Presented at the 2nd People's Education Congress, Homi Bhabha Centre for Science Education, Mumbai, 5-9 Oct 2009, in both panels on (a) Mathematics Education and (b) History and Philosophy of Science in Science Education.

Calculus Without Limits: Report of an Experiment

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Aim

- To report on a new system of teaching mathematics, and some recently conducted experiments on it.

Outline of the new system

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- and new philosophy of math ("Eliminate theology from math").

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- which required a table of secants (for Mercator chart) and a precise calendar (for latitude determination; hence calendar reform of 1582).

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- Accepted by Newton and Leibniz whose work was torn to bits by Berkeley.

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- Which used Cantor's set theory, formalised in the 1930's.
- Leads to present-day teaching of calculus with limits as in current school texts.

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- The calculus developed in one cultural context and was absorbed in another.
- Calculus arose as *gaṇita* and was sought to be absorbed within the religious tradition of mathematics (as science of the soul)

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  - Empirical methods must be admitted in proof since they are MORE certain than metaphysics (deduction).

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  - Racist history of science ("Euclid" etc) key part of this agenda of domination through soft power. (Children should grow up in awe of the West.)

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- Zeroism. This also involved a new philosophy of zeroism, similar to Nagarjuna's *śūnyavāda*.
- For more details on zeroism see my paper `http://ckraju.net/papers/Zeroism-and-calculus-without-limits.pdf`, 4th Nalanda conference, on my website.

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- Background: Students admitted after 8th std. Some have monastic education.
- Very poor performance in pre-test even on elementary arithmetic.

Pre-test
Calculus without Limits

Notes:

1. This is NOT a competitition. The aim of this test is only to provide *feedback* regarding your current knowledge of mathematics.
2. Some questions may be beyond your current knowledge. Please don't be anxious about it. It is expected that you do *not* know the answers to all questions, and those questions are there only to establish the limits of your knowledge.

I : Arithmetic

1. Find $124 + 568$.
2. Find $532 - 319$.
3. Calculate 3542×213.
4. If 2184 is divided by 17 what is the quotient and what is the remainder?
5. Which is the greatest among the following four numbers: $\frac{5}{8}, \frac{2}{3}, \frac{3}{7}, \frac{4}{10}$?
6. Write $\frac{3}{4}$ as a decimal.
7. Write 0.4352 as a proper fraction.
8. What is the square of 23?
9. A trader bought an item for Rs 26 and sold it for Rs 38. What percentage profit did he make?
10. The Rajdhani express travels from Delhi to Mumbai in 18 hours and 30 minutes with stops of 10 minutes each at Kota, Ratlam and Baroda. If its average speed is 81 km, what is the distance from Delhi to Mumbai?
11. If 3 kg of flour sells for Rs 32 how much does 5 kg of flour sell for?

II : Alegbera

12. If $x = 5$ what is the value of x^2?
13. If $2x + 3 = 10$ what is the value of x?
14. If $2x + 3y = 40$ and $x = 7$ what is the value of y ?
15. If $x^2 - x - 6 = 0$ what are the possible values of x?

III : Geometry

16. If one angle of a right-angled triangle is 30° write the other two angles in degrees.
17. A rectangle has length 1 and width 2. What is the length of it diagonal?
18. Give an approximate figure for the circumference of a circle whose radius is 1.
19. Plot a straight line through the points $(2,\ 3)$ and $(2,\ -3)$.

IV : Elementary Calculus

20. What is $\frac{d}{dx}\sin(x)$?
21. What is $\int\ x^2 dx$?

V: Calculus questions from question bank

22. Differentiate $\sqrt{\frac{\sec x - 1}{\sec x + 1}}$ with respect to x.
23. Differentiate $\log \frac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}+x}$ with respect to x.
24. Evaluate the integral $\int \frac{x^2+1}{x^4+1}\ dx$.
25. Evaluate the integral $\int x^2 \tan^{-1} x\ dx$.

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- And also solve ordinary differential equations.

Post-test
Calculus without Limits

I : Elementary computations

1. Convert 30 deg to radians.
2. Convert 2 radians into degrees.

II : Elementary Calculus

3. What is $\frac{d}{dx}\sec(x)$?
4. Evaluate $\int \cos(3x+1)dx$
5. Find the second derivative of $x\sin x$.
6. Find
$$\int_0^1 xe^x dx$$
7. Numerically integrate
$$\int_0^{0.5} \frac{1}{\sqrt{1-x^2}\sqrt{1-x}}\, dx$$
.

III: Questions from question bank (differentiation)

Differentiate the following functions with respect to x.

8. $\sqrt{1-x^2}$.
9. $x^2 e^{\sqrt{x}}$.
10. $x^2 \sin^3 x\ \cos^4 x$

(continued from page 1: differentiate the following with respect to x)

11. $$\log \sqrt{\frac{1+x\cos x}{1-x\cos x}}$$

12. $$\tan^{-}1\left(\frac{e^{2x}+1}{e^{2x}-1}\right)$$

IV: Questions from question bank (integration)

Evaluate the following integrals.

13. $$\int \frac{1}{1-x^2}\,dx$$

14. $$\int \frac{1}{x^3+x^2+x+1}\,dx$$

15. $$\int \frac{\sqrt{2}-\sqrt{x}}{1-\sqrt{2x}}\,dx$$

16. $$\int \sec^{-1}\sqrt{x}\,dx$$

17. $$\int \cot^5 x\,dx$$

V : Ordinary differential equations

18. Solve the differential equation $y' = 2y$, with $y(0) = 1$ and hence find $y(4)$.

19. Solve the differential equation $y' = x\sin(x)$ with $y(0) = 1$ and find the value of $y(10)$.

20. Solve the differential equation $y'' = -3y$ with $y(0) = 1$ and $y'(0) = 0$, and find the value of $y(20)$.

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- Below 35% —-15

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- In the post-test this student got nearly 100%. About half the class managed to clear the test.
- The bottom half of the class performed poorly.
- As clear from the pre-test some of the students (and faculty) did not fulfil the starting criterion of knowing school math at 8th std. level. They are being given remedial coaching in school math.


Central University of Tibetan Studies
Sarnath, Varanasi
Workshop on "Calculus without limits"
22nd - 26th September, 2009