Decolonized calculus

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Abstract

Calculus is difficult hence California recently canceled calculus. Resolving those difficulties involves rejecting a false history and a bad philosophy of mathematics, both long used to claim "superior" colonial knowledge. 16th c. Cochin-based Jesuits stole calculus, for the European navigation problem which needed accurate trigonometric values, available in Indian texts to 9 decimal place precision. On my epistemic test, knowledge thieves fail to *fully* understand what they steal: neither Newton nor Leibniz knew how to sum infinite series. 19th c. real numbers, and 20th c. axiomatic set theory only partly solved the problem. Calculus originated in India with the 5th c. Aryabhata as a numerical method ("Euler's" method) to solve differential equations. Combined with the Indian method of summing infinite series using non-Archimedean arithmetic and the philosophy of zeroism makes calculus without limits very easy. It enables students to solve harder problems not covered in usual calculus courses. This has been demonstrated in teaching experiments in 3 countries.

Keywords: Calculus without limits, non-Archimedean arithmetic, zeroism, Indian origin of calculus, decolonization of math

California recently "canceled" calculus teaching in schools (Blume, 2021; Evers, 2021; Evers & Wurman, 2018; Replace the Proposed New California Math Curriculum Framework, n.d.; Wood, 2021). This is a symptom of a deeper malaise. Obviously, students in school and university find calculus difficult. But the solution to those difficulties involves standing up to a false history(Raju, 2021c) and a bad philosophy(Raju, 2021d) of mathematics, long used to promote the false claim of "superior" colonial knowledge. Thus, there is an old controversy about who invented calculus: Leibniz or Newton? This is astonishing because, for some two centuries, the West has known that the infinite series claimed by Leibniz, like the sine series claimed by Newton, were known from centuries before them in India.(Whish, 1835)

More recently, the issue of the Indian origin of calculus, and its transmission to Europe by Cochin-based Jesuits in the 16th c., has been exhaustively documented (Raju, 2001, 2007, 2015b, 2016a, 2016b) and further new facts have emerged. Calculus (and calendrical knowledge) was stolen from India to solve the European navigational problem, which required precise trigonometric values, the most precise of which (precise to the third sexagesimal minute) were then available in Indian calculus-related mathematics and astronomy texts. These precise values were obtained starting with Aryabhata (5th c.) and his numerical method of solving differential equations ("Euler" method) later extended to 11th and 12th order polynomials, and infinite series by the 14th c. Madhava.

The Jesuit general Christoph Clavius (head of the Gregorian calendar reform committee) first published an interpolated version of these precise Indian trigonometric values in his own name.(Bambergensis, 1607) (A carbon copy of the Indian astronomical model by Nilakanth is known as "Tychonic" after Tycho Brahe, the Astronomer Royal to the Holy Roman Empire). In

short, there is ample circumstantial evidence that Indian calculus texts, stolen by Jesuits, were circulating in Europe, beginning with the top Roman Catholic functionaries.

Kepler, Galileo (and his student Cavalieri), Fermat and Pascal showed insufficient interest, and, these Indian infinite series diffused to Protestants. Eventually, Newton and Leibniz claimed credit for "discovery" of calculus, the way Columbus claimed to have discovered America. These claims of "discovery" involved an extremely Christian chauvinistic and genocidal church dogma: "the doctrine of Christian discovery", according to which any piece of land or knowledge belongs to the first Christian to sight it.(Raju, 2015a, 2020b)

Granting the Indian origin of calculus, what difference does that make to calculus teaching today? This involves my epistemic test:(Raju, 2020c) those who steal knowledge. do not fully understand it, like students who cheat in a test. (Therefore, lack of understanding is proof of copying, and this applies especially under suspicious circumstances such as claims of late "independent rediscovery".) Thus, the fact is that neither Newton nor Leibniz *understood* how to sum the infinite series such as the "Leibniz series". Because credit was falsely appropriated, **a poor understanding of calculus** spread throughout Europe, and was eventually globalised by colonial education.

That there was persistent lack of understanding of the calculus in Europe, long after its purported "discovery" by Newton and/or Leibniz, is indisputable: many European thinkers (including Karl Marx (Raju, 2020a)) recognized this lack of understanding, and eventually real numbers were invented by Dedekind in the late 19th century,(Raju, 2020a) to try and correct this lack of understanding of calculus.

However, despite real numbers, this Western lack of understanding of calculus continues to cause difficulties in applications today (e.g., discontinuous functions cannot be differentiated, and the Schwartz theory of distributions is linear, so it cannot readily be used to make sense of discontinuous solutions of non-linear differential equations, which naturally arise, such as shock waves). These difficulties of calculus were earlier partly resolved using non-standard analysis,(Raju, 1989) applied to the Schwartz theory of distributions.

However, the only essential feature of non-standard analysis used in the above solution was "non-Archimedean" arithmetic, already present in the Indian way of summing infinite series. The infinite geometric series was first summed by Nilakanth in the 15th-16th c.(Raju, 2016a) and that involved "non-Archimedean" (polynomial) arithmetic of the 7^{th} c. Brahmagupta (Raju, 2018a), who referred to polynomials as "unexpressed numbers". With non-Archimedean arithmetic, due to the presence of infinities and infinitesimals, limits are not possible. (Real numbers have the "Archimedean" property.)

Therefore, also, because of the presence of infinitesimals, the doctrine of the exactitude of mathematics has to be dropped and replaced by the alternative philosophy of zeroism.(Raju, 2016c) This causes not the least difference in *applications* of mathematics, especially those involving computers: for example, all calculations on computers are done using floating point numbers, not real numbers, which *cannot* be represented on a real computer (with finite memory, not a Turing machine). Floats do not obey even the associative "law" for addition.(Raju, 2001)

On the principle that "phylogeny is ontogeny", that history repeats in fast forward mode in the classroom: the persistent European historical difficulties with calculus are reflected in the present-day difficulties of calculus teaching. It is in acknowledgment of these difficulties, that the California state board recently cancelled the calculus. But that is a hack solution, because technology development needs proper understanding, and teaching statistics without calculus results in an

improper understanding also of statistics. (E.g., in common texts, probability theory begins with a standard Borel probability space, so the Lebesgue integral and measure is assumed as basic to probability, though this obviously does not apply to quantum probabilities,(Raju, 1994) we omit this technical topic.) Bad understanding of statistics may result in dangerously wrong machine-learning programs tomorrow.

The solution is to abandon the false history and bad philosophy of calculus. **The use of non-Archimedean arithmetic, as in teaching decolonized "calculus without limits", makes calculus extremely easy, as has been demonstrated by teaching experiments in 3 countries. (Raju, 2011a, 2011b. 2018a)**

Because this is contrary to centuries of false colonial pride, the West is unable to reconcile itself to this loss of face arising from this devastating non-Western critique of its systematically false history and related bad philosophy of mathematics,(Raju, 2017b, 2017a, 2018b) except by using its authority to censor it, and vilify it. However, the issue really is about the extreme Christian chauvinistic history and philosophy(Raju, 2018a, 2022) which developed in the West under centuries of church hegemony. The West (especially the American Mathematical Society(Raju, 2019a, 2019b, 2019c)) continues to ignore this non-Western critique, by vilifying it, and striking the same old racist pose of "superiority", despite its persistent inability to engage with the evidence or arguments in this decolonial critique of mathematics.

Finally, as the California attempt to cancel the calculus shows, while church superstitions such as those about racism (*'Euclid' Must Fall*, 2021; Raju, 2021a, 2021b) may have benefited the West in the past, and helped it to acquire wealth (e.g., by justifying slavery), the persistence of church superstitions in mathematics will inevitably result in a decline in Western technological dominance, and possible sudden collapse like that of the British empire, unless it quickly corrects its struthious attitudes.

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References

Bambergensis, C. C. [Christophori C.] (1607). *Tabulae Sinuum, Tangentium et Secantium ad partes radij 10,000,000...* Ioannis Albini.

Blume, H. (2021, May 20). *California's controversial math overhaul focuses on equity*. Los Angeles Times. https://www.latimes.com/california/story/2021-05-20/california-controversial-math-overhaul-focuses-on-equity

'Euclid' must fall. (2021, May 13). https://www.youtube.com/watch?v=kAP1BcK8mLE

Evers, W. M. (2021, May 18). Opinion | California Leftists Try to Cancel Math Class. *Wall Street Journal*. https://www.wsj.com/articles/california-leftists-try-to-cancel-math-class-11621355858

Evers, W. M., & Wurman, Z. (2018, May 9). *California's Common Core Mistake* [Text]. Hoover Institution. https://www.hoover.org/research/californias-common-core-mistake

Raju, C. K. (1989). Distributional matter tensors in relativity. In D. Blair & M. J. Buckingham (Eds.), *Proceedings of the 5th Marcel Grossman meeting* (pp. 421–423). World Scientific. arXiv:0804.1998

Raju, C. K. (1994). Quantum-mechanical time. In *Time: Towards a Consistent Theory*. Kluwer Academic.

Raju, C. K. (2001). Computers, mathematics education, and the alternative epistemology of the calculus in the Yuktibhāṣā. *Philosophy East and West*, *51*(3), 325–362. http://ckraju.net/papers/Hawaii.pdf

Raju, C. K. (2007). *Cultural Foundations of Mathematics: The Nature of Mathematical Proof and the Transmission of Calculus from India to Europe in the 16th c, CE*. Pearson Longman.

Raju, C. K. (2011a). Teaching mathematics with a different philosophy. 1: Formal mathematics as biased metaphysics. *Science and Culture*, *77*(7–8), 274–279. arXiv:1312,2099

Raju, C. K. (2011b). Teaching mathematics with a different philosophy. 2: Calculus without limits. *Science and Culture*, *7*(7–8), 280–285. arXiv:1312,2100

Raju, C. K. (2015a). The meaning of Christian 'discovery'. *Frontier Weekly*, *47*(29), 25–31. http://www.frontierweekly.com/archive/vol-number/vol/vol-47-2014-15/47-29/47-29-The%20Meaning%20of%20Christian%20Discovery.html

Raju, C. K. (2015b). *Calculus: The real story", talk at MIT*. http://ckraju.net/papers/Calculus-story-abstract.html. Video: https://youtu.be/IaodCGDjqzs.

Raju, C. K. (2016a). Calculus. In *Encyclopedia of Non-Western Science, Technology and Medicine* (pp. 1010–1015). Springer. http://ckraju.net/papers/Springer/ckr-Springer-encyclopedia-calculus-1-final.pdf

Raju, C. K. (2016b). Calculus transmission. In H. Selin (Ed.), *Encyclopedia of Non-Western Science, Technology and Medicine* (pp. 1016–1022). Springer. http://ckraju.net/papers/Springer/ckr-Springer-encyclopedia-calculus-2-final.pdf

Raju, C. K. (2016c). Zeroism. In *Encyclopedia of Non-Western Science, Technology and Medicine* (pp. 4604–4610). Springer. http://ckraju.net/papers/Springer/zeroism-springer-f.pdf

Raju, C. K. (2017a). Black Thoughts Matter: Decolonized Math, Academic Censorship, and the "Pythagorean" Proposition. *Journal of Black Studies*, *48*(3), 256–278. https://doi.org/10.1177%2F0021934716688311

Raju, C. K. (2017b). *Mathematics, decolonisation and censorship*. https://kafila.online/2017/06/25/mathematics-and-censorship-c-k-raju/

Raju, C. K. (2018a). Decolonising mathematics. *AlterNation*, *25*(2), 12–43b. https://doi.org/10.29086/2519-5476/2018/v25n2a2

Raju, C. K. (2018b). To decolonise math stand up to its false history and bad philosophy. In *Rhodes Must Fall: The struggle to decolonise the racist heart of empire* (pp. 265–270). Zed Books.

Raju, C. K. (2019a, November). *Plagiarism by ex-President of the Royal Society. 1: The facts.* http://ckraju.net/blog/?p=183

Raju, C. K. (2019b, November). *Plagiarism by ex-President of the Royal Society. 2: The cover-up by the American Mathematical Society*. http://ckraju.net/blog/?p=184
Raju, C. K. (2019c, November). *Plagiarism by ex-President of the Royal Society. 3: Lessons for decolonisation*. http://ckraju.net/blog/?p=185
Raju, C. K. (2020a, August 28). Marx and mathematics-1 Marx and the calculus. *Frontier Weekly*. https://www.frontierweekly.com/views/aug-20/28-8-20-Marx%20and%20mathematics-1.html
Raju, C. K. (2020b, August 31). Marx and mathematics. 2: 'Discovery' of calculus. *Frontier Weekly*. https://www.frontierweekly.com/views/aug-20/31-8-20-Marx%20and%20mathematics-2.html
Raju, C. K. (2020c, September 8). Marx and mathematics. 4: The epistemic test. *Frontier Weekly*. https://www.frontierweekly.com/views/sep-20/8-9-20-Marx%20and%20mathematics-4.html
Raju, C. K. (2021a). "Euclid" Must Fall: The "Pythagorean" "Theorem" and The Rant Of Racist and Civilizational Superiority—Part 1. *Aṛumaṛuka: Journal of Conversational Thinking*, *1*(1), 127–155. https://doi.org/10.4314/ajct.v1i1.6
Raju, C. K. (2021b). "Euclid" must fall: The "Pythagorean" "theorem" and the rant of racist and civilizational superiority—Part 2. *Arumaruka: Journal of Conversational Thinking*, *1*(2), 57–105. https://doi.org/10.4314/ajct.v1i2.5
Raju, C. K. (2021c, December 11). California, Indian Calculus and the Technology Race. 1: The Indian origin of calculus and its transmission to Europe. *Boloji.Com*. https://www.boloji.com/articles/52924/california-indian-calculus
Raju, C. K. (2021d, December 24). California, Indian calculus and the technology race. 2: Don't cancel the calculus, make it easy! *Boloji.Com*. https://www.boloji.com/articles/52950/california-indian-calculus-and
Raju, C. K. (2022, June 21). *The church origins of (axiomatic) math*. Medium. https://medium.com/@c_k_raju/the-church-origins-of-axiomatic-math-e08036dbe29d
Replace the Proposed New California Math Curriculum Framework: Open Letter to Governor Gavin Newsom, State Superintendent Tony Thurmond, the State Board of Education, and the Instructional Quality Commission. (n.d.). The Independent Institute. Retrieved 6 November 2021, from https://www.independent.org/news/article.asp?id=13658
Whish, Charles M. (1835). On the Hindu quadrature of the circle and the infinite series of the proportion of the circumference to the diameter exhibited in the four Shastras, the Tantrasamgraham, Yukti-Bhasa, Carana Padhati and Sadratnamala". *Trans. R. Asiatic Soc. Gr. Britain and Ireland*, *3*, 509–523.
Wood, R. (2021, June 21). *Calculus Is Canceled: A Legacy Betrayed?* California Globe. https://californiaglobe.com/section-2/calculus-is-canceled-a-legacy-betrayed/