excessively wrong values.[31])

The "Pythagorean theorem": square roots and sine values

In ancient Egypt,[32] Iraq,[33] and India[34] the "Pythagorean theorem" is found as a proposition about the *diagonal* of a *rectangle* in relation to its sides. Attention has been focused on the fact that these formulations predate the mythical Pythagoras. However, all these early formulations, in contrast to a mere deductive proof (not actually found in "Euclid's" *Elements*), also provided a way to *calculate* the diagonal from a knowledge of the sides using *square roots.* The Manava *sulba sutra* explicitly states the "Pythagorean" proposition using square roots.[35] Some formulations also provide a way to calculate the sides from a knowledge of the diagonal (and one angle) using *sine values,* as needed for navigation. (With sine values any triangle—not necessarily right-angled—can be solved; spherical triangles too.)

However, square roots (as in the *sulba sutra,* and *Aryabhatiya*[36]) and sine values (as in *Aryabhatiya*[37]) create a fundamental philosophical problem, for they can almost never be specified *exactly*, as, for example, in the case of $\sqrt{2}$ or sin 1. In this context of square roots and the value of π, though better numerical values are found in Egypt and Iraq, the philosophy seems to have survived only in the *sulba-sutra*-s, which explicitly use the terms inexact (सविशेष)[38] and non-eternal (अनित्य[39]). Likewise, Aryabhata invokes the same philosophy when he uses the term आसन्न[40] (near value) for his precise value of π still used in Europe in the 16th c.

The "Pythagorean theorem" in this form, suited to calculation, was unknown to Europe. Square roots and sine values are absent from European tradition until the 12th c. Toledo translations. This is shown by the very terms in current use, "surd" (for $\sqrt{2}$) and "sine", both of which are foolish translations (from Sanskrit via Arabic to Latin to English). Thus, surd derives from the Latin *surdus*, meaning deaf; bad *karna* in the sense of bad diagonal was mistranslated as "bad ear", since *karna* also means ear in Sanskrit. Likewise, sine derives from the Latin *sinus* (meaning fold) from the Arabic जेब a misreading

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Mathematics, cited above, chp. 5. Navigation: Kamal or Rapalagai.

29 It is in this context that Bhaskar 1, महाभास्करीय, 2.5, attributes this critique (neglect of the sphericity of the earth for solving longitude triangles) to the "disciples of the bhata", (भटस्य शिष्या:). However, those disciples are stated to be experts in *ganita* (गणितविद), and it is Brahmagupta's rule which is being criticised.

30 ब्राह्मस्फुटसिद्धांत 11.15-16. "भूव्यासस्याऽज्ञानाद् व्यर्थं देशान्तरं" ("ignorance of the radius of the earth makes longitude calculations futile").

31 See, *Cultural Foundations of Mathematics*, cited above, chp. 5

32 Berlin Papyrus, problems 1 and 2.

33 Tablet YBC7289.

34 E.g., Baudhayana, 1.12, Apastamba 1.4, Katyayana 2.7.

35 Manava *sulba sutra*, 10.10.

36 गणित, 4.

37 गीतिका, 12, गणित, 12

38 Baudhayana *sulba sutra* 2.12

39 Apastamaba *sulba sutra*, 3.2

40 गणित 10.

of जीबा (written as the consonantal skeleton jb, without nukta-s) from the Sanskrit जीवा, which means half-chord (or अर्ध ज्या as Aryabhata calls it).

This sort of linguistic misunderstanding was accompanied by conceptual misunderstanding, in Europe. That misunderstanding persists in the way sine is wrongly defined in school today, in "trigonometry", as "opposite side upon hypotenuse". Sine, as (half-) chord, relates to a *circle*, a curved line, *not* primarily a triangle made up of straight lines. In particular, the "trigonometric" functions are actually circular functions, and cannot be correctly defined without reference to a circle. But our colonial education system still blindly imitates that European misunderstanding. Indeed, the value of π is not meaningful in a triangle; it involves the length of a curved lines which can be measured with a flexible *sulba* or string, as also used in Egypt, but not with the geometry box used today to teach geometry. The length of a curved line is not conceptually defined until the calculus. Hence, our school students today remain as conceptually confused about it as Descartes was about ratios of curved and straight lines.[41]

How should we teach mathematics today

A fundamental question that arises then is this: *how* should we teach mathematics today, especially to dalits, engineers, and scientists, economists, social scientists, and all those engaged in practical[42] professions? Should we teach mathematics as concerning inexact calculations of practical value, as it arose in the non-West, or as a matter of formal theorem-proving of unclear practical (or theoretical) value, but which purports to be eternal and exact truth, as claimed in the West?

As the references to such unverifiable notions of "eternal" and "exact truth" suggest, some religious belief lurks in the background. Indeed, mathematics is explicitly related to soul-arousal by Plato, and *hence* to eternal truth. Through the "Neoplatonic" tradition this carried over into the *falsafa* and *aql-i-kalam* in Islam. During the Crusades, in its greed to grab Muslim wealth, the church abruptly changed the entire Christian belief system and switched to the Christian theology of reason. This was adapted from the Islamic *aql-i-kalam*, since the switch was done with a view to persuade Muslims to convert.

Since the Bible says nothing about reason, the church invented a false history to appropriate reason and project it as a Christian inheritance. This was done by a simple trick: late Arabic and Byzantine Greek texts were anachronistically attributed to *early* (pre-Christian) Greeks, declared "friends" of Christians since Eusebius. (These early Greek authors were depicted as White and "West", by racist and colonial historians, respectively.) Thus, an early Greek called "Euclid" was concocted to appropriate reason (from "Neopolatonists" and Islam). To suit the new church theology of reason, the book *Elements* was "reinterpreted" as solely related to persuasive proofs based on reason. This enabled the church to use that reinterpreted book to teach reasoning to its priests. The church not only propagated myths such as "Euclid", it also glorified pure deductive proofs. The beliefs of formal mathematics today are closely related to the dogmas and practices of the Crusading church.[43] Reason was declared to be "universal" by Western philosophers, and it was argued that even god was bound by reason.

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41 For a detailed account of string geometry and Descartes' confusion, see C. K. Raju, "Towards equity in mathematics education. 2: The Indian rope trick", *Bharatiya Samajik Chintan* **7** (4) (2009) 265–269. http://ckraju.net/papers/MathEducation2RopeTrick.pdf.

42 Dalits are clubbed with engineers and scientists because if there is anything like "dalit philosophy" it must be a practical philosophy, since the overwhelming majority of dalits are engaged in practical professions, like scientists and engineers.

43 Some people argue that Russell, one of the founders of formal mathematics, was an atheist, hence could not have furthered church dogmas and myths. This argument is specious and based on the foolish presumption that the church is concerned with god, whereas it has actually been concerned with political power ever since it married the state in the 4th c. Russell served church interests by gullibly furthering the myth of Euclid. Likewise, even more determined opponents of the church, such as Newton (a fanatic Christian) and Nietzsche, fell victims to church propaganda: for example, Newton spoke of "laws" of nature, a dogma of Aquinas.

The problem with continuing the church practice of using mathematics to teach reason is this: reason is NOT universal. Reason is based on logic, but on the Indian experience of long debates between Naiyayikas and Buddhists, logic is *not* culturally universal, the Buddhist logic of *catuskoti,* differs from the 2-valued logic used for deductive proofs.[44] Which logic should one use in formal math? Changing the logic used in mathematics would change the theorems of mathematics. That is, mathematical theorems are actually relative truths *relative to both postulates and logic*. Innumerable combinations of the two are possible, so theorem proving is an idle pastime irrelevant to valid knowledge! However, teaching a particular logic (or method of reasoning) as universal teaches a cultural bias. This is an anti-Buddhist bias, and to that extent anti-dalit.

Since even some seemingly intelligent Western philosophers seem not to have understood this point about logic, in two decades, I clarify that my objection to 2-valued logic is only an objection to deciding the logic underlying mathematics on *a priori* grounds or on biased cultural grounds: I have absolutely no objection to 2-valued logic taught as a *practical* matter. But doing so would entail *empirical proof* for the use of two-valued logic. That commonsense act admits that deductive proof is useless without the help of empirical proof, an admission that instantly punctures the centuries-old church/racist/colonial boast of "superior" deductive proofs. For if the choice of logic itself requires empirical proof, and it cannot be justified on some god-given intuition, or Kantian *a priori* (or theological dogma that a particular logic binds god), then deductive proofs cannot be "stronger" or less fallible than empirical proofs, as Western philosophy has wrongly maintained.[45]

Further, choosing logic on empirical grounds is non-trivial since the nature of logic depends upon the physical nature of time,[46] which is a subtle matter. Time is simplistically visualised as a featureless (straight!) line in Western tradition, but that is unlikely to be the case in fact. Time may have a microphysical structure as in my structured-time interpretation of quantum mechanics.[47] In that case, the Buddhist logic of *catuskoti* may be understood as a quasi truth-functional logic,[48] or a temporal logic corresponding to that structure of time. Such a logic is of importance also at the macrophysical level. It is even of technological value for the semantics of computer programs for existing parallel computers[49] (such as those based on I5, I7 etc.) , and the future technology of quantum computing. (I do not go into the question of how a structured time naturally arises in physics, and of its connection to the Buddhist notion of पतिच्च समुप्पाद, on the one hand, and to the notion of शील, on the other, which is explained in my other writings and incorporated in the ethics of the harmony principle.[50])

A little known but great advantage of deductive proofs, from the church viewpoint, is this: unlike empirical proofs, absolutely any nonsensical proposition, anything at all (such as "there exists a rabbit with two horns"), can be proved deductively as a mathematical theorem by starting from equivalent assumptions. (That was how Russell and Hilbert eventually gave a deductive proof of the "Pythagorean theorem" by the trick of starting from an equivalent assumption—the original side angle side theorem —which they assumed as a postulate.) This trick of being able to provide a "rigorous" proof of

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44 See article on "Logic" for the Springer *Encyclopedia of Non-Western Science, Technology and Medicine*, http://ckraju.net/papers/Nonwestern-logic.pdf. For a more detailed exposition, see C. K. Raju, *The Eleven Pictures of Time*, Sage, 2003.

45 See, e.g., C. K. Raju, "The religious roots of mathematics", *Theory, Culture & Society* **23**(1–2) Jan-March 2006, Spl. Issue ed. Mike Featherstone, Couze Venn, Ryan Bishop, and John Phillips, pp. 95–97. http://ckraju.net/papers/Religious-roots-of-math-TCS.pdf.

46 C. K. Raju, *Time: Towards a Consistent Theory*, Kluwer Academic, Dordrecht, 1994.

47 *Time: Towards a Consistent Theory,* cited above

48 *The Eleven Pictures of Time*, cited above.

49 *Time: Towards a Consistent Theory*, cited above.

50 C. K. Raju, "The harmony principle", in *Philosophy East and West*, **63** (4) 2013, pp. 586-604. http://www.ckraju.net/papers/Harmony-principle-pew.pdf.

absolutely anything was obviously of great value to the church in spreading all sorts of superstitions among the ignorant (especially the educated colonised elite who never even wonder why their education kept them ignorant of mathematics).

Finally, it is noticeable that of Ambedkar's 22 vows the first six and No. 8 (or about 32%) are directed against superstitions. This *explicit* opposition to superstition is the characteristic feature which distinguishes Ambedkar's Navyana from classical Buddhism where reliance on the two principles of evidence, प्रत्यक्ष and अनुमान, implicitly excludes all superstition. However, those superstitions are restricted to Hindu superstitions. That list of vows should be expanded to include the explicit rejection also superstitions which the church brought in along with colonial education. The most dangerous of those superstitions are the dogmatic or superstitions beliefs which have crept into mathematics and science, for example the belief that deductive proof (using 2-valued logic) is "superior" to empirical proof.

Due to all these reasons, it is clear how to answer the question about what mathematics we ought to teach today. We should teach *ganita* for the benefit of those like dalits, engineers, scientists, economists etc. engaged in practical professions, who learn mathematics for its practical applications. We should *not* teach formalist/church/Western mathematics. We should certainly not teach it as a compulsory subject in school, to save children from indoctrination into church superstitions.

Actually implementing that program in India would require the overthrow of the colonial education system, the Brahmins controlling the TIFR math school and the Westerners who control *them* behind the scenes. This can be achieved by forcing them to debate their wretched philosophical beliefs publicly. While there is nothing Vedic in "Vedic mathematics", there is church dogma in formal mathematics. It remains to be seen how difficult it is for our "experts" to explain publicly why a certain church dogma is taught in schools and universities alongside some practical *ganita* (almost all of Indian origin).

As a matter of fact, actual pedagogical experiments have been successfully performed on teaching calculus as it originated with Aryabhata. These experiments have been performed with 8 groups in 5 universities in 3 countries and the results reported in scholarly forums.[51] This way of teaching math makes it so easy that calculus can be taught in 5 days, even to social scientists, as was done in Ambedkar University Delhi,[52] and to those who did not study math beyond the $8^{th}/10^{th}$ standard, as was done in Sarnath. The test of learning was that students should be able to do problems drawn at random from the typical calculus text of some 1350 pages (in two-columns and small type) or from a related question bank.[53]

Zeroism

Teaching mathematics in this new way requires a whole philosophy of mathematics which cannot be based just on a few words in ancient texts, such as सविशेष, अनित्य, and आसन्न. Accordingly, I have embedded that mathematical practice in the Buddhist philosophy of शून्यवाद. Since the texts of Nagarjuna are hard to read, and interpretations differ widely, and since my concern is limited to the

51 "Teaching mathematics with a different philosophy. Part 1: Formal mathematics as biased metaphysics." *Science and Culture* **77** (7-8) (2011) pp. 274–279. http://www.scienceandculture-isna.org/July-aug-2011/03%20C%20K%20Raju.pdf, arxiv:1312.2099. "Teaching mathematics with a different philosophy. Part 2: Calculus without limits", *Science and Culture* **77** (7-8) (2011) pp. 280–85. http://www.scienceandculture-isna.org/July-aug-2011/04%20C%20K%20Raju2.pdf. arxiv:1312.2100.

52 "Calculus for social scientists", http://ckraju.net/blog/?p=83.

53 "Calculus without limits" article for Second People's Congress on Education, 2009, (to appear) in Proc. http://ckraju.net/papers/calculus-without-limits-paper-2pce.pdf

practical consequences of that शून्यवाद philosophy, and not with any authoritative exegesis, I have renamed it zeroism.[54] Particularly, there is a common misconception that शून्यवाद denotes "emptiness". However, I use zeroism to denote *inexactitude* or the zeroing of small differences.

This is a realistic philosophy, more sophisticated than Lokayata. It accepts inference, as in science, but nevertheless does not accept pure metaphysics, devoid of any empirical base, as valid knowledge. Thus, zeroism rejects the idea of $\sqrt{2}$ as a unique metaphysical "real" number, for that number can *never* be specified in practice. All we can do is to write down $\sqrt{2}=1.4142135...$ in so many different ways, accurate to a certain large number of decimal places. On the contrary, as a realistic philosophy, zeroism is anti-idealist, and holds that it is the idealised notion of a unique number $\sqrt{2}$, like the idealised notion of a geometric point, which is an error, the only virtue of which may be simplification.

The discarding or zeroing of small differences irrelevant to the context is an essential aspect of *all* abstract representations, on the understanding of पतिच्च समुप्पाद (co-origination conditioned by the past) as implying continuous creation (wrongly called "Buddhist doctrine of flux"). Consequently, even a "single person", corresponds, in actual fact, to a whole procession of individuals, one for each instant, who differ from each other in "inessential" ways. That is, in everyday life we constantly deal with inexactitude by discarding inessential differences. This discarding of inessential differences in ostensive empirical referrants is true for all abstractions. Most people understand 2+2=4 in natural language by pointing to pairs of objects which are never identical. Most people definitely do *not* understand 2+2=4 as a theorem deduced from Peano's axioms!

A clear advantage of teaching practical math with zeroism, in the way of Aryabhata and the *sulba sutra*, is this: students can solve harder problems. It is necessary to solve those harder mathematical problems even to teach elementary science properly in school. For example, the first serious science experiment in school concerns the simple pendulum, which most students and teachers confound with simple harmonic motion.[55] The actual time period of the simple pendulum varies with amplitude as can be easily *observed*, but *calculating* it requires knowledge of elliptic integrals not taught in typical calculus courses. Going by simplified theory instead of actual observation is to negate the basic principle of science that theory must correspond to observations. (Teachers, incidentally, trust the wrong formula, stated in Western books, not the observation! The new method also negates the blind trust in Western authority taught by formal mathematics.)

Advantages of teaching calculus the Aryabhata way

The new way of teaching mathematics enables students to do those harder problems involving non-elementary elliptic integrals. This was demonstrated, for example, in my elder son's school project on how the time period of a simple pendulum varies with amplitude.[56] (Google: "how does the time period of a simple pendulum vary with amplitude".) Likewise, my younger son demonstrated in 9th std, the solution of missiles with air resistance which is needed to explain why a cricket ball can be thrown further than a tennis ball. He also demonstrated and later published the solution of the brachistochrone with resistance.[57] Thus, teaching math in this practical way not only makes math easy, it *enhances* the

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54 "Zeroism" Article for Springer *Encyclopedia of Non-Western Science, Technology, and Medicine*, http://ckraju.net/papers/zeroism-springer-f.pdf.

55 C. K. Raju, "Time: what is it that it can be measured", *Science & Education*, **15**(6) (2006) pp. 537–551. Draft available from http://ckraju.net/papers/ckr_pendu_1_paper.pdf.

56 Suvrat Raju, "Pendulum project", http://ckraju.net/11picsoftime/pendulum.pdf

57 Archishman Raju, "A simple way to solve the brachistochrone problem with resistance" *Physics Education* (India) **28**(3) July-Sep 2012, http://www.physedu.in//uploads/publication/3/65/Archishman_Brachistochrone13July.pdf.

abilities of the students to do harder problems and thus relate theory to actual experience. Thus, it also enhances the teaching of science (and social science).

As for bigger real life practical problems in science and engineering, my survey of problems of national importance for C-DAC showed that most of them involve numerical computation. As already pointed out, such computations are still done in the way of Aryabhata, and effectively involve zeroism. These involve doing calculus inexactly using not formal real numbers but floating point numbers, and small numbers must be discarded. So, these practical applications are certainly unaffected by the change in the teaching of mathematics.

Nevertheless, students of my new decolonised course on the history and philosophy of science,[58] as also some colleagues in the decolonisation movement, have this doubt. "It works", they say of formal math. Others ask: granting that practical applications to engineering and most scientific problems are unaffected, what happens to advanced scientific applications? Don't those need formal mathematics? (Let me help their question along. Formal mathematics such as Hilbert space, is used in axiomatic quantum mechanics. Or operator-valued tempered distributions are used in axiomatic quantum field theory.) Now the question itself arises because colonial education ensures that the vast majority are ignorant of mathematics.[59] Being ignorant, they favour a ritualistic approach: if it works don't change any aspect of it for that might make it fail. Now a ritualistic approach, even with regard to science, is no less a superstition.

My simple answer to "it works" is that "it works better" if we change math along the suggested lines. "It works better" is true even for advanced scientific applications. However, it is hard to explain this to those who do not understand even university calculus or its deficiencies which necessitated alternatives such the Schwartz theory of distributions.[60] One can understand why "it works better" with a historical approach which separates what works from what does not. What "works" is calculus as a technique of practical numerical computation, as it developed in India starting from Aryabhata. While Europeans imported calculus for its practical value, they did not *understand* it when it first arrived, just as in the case of square roots and sine values. (Unlike square roots and sine values the calculus is not properly understood even today!) This lack of understanding led to the imposition of a huge layer of metaphysics (related to infinity and eternity[61]) which is irrelevant to the practical applications of calculus. Removing that redundant layer of metaphysics is what makes calculus easier and better.

The simplest concrete example of how "it works better" and leads to better contemporary science is the following. Newtonian physics failed just because because of the difficulty with time;[62] Newton made time metaphysical.[63] He did that just because he misunderstood the calculus and tried to make it "perfect" through his confused theory of fluxions,[64] Correcting Newtonian physics requires us to correct also the theory of gravitation, as in my retarded gravitation theory.[65] That works better than

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58 See blogs on the decolonised course: http://ckraju.net/blog/?p=89, and http://ckraju.net/blog/?p=73. Also, short video http://youtu.be/ozQRBNk2alg.

59 This ignorance is by design of colonial education which was church education. See, e.g. "Education and Church: Decolonising the hard sciences", *Frontier Weekly* **46** (7) 25-31 Aug 2013. http://ckraju.net/papers/education-and-counter-revolution.pdf.

60 *Cultural Foundations of Mathematics*, cited above, Appendix on "Renormalization and shocks"

61 C. K. Raju, "Eternity and Infinity: the Western misunderstanding of Indian mathematics and its consequences for science today." *American Philosophical Association Newsletter on Asian and Asian American Philosophers and Philosophies* **14**(2) (2015) pp. 27-33. Draft at http://ckraju.net/papers/Eternity-and-infinity.pdf.

62 *Time: Towards a Consistent Theory*, cited above.

63 "Time: what is it that it can be measured" cited above.

64 For a detailed account of that confusion about Newton's fluxions as it emerged in the debate with Berkeley, see *Cultural Foundations of Mathematics,* cited above, chp. 8 Numbers in calculus, algorismus, and computers.

65 For an expository account of my retarded gravitation theory, see C. K. Raju, "Functional Differential Equations. 4: Retarded gravitation", *Physics Education* (India) **31**(2) April-June, 2015,

Newtonian gravitation in two ways. Though the differences for the solar system are tiny, the new theory explains the observed tiny flyby anomaly of NASA satellites as an effect due to the rotation of the earth. This is inexplicable on Newtonian gravitation, where gravitational force depends only on distance and never on velocity.

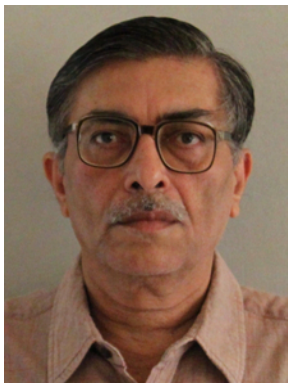
Further, Newton's "universal law of gravitation", back-calculated from observations of planetary motion, fails if we go beyond planetary motion to the galaxy. The theory is commonly "saved" by piling on additional hypotheses such as dark matter and its peculiar distribution like a halo. (Newtonian physics is what is applied to the galaxy since it is impractical to use general relativity in this context of the billion body problem.) For the galaxy, my retarded gravitation theory works distinctly better for it needs no additional hypotheses to explain why the rotational velocities of stars in spiral galaxies *increas*e with distance from the galactic centre, and then become constant.

A more general way in which the Indian calculus, as done by the Aryabhata school, "works better" is through the use of non-Archimedean arithmetic, but its relation to formal math is too technical to be discussed here.[66]

http://www.physedu.in/uploads/publication/19/309/1-Functional-differential-equations-4-Retarded-gravitation-(2).pdf.

66 See, for example, my talk at MIT, "Calculus: the real story". Abstract (http://ckraju.net/papers/Calculus-story-abstract.html), blog (http://ckraju.net/blog/?p=106), and video (https://youtube.com/IaodCGDjqzs). Or see the Appendix to *Cultural Foundations of Mathematics*.

About the author



C. K. Raju holds an MSc in math from Mumbai, and a PhD from the Indian Statistical Institute, Kolkata. He initially taught at Pune University, and researched in formal math (functional analysis), for several years. He was responsible for porting applications on India's first supercomputer Param, and has long been a Professor of mathematics and computer science, in various universities, in India and abroad. He has developed and maintains software for educational and industrial use. In 2010 he received the TGA gold medal in Hungary.

He has authored several critically acclaimed books, putting forward novel theses. In *Time: Towards a Consistent Theory* (Kluwer, 1994) he argued for a fundamentally new physics using functional differential equations, overlooked by Einstein. In *Cultural Foundations of Mathematics* (Pearson Longman, 2007) he proposed a new philosophy of math, called zeroism, and related it to the development of calculus in India while compiling evidence for its transmission to Europe in the 16th c. He later showed how correcting Newton's mistake about calculus also leads to a better theory of gravitation, using functional differential equations. In the *Eleven Pictures of Time* (Sage, 2003) he related science and various religions through the shared interface of time. His shorter books include *Is Science Western in Origin?* and *Ending Academic Imperialism*. In *Euclid and Jesus* he explained for the layperson how religious beliefs enter into Western mathematics.

He has demonstrated through teaching experiments in 5 universities in 3 countries that teaching calculus with zeroism, as it developed in India, makes it easy enough to be taught in five days, and enables student to do harder problems not covered in usual calculus courses. He has also developed and taught other decolonised courses in math, physics, statistics, computers, and the history and philosophy of science.

In his research he has challenged a number of leading Western icons, including Isaac Newton, Albert Einstein, Richard Feynman, Stephen Hawking, Bertrand Russell, Karl Popper, Thomas Kuhn. He has also debunked many Western myths including those of Euclid, Archimedes, Aristotle, Claudius Ptolemy, Copernicus, Kepler, etc. His research has been coveted and repeatedly plagiarised by Westerners, including by a former President of the Royal Society.

As regards social sciences and humanities, he has been on the editorial board of *Journal of Indian Council of Philosophical Research*, etc., an Editorial Fellow of the Project of History of Indian Science, Philosophy and Culture, a Fellow of the Indian Institute of Advanced Study, and Associate Fellow of Nehru Memorial Museum and Library, etc.