19. Park, *Africa, Asia, and the History of Philosophy*, 94.
20. Robert Bernasconi, "Philosophy's Paradoxical Parochialism," 218ff.; Park, *Africa, Asia, and the History of Philosophy*, esp. 11–17 and 76–95.
21. It is perhaps worth noting that Martin Bernal identifies Meiners as a crucial player in this debate as well; see *Black Athena: The Afroasiatic Roots of Classical Civilization, Vol. I: The Fabrication of Ancient Greece, 1785–1985* (New Brunswick, NJ: Rutgers University Press, 1987), 217-18; and Mikkelsen includes one of Meiners's essays ("Of the Varieties and Deviate Forms of Negroes" [1790]) in *Kant and the Concept of Race*, 198–207.
22. Park, *Africa, Asia, and the History of Philosophy*, 76-77.
23. Ibid., 95.
24. Ibid., 151.
25. Robert Bernasconi, "With What Must the History of Philosophy Begin? Hegel's Role in the Debate on the Place of India within the History of Philosophy," *Hegel's History of Philosophy: New Interpretations*, ed. David A. Duquette (Albany: SUNY Press, 2003), 35–49, at 36.
26. Bernasconi, "With What Must the History of Philosophy Begin?" 37. (See also Hegel, *Lectures on the History of Philosophy 1825-6*, Vol. I, 46, 49, and elsewhere.)
27. For details, see Bernasconi, "With What Must the History of Philosophy Begin?" 38–43.
28. Cited in ibid., 43.
29. Ibid., 45-46.
30. See, for example, Copleston, *A History of Philosophy*, vol. I, 10-11, 15-16. The quoted phrase about Copleston being splendid for facts comes from David Hamlyn, *A History of Western Philosophy* (London: Penguin, 1987), 334.
31. Walter Burkert, *Babylon, Memphis, Persepolis: Eastern Contexts of Greek Culture* (Cambridge, MA: Harvard University Press, 2004), 50-51.
32. Edward Zeller, *Outlines of the History of Greek Philosophy* [1883], trans. Sarah Frances Alleyne and Evelyn Abbott (New York: Henry Holt and Co., 1890), 6, 5.
33. W. K. C. Guthrie, *A History of Greek Philosophy*, Vols. I-VI (Cambridge: Cambridge University Press, 1962–1981). (All the quotes in this paragraph are from Volume I: The Earlier Presocratics and the Pythagoreans.)
34. For the work of Burkert, West, Kingsley, and Bernal, see (among others) Walter Burkert, *Babylon, Memphis, Persepolis*, esp. 49–70; M. L. West, *Early Greek Philosophy and the Orient* (Oxford: Clarendon Press, 1971); Peter Kingsley, "Meetings with Magi: Iranian Themes among the Greeks, from Xanthus of Lydia to Plato's Academy," *Journal of the Royal Asiatic Society* III:5 (1995): 173–209; and Bernal, *Black Athena*, esp. vol. I. In addition, for an assessment of Bernal's *claims* through Volume II (and separate from an evaluation of his *evidence*, which is set aside in favor of considering whether his assertions might be true), see Dan Flory, "Racism, *Black Athena*, and the Historiography of Ancient Philosophy," *The Philosophical Forum* 28 (1997): 183–208.
35. See, for example, Burkert, *Babylon, Memphis, Persepolis*, 113; and Kingsley, "Meetings with Magi."

Eternity and Infinity: The Western Misunderstanding of Indian Mathematics, and Its Consequences for Science Today

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PRACTICAL INDIAN MATH

Most students today study mathematics for its practical applications. And it was for its practical applications that *ganita* developed in India: arithmetic and algebra for commerce, permutations and combinations for the theory of metre, probability theory for the game of dice,[1] "trigonometry" and calculus, or rather the study of the circle and the sphere, for astronomy and navigation. Navigation was important for overseas trade, which stretches back 5,000 years in India, and was an important source of wealth. Astronomy was needed for the calendar and to determine the seasons, since the rainy season is critical to Indian agriculture, the other key source of wealth in India.[2]

Since *ganita* was done for its practical applications, Indian texts from the ancient *sulba sutra*-s, through the fifth century Aryabhatiya to the sixteenth century Yuktidipika, all admit empirical proofs in *ganita*.[3] An empirical proof is one that involves anything we can perceive with our senses. For example, Aryabhata states that a plumb line is the test of verticality. Secondly, all practical applications invariably involve a tolerance level, or an "error margin." Thus, all the above three texts give the ratio of the circumference of a circle to its diameter, or the number today designated by π, as 3.1415.... The sulba sutra-s declare its value to be non-eternal (*anitya*)[4] and imperfect (*savisesa*),[5] with something left out). Aryabhata, who numerically solves a differential equation, to derive his sine values precise to the first sexagesimal minute (about five decimal places), declares his value of B to be *asanna* (near value).[6] That is, Indians accepted both (1) empirical proofs, and (2) imperfections as part of *ganita* or mathematics.

Discarding insignificant quantities naturally extends to the discarding infinitesimals. This latter enters essentially in the way Indian texts treat infinity and the sum of infinite series. By the fourteenth century, Aryabhata's method was extended in India to an infinite "Taylor" series for the sine, cosine, and arctangent functions, to derive their values accurate to the third sexagesimal minute (about ten decimal places), and Nilakantha in his commentary also explains why the near value of B is given and not the real value (*vasatavim sankhya*).[7] The fifteenth century, Nilakantha is also the first source for the formula for the sum of an infinite geometric series.[8] Discarding infinitesimals involves a rigorous way to sum infinite series, a way that was not understood until recently in the West.

RELIGIOUS WESTERN MATH

In the West mathematics was long valued for its religious connections. The very word mathematics derives from mathesis, which means learning.[9] In Plato's *Meno*, Socrates explains that learning is achieved by arousing the soul, for "all learning is recollection" of the eternal ideas in the soul. Having demonstrated a slave boy's innate knowledge of mathematics, he claims he has proved the existence of the soul and its past lives: for, he argues, if the slave boy did not learn mathematics in this life, he must have learned it in a previous life.[10] The Greeks had imported Egyptian mystery geometry, which had the spiritual aim of arousing the soul by turning the mind inward.[11] The belief was that math contains eternal truths and hence arouses the eternal soul by sympathetic magic. This notion of soul became unacceptable to the post-Nicene church, which cursed the belief in past lives,[12] and banned mathematics, in the sixth century. However, those "Neoplatonic" beliefs survived in

Islam as part of what Muslim scholars called "the theology of Aristotle," and were influential in the *aql-i-kalam* or Islamic theology of reason. From there it came to Europe as part of Christian theology.

When the wealthy Khilafat of Cordoba splintered and became weak, in the eleventh century, the church saw an opportunity, and launched the Crusades with a view to conquer and convert Muslims by force, the way Europe was earlier Christianised by force. However, the Crusades failed militarily (beyond Spain and after the first Crusade). Nevertheless, Muslim wealth was so tempting that the church changed its entire theology to the Christian theology of reason promoted by Aquinas and his schoolmen. This was a modified form of the Islamic theology of reason. Reason was declared universal, since Muslims too accepted it so it helped to persuade them. However, not wishing to acknowledge that this major change in theological beliefs arose from an adaptation of Islamic beliefs, and not finding any sources in the Bible to support rational theology, the church claimed ownership of reason by attributing its origin to an early Greek called Euclid. Alongside it reinterpreted the *Elements* and its geometry, supposedly authored by Euclid, as concerned not with the soul, but solely with metaphysical (deductive) proofs or methods of persuasion, to align it with the post-Crusade theology of reason.

There is no evidence for "Euclid." While my book *Euclid and Jesus* goes into all the details of this spurious myth, to avoid having to do so repeatedly, I instituted the "Euclid" prize of USD 3300 for serious evidence about "Euclid." Needless to say, the challenge has not been met. If the book was written by someone else in another era it might admit a totally different interpretation. Accordingly, one needs to go by the book *Elements* itself, and not by the story told about the book. The two are quite different, as clarified below.

FROM CONCOCTED EUCLID TO FORMALISM

Post-Crusade, the belief in the eternal truths of mathematics persisted for new theological reasons. Western theologians who always understood how God worked said that logic bound God who could not create an illogical world, but was free to create the facts of his choice.[13] Hence it came to be believed in the West that mathematics, as truth which binds God, or eternal or necessary truth, must be "perfect" and cannot neglect even the tiniest errors (which are bound to surface some time during eternity!) It was further believed that this "perfection" could be achieved only through metaphysics: a "perfect" mathematical point is never a real dot on a piece of paper, howsoever much one may sharpen the pencil.

Carried away by the story that this metaphysical understanding of "real" math originated with "Euclid" and his "irrefragable" proofs, European scholars did not notice the fact that the very first proposition of the *Elements* uses an empirical proof! The proof involves the intersection of two arcs: we *see* the arcs intersecting, so it is an empirical proof. But there is no axiom from which this intersection can be deduced, so there is no axiomatic proof. This error in the supposedly infallible proofs in the *Elements* went unnoticed for some 700 years. This error was finally admitted in the nineteenth century, and it was further admitted that other empirical proofs (such as the proof of Proposition 4 or the side-angle-side theorem) are essential to the proof of the "Pythagorean" theorem in the *Elements*. So, not even the *Elements* ever had a non-empirical proof of the Pythagorean theorem, which was, of course, empirically known long before "Pythagoras" or Pythagoreans.

But what happened subsequently was even more amusing. For metaphysicians, the story naturally proved to be stronger than the facts! Instead of accepting that the whole story was false, a new story was added. Those empirical proofs were attributed to an error by the supposed "Euclid" in executing his purported intentions. Bertrand Russell and David Hilbert then rewrote the *Elements* to correct "Euclid" and make his book 100 percent metaphysical! That rewriting does not fit[14] the *Elements*, but it led to the present-day formal mathematics[15] of Russell and Hilbert, which makes all mathematics 100 percent metaphysics.

TRANSMISSION OF INDIAN MATH AND ITS EUROPEAN MISUNDERSTANDING

The two streams of mathematics, religious and practical, collided when the West started importing Indian mathematics for its practical applications from the tenth century.[16] Earlier, on the "Neoplatonic" belief that knowledge is virtue, the Baghdad House of Wisdom had imported numerous texts from all over the world, especially India, in the ninth century. Muslims frankly acknowledged those imports as in al Khwarizmi's book titled *Hisab al Hind*. When the techniques in this book traveled to Europe, they were called algorismus or algorithms (after al Khwarizmi's Latinized name) Again, the algebra from Brahmagupta[17] came to be known as algebra after al Khwarizmi's *Al jabr waa'l Muqabala*. These arithmetic and algebraic techniques were adopted by Florentine merchants because of their immense practical advantage for commerce.

Transmission of knowledge often results in misunderstanding, and the hilarious story of the persistent European misunderstanding of imported Indian math is told by the very words like "zero," "surd," "sine," "trigonometry," etc., in common use today. Zero (from cipher, meaning mysterious code) created conceptual difficulties for Europeans for centuries, since it involved the sophisticated place value system, different from the primitive Greek and Roman numerals which were additive and adapted to the abacus. Thus, in 976, Gerbert, who later became the infallible pope Sylvester, had a special abacus constructed for "Arabic numerals," which he imported from Cordoba, for he thought the abacus was the only way to do arithmetic![18] Due to these conceptual difficulties among Europeans, elementary arithmetic algorithms (for addition, subtraction, multiplication, division, etc.) entered the Jesuit syllabus as "practical mathematics" only as late as 1572.

Similar amusing European confusion underlies the term "surd" from the Latin *surdus* meaning deaf, applied today to the square root of two. That was calculated since the *sulba sutra*-s as the diagonal (*karna*) of the unit square, and the term *surdus* is a mistranslation of bad *karna*, meaning bad diagonal but misunderstood as bad ear, for the word *karna* also means ear.

Similarly, the term sine is a translation error from Toledo. It arose from the Arabic *jaib*, meaning pocket, as a misreading of *jiba* from the vernacular *jiva*, from the Sanskrit *jya* meaning chord. Since the chord relates to the circle, not the triangle, the word "trigonometry" (or measurement of a triangle) indicates a European conceptual misunderstanding for what should properly be called circlemetry (or measurement of the circle), and was studied in Indian texts in chapters on the circle.

THE PROBLEM OF INFINITE SERIES

While early imports of Indian mathematics in Europe came indirectly via Arabs from Baghdad, Cordoba, and Toledo, calculus and probability went directly to Europe through Jesuits based in Cochin in the sixteenth century.[19] The maximum confusion and misunderstanding attended the transmission of the infinite series of the Indian calculus to Europe. As already indicated, the most elementary circlemetric ratio, the ratio of the circumference of a circle to its diameter, necessarily involves an infinite series, as in B = 3.1415...., which decimal representation is an infinite sum 3 + 1/10 + 4/100 + 1/1000 + 5/10000 +.... These infinite series were used in India to derive sine, cosine, and arctangent values accurate to the third sexagesimal minute (about ten decimal places).[20] These values ("tables of secants") were of great practical importance to the navigational problem of determining latitude and longitude at sea. They were also critical for the (specifically) European navigational problem of determining loxodromes: Europeans navigated with charts, and since the surface of the earth is curved, setting a straight course by the compass did not result in a straight line course on the chart. Recall that navigation was, for centuries, the principal scientific challenge facing Europeans, who dreamed of wealth through overseas trade. The Royal Society, and the French Royal Academy were set up around this problem. The overwhelming practical value of precise trigonometric values from India meant that the related infinite series could not simply be abandoned.

Now for practical purposes, related to navigation and astronomy, a precision of, say, eight decimal places was ample. But the infinite series presented a conceptual difficulty on the European faith in mathematics as "perfect." Thus, the infinite series of the imported Indian calculus could not be "perfectly" summed. Practically speaking, even today, one typically states the number π only to a few decimal places as B = 3.14. But this means that there is some error: about 0.0015. One can make the error much smaller by proceeding to 100 or 1,000 places after the decimal point, with the understanding that one can go on further if one really needs to do so. The resulting tiny error is of no practical consequence. While this process is adequate for *all* practical purposes, as an infinite sum it is nevertheless not "perfect," since some tiny error would still remain neglected. On the other hand, it is evidently impossible to sum the series "perfectly" by adding all terms, physically, for that would take an eternity of time, no matter how fast one does the addition.

Hence, on the deep-seated Western faith in mathematics as perfect, Descartes[21] declared that the ratio of curved and straight lines was beyond the human mind. "[T]he ratios between straight and curved lines are not known, and I believe cannot be discovered by human minds, and therefore no conclusion based upon such ratios can be accepted as rigorous and exact." Coming from a leading Western mind, this was curious, since, from the days of the *sulba sutra*-s, Indian children were taught to measure curved lines using a string, and to compare them with straight lines just by straightening the string. This was not Descartes' individual problem. Galileo in his letters to Cavalieri[22] concurred with Descartes, and Newton's posthumous opponent Berkeley[23] thought that this was good reason to reject the calculus. He asserted, "It is said, that the minutest Errors are not to be neglected in Mathematics." Thus, the argument from Descartes to Berkeley was that summing infinite series involved either an eternity of time or minute errors; that was imperfect, hence not mathematics, which, they took for granted, ought to be perfect. Why ought mathematics to be perfect? Why not an "imperfect" mathematics good enough for all practical applications? This issue seems never to have been debated in the West since it related to the hegemonistic religious faith.

Indeed, though calculus began as circlemetry, this Cartesian difficulty with curved lines is still part of Western mathematical education today. The geometry box which every child carries to school has nothing with which to measure curved lines, although an angle is better defined as a circular arc rather than somehing (what thing?) between two straight lines.[24] These objections regarding the purported "imperfections" of the calculus created a problem for Newton, whose physics could not do without calculus. Newton thought that Descartes' objection could be met, and (d/dt) could be "rigorously" or "perfectly" defined by making time "flow" metaphysically.[25] That idea of time itself flowing is a statement explicitly recognized as meaningless by Indians at least since Sriharsa.[26] Newtonian physics failed just because of this conceptual error about the nature of time.[27]

INFINITY AND ETERNITY

Though Newton's fluxions were eventually abandoned, the West still maintained that metaphysical "real" numbers are the solution to the specifically European problem of "perfectly" summing an infinite series. At least this is the solution that is taught in schools and universities today: that real numbers are essential to calculus. (Hence, the formulation of physics using calculus forces physical time to be represented by the real line.) In the nineteenth century, the Western solution to the problem of infinite sums moved towards metaphysical "real numbers," or the continuum, an uncountable infinity of numbers constructed using Cantorian set theory and its transfinite cardinals.[28] In schools and universities today, calculus is taught by appealing to the continuum and the metaphysical "limits" that make it possible to "perfectly" sum infinite series. Actually, all that metaphysics is too difficult to teach in high school and even most undergraduate courses for non-mathematics majors, so students are only told about it, not actually taught.

That is, not only was calculus wrongly attributed to Newton and Leibniz, it is today taught in universities and schools by falsely claiming that its infinite series can be summed "rigorously" only by using a particular metaphysics of infinity.

There are two issues here. First, it should be clearly noted that there is nothing unique or "universal" about metaphysical notions such as infinity and eternity. The notion of *atman* in the Upanishads, so fundamental to Hinduism, is embedded in an underlying *physical* belief[29] in quasi-cyclic time: eternity is not "linear" like the real line. The same is true of the notion of soul according to Egyptians, Socrates, or early Christians. Indeed, the primary conflict in Christian theology was over the nature of eternity, whether it is quasi-cyclic as Origen thought, or whether it is metaphysical and apocalyptic as believed in post-Nicene theology.[30] It was this fundamental religious conflict over the nature of eternity which culminated in the church's ban on mathematics (for "pagans" like Hypatia and Proclus still understood mathematics as concerning the soul).

This conflict over the nature of eternity was also the basis of the subsequent curse on "cyclic" time called the anathemas against pre-existence. It is also reflected in the first creationist controversy, which concerned the nature of eternity, not evolution. Thus, Proclus stated, in *his* book, also called *Elements*, that eternity turns back on itself, as in the uroburos, or a snake eating it own tail. This was the ancient Egyptian symbol for quasi-cyclic time and is still the modern symbol for infinity, ∞. In contrast, John Philoponus[31] maintained that would make one time creation, as in the Bible, impossible, and also make apocalypse impossible, depriving the church of a valuable weapon of terror ("doomsday is round the corner").[32]

Formally speaking, infinite sums have no intrinsic meaning "out there," in some Platonic sense, and can be defined in surprising ways. For example, the Ramanujan sum of $1 + 2 + 3 + 4 + \ldots = -1/12$.

THE METAPHYSICAL CONTINUUM NOT ESSENTIAL FOR CALCULUS

Further, contrary to what is taught in schools and universities, calculus and the summation of infinite series can be done using number systems both smaller and larger than the continuum. Formally speaking, the continuum or the field of real numbers, R, is the largest "Archimedean" ordered field. Therefore, any ordered field, F, larger than R must be non-Archimedean. The failure of the Archimedean property in F means that F must have an element x such that $x > n$ for all natural numbers n. (Any ordered field must contain a copy of the natural numbers, and also fractions or "rational" numbers.) Such an x may be called an infinite number. Since F is a field, the positive element x must be invertible, and the inverse too must be positive, so we must have $0 < (1/x) < (1/n)$ for all natural numbers n. Such a number $(1/x)$ may be called an infinitesimal. Note that, unlike non-standard analysis, where such infinities and infinitesimals appear only at an intermediate stage, the infinities and infinitesimals in a non-Archimedean field are "permanent."

On the other side, of a number system smaller than R, a computer can only work with a finite number system. A computer cannot handle infinity or the continuum and uses instead floating point numbers. These numbers do not even obey the associative "law," and hence do not constitute a field.[33]

Both approaches (with a larger or smaller number system) fit into the *sunyavada* philosophy, which I call zeroism, which tells us that in representing an entity (any real entity, not merely a "real" number or integer) we are compelled to discard or "zero" some small aspect as "non-representable" on the grounds that "we don't care." This happens because any real entity constantly changes, though we usually neglect those changes as too tiny to care about. Likewise, when we speak of "two dogs" we do not thereby imply that the two dogs are identical but only that we don't care to describe the differences. The difference is that, on zeroism, it is not the representation that is erroneous, but the idealistic belief in "perfection" that is erroneous. This point of view is not found in Western philosophical thought about mathematics. The representation (of, say, B) can always be improved, but achieving "perfection" is impossible. This makes no difference to any practical applications: computer arithmetic is good enough for most practical applications of mathematics, and even calculations done by hand involving say, B, can only use a finite number of digits to represent B. Even theoretically zeroism has a clear advantage in the case of probability,[34] for probability cannot be recovered as the conventional limit of relative frequency.

Historically speaking, Indians used both rounding and discarding of infinitesimals, which are similar but not identical processes. The formula for an infinite geometric series was first developed using exactly such non-Archimedean arithmetic. From the time of the sixth century, Brahmagupta, Indians used polynomials, which they called unexpressed numbers. This naturally led to "unexpressed fractions" or ratios of polynomials, corresponding to what are today called "rational functions." These are an example of non-Archimedean arithmetic.[35] What are today called "limits" were determined in that non-Archimedean arithmetic using order counting or discarding infinitesimals very similar to discarding small numbers.[36] (Formally speaking, limits in a non-Archimedean field are not unique, as in R, but involve discarding infinitesimals. Thus, the best one can say is that for any infinite n, the inverse, $(1/n)$, is infinitesimal, not zero.) This was too sophisticated for Western mathematicians of the seventeenth century to understand: who lacked even a precise idea of infinitesimal and naively thought of it as a very tiny quantity.

It is well known that constructing the continuum required Cantor's set theory, which was full of holes exposed by paradoxes such as Russell's paradox. While the axiomatization of set theory resolved Russell's paradox, other paradoxes like the Banach-Tarski paradox still persist, though they are not so well known. According to this paradox, using set theory, one ball of gold can be cut into a finite number of pieces that can be reassembled into two balls of gold of the exact same size! Western mathematicians believe that to be a form of truth higher than empirical truth, hence one on which they base present-day math! More fundamentally, the consistency of set theory is maintained by using double standards typical of theology: adopting separate standards of proof for metamathematics and mathematics. If transfinite induction were permitted in metamathematics, as it is in mathematics, that would make set theory decidable, hence inconsistent. If transfinite induction is not solid enough for metamathematics, why should it be acceptable in mathematics? Thus, it is only an agreement between Western scholars, an agreement which is sustained by a system of "authorized knowledge."

To reiterate, eliminating the Western metaphysics of infinity in present-day mathematics does not affect any practical applications of mathematics to science and engineering, which must all be done in the old way. For example, as already noted, all practical applications of the calculus to physics, such as sending a rocket to Mars, still involve Aryabhata's method of numerical solution of differential equations, or its variants.[37] This numerical solution is typically obtained today by using a computer that cannot handle the continuum.

SPREADING RELIGIOUS BIASES THROUGH MATH

Ironically, however, this cocktail of practical Indian mathematics and Western metaphysics was declared "superior" to the original, and returned to India, and globalized through colonial education. The claim of "superiority" is a fake one: one could, with stronger reason, maintain that empirical proofs are more reliable than the metaphysical claims of Western theologians about infinity, and reject formalism. This colonial story of "superiority" is central to Christian triumphalist history from Orosius to Toynbee, which predates also the racist claim of superiority put forward by Kant, for example.

Indeed, along with the practical value of mathematics, children today learn at an early age that empirical proofs are inferior. Now, all systems of Indian philosophy accept the *pratyaksa*, or empirically manifest, as the first means of *pramana*. This applies also to Indian *ganita*, which accepts empirical proofs. This means that along with mathematics, children today are implicitly taught in school that all systems of Indian philosophy are "inferior" compared to "superior" church metaphysics. Since Islamic philosophy too accepts *tajurba* as a means of proof, this bias is against all non-Christian beliefs.[38] Note, once again, that science too prefers empirical proofs to metaphysics, so accepting empirical proofs does not damage any practical applications of mathematics to science. However, Western metaphysical mathematics just gives our children that foolish sense of "superiority" and teaches them that they must reject all Indian traditions as "inferior."

Indian philosophers too have swallowed the story that mathematics involves some superior kind of knowledge ("binding on God," true in all possible worlds which God could create, true in all possible Wittgensteinian worlds on possible-world semantics). "As certain as 2+2=4," as even the late Daya Krishna once said to me. But why exactly is deduction a "superior" form of proof? Why should non-Christians accept that belief in superiority? When Western theologians claimed that logical proofs are "superior" (since logic binds God), they neglected to ask "which logic"? Logic is not universal. The Buddhist logic of *catuskoti* or the Jain logic of *syadavada* are not 2-valued, and not even truth-functional. Therefore, the theorems of mathematics are at best cultural truths relative to a culturally biased axiom set[39] and a culturally biased logic: in other words, mathematical theorems may be Christian truths, but they are far from being universal truths. The quickest way to show how a different logic would lead to a different mathematics is to see that proofs by contradiction would fail with a quasi truth-functional logic, though the consequences of changing logic obviously extend far beyond intuitionism.

CRITICALLY RE-EXAMINING WESTERN MATH

So, it is important to carefully examine the "superior" way of doing 2+2=4 as metaphysics, the links of this metaphysics to church theology, and whether that really is a superior way of doing mathematics or just an inferior misunderstanding that should be abandoned. The "superior" way to do 2+2=4 is to prove it as a theorem starting from Peano's axioms. However, Peano's axioms bring in infinity by the backdoor. The quickest way to see this to note that a computer can never do Peano arithmetic,[40] since that involves a notion of infinity. Once again, a computer can do all integer arithmetic needed for practical purposes; what it cannot do is handle the entire infinity of natural numbers, for any integer arithmetic on a computer will fail beyond some large number.

A critical examination of the Western philosophy of mathematics from a non-Western perspective was not even attempted for the almost two centuries since colonialism globalized Western education. That system of education, originally designed for missionaries, makes it almost impossible for anyone to carry out such a critical examination. The ordinary way of doing 2+2=4 is to point to two pairs of apples to make four apples. Most people think this is the only way. They are unaware that this empirical way is regarded as erroneous on the "perfect" or "superior" Western way of deducing 2+2=4 as a consequence of Peano's axioms or set theory. Most people never learn this "perfect" and "superior" way, perhaps because it is too complicated to teach axiomatic set theory at the high-school level.

Thus, when it comes to mathematics, for even a simple thing like 2+2=4, the Western-educated have no option but to confess their ignorance and rely on authority which is located in the West. Consequently, they accept Western mathematics as a package deal, and it does not strike them that it is possible to separate the original practical value of mathematics from its add-on, metaphysics. More people need to be informed about this cocktail of practical value and religious belief, which indoctrinates millions of children into religious biases, though they come to school to learn only the practical applications of mathematics.

Recently, a serious challenge to the Western philosophy of mathematics as metaphysics did come up, through my philosophy of zeroism, Western mathematicians, their followers, and Western philosophers of mathematics are the ones now reduced to silence, for there is no answer to these potent objections.

WHAT IS THEORETICALLY NEEDED TO APPLY CALCULUS TO SCIENCE?

So far as practical applications of mathematics to science are concerned, we have seen that the continuum is a redundant piece of metaphysics. However, the theoretical defects in the Western misunderstanding of the Indian calculus, even from within formalism, were exposed long ago. On university-text calculus, a differentiable function must be continuous, so a discontinuous function cannot be differentiated. However, long before the axiomatization of set theory, which supposedly made calculus "rigorous" by giving an acceptable basis to the continuum, Heaviside was merrily differentiating discontinuous functions in his operational calculus, for the need to do so arises in science and engineering. The formalized version of Heaviside's theory is known as the Schwartz theory of distributions. This permits a discontinuous function to be infinitely differentiated.

So what exactly is the definition of the derivative one must use in physics? The one on which a discontinuous function is not differentiable, or the one on which it is infinitely differentiable? "Choose what you like" is the typical response of a formal mathematician. This may sit well with the belief that mathematics is metaphysics, but the slightest reflection shows that, since mathematics is an integral part of physics, this "choose what you like" response makes the resulting physics irrefutable, hence unscientific in a Popperian sense.

Worse, we *cannot* choose what we like, since *both* definitions are inadequate. The inadequacy of the Schwartz theory was established even before its birth, for products of distributions arise in the S-matrix expansion in quantum field theory, and such products are not defined on the Schwartz theory.[41] Many equations of physics, such as the equations of fluid dynamics, or of general relativity, are non-linear partial differential equations. Shocks arise naturally, and represent a (hyper)surface of discontinuity. If we use university-text calculus, based on the continuum, then the derivative of a discontinuous function is not defined, so the "laws of physics" break down, as, for example, in Stephen Hawking's creationist claim that a singularity represents the moment of Christian creation when the "laws of physics" break down.[42] If we use the Schwartz theory, then derivatives are defined, but not products, so there is again a problem. (A similar problem arises in quantum field theory, and is known as the renormalization problem.)

To be sure, there are umpteen definitions of the product of Schwartz distributions, including one that I proposed long ago, using non-standard analysis.[43] The problem is which one to choose. There are two ways of deciding: 1) consult an authoritative Western mathematician, and 2) choose the definition which best fits the widest spectrum of practical applications, where the "best fit" is to be decided by empirical proof or an empirical test of the consequences. Most mathematicians will prefer the first method, for formal mathematics, like theology, is all about authority. But this method does not suit science, so I prefer the second one of relying on practical applications. That makes mathematics just an adjunct physical theory. This also selects out my definition, which is the only one which works for both classical physics[44] and quantum field theory.[45]

The interesting thing is this. While my definitions initially used non-standard analysis, it can all be done just as easily using a non-Archimedean ordered field.[46] That brings us back full circle to the original Indian understanding of the calculus as best suited even to present-day science. So, we should discard formalist mathematics as merely a biased metaphysics of infinity, based on Western notions of eternity, and a Western misunderstanding of Indian calculus, which does not properly fit either the calculus or its applications to current science.

NOTES

1. C. K. Raju, "Probability in Ancient India", ch. 37 in *Handbook of the Philosophy of Science, vol 7. Philosophy of Statistics*, ed. Prasanta S. Bandyopadhyay and Malcolm R. Forster, General ed. Dov M. Gabbay, Paul Thagard and John Woods (Elsevier, 2011), 1175–96, http://www.ckraju.net/papers/Probability-in-Ancient-India.pdf.
2. For a short account, see "Cultural Foundations of Mathematics," Ghadar Jari Hai, 2, no. 1 (2007): 26–29, http://ckraju.net/papers/GJH-book-review.pdf.
3. For a detailed discussion of this issue of empirical vs deductive proofs, see C. K. Raju, "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhâsâ," *Philosophy East and West* 51, no. 3 (2001): 325–62, http://ckraju.net/papers/Hawaii.pdf.
4. Apastamba sulba sutra 3.2.
5. Baudhayana sulba sutra 2.12.
6. *Aryabhatiya*, Ganita 10.
7. Nilakantha, *Aryabhatiyabhasya*, commentary on Ganita 10, Trivandrum Sanskrit Series, 101, reprint 1977, p. 56, and its translation in C. K Raju, *Cultural Foundations of Mathematics* (Pearson Longman, 2007), 125-26.
8. Nılakantha, *Aryabhatıyabhasya*, cited earlier, commentary on Ganita 17, p. 142.
9. Contrary to the wrong derivation of mathematics from "mathema," given currency by the Wikipedia, Proclus clearly derives mathematics from "mathesis." "This, then, is what learning (μάθεσιή) [mathesis] is, recollection of the eternal ideas in the soul; and this is why the study that especially brings us the recollection of these ideas is called the science concerned with learning (μαθέματικζ) [mathematike]). Its name thus makes clear what sort of function this science performs. It arouses our innate knowledge . . . takes away the forgetfulness and ignorance [of our former existence] that we have from birth, . . . fills everything with divine reason, moves our souls towards Nous, . . . and through the discovery of pure Nous leads us to the blessed life." Proclus, *Commentary on the Elements* [Corrected title], trans. Glenn R. Morrow (Princeton University Press, Princeton, 1992), 47, p. 38.
10. Plato, Meno, In *The Dialogues of Plato*, trans. B. Jowett, Encyclopedia Britannica (Chicago, 1994), 179–80.
11. C. K. Raju, *Euclid and Jesus: How and Why the Church Changed Mathematics and Christianity across Two Religious Wars* (Multiversity, 2012).
12. C. K. Raju, "The Curse on 'Cyclic' Time," in *The Eleven Pictures of Time* (Sage, 2003), ch. 2.
13. C. K. Raju, "The Religious Roots of Mathematics," *Theory, Culture & Society* 23, no.1–2 (Jan–March 2006): 95–97, http://ckraju.net/papers/religious-roots-of-math-tcs.pdf.
14. C. K. Raju, "Euclid and Hilbert," in *Cultural Foundations of Mathematics* (Pearson Longman, 2007), ch. 1.
15. For the purposes of this article, the fine distinction between logicism and formalism is irrelevant, and distracting, for my key concern is with the issue of empirical proofs and "perfection"

of mathematics. Both Russell and Hilbert followed a similar historical trajectory in first analyzing the *Elements*, and writing books, both titled *Foundations of Geometry*, and then proposing to get rid of empirical proofs not only in the *Elements* but in all mathematics. I will refer to that sort of mathematics, divorced from the empirical, which was globalized by colonialism, and is taught today in schools and universities as "formal mathematics."

16. C. K Raju, "Math Wars and the Epistemic Divide in Mathematics," ch. 8 in *Cultural Foundations of Mathematics*, cited above.

17. See, e.g., *Algebra....from the Sanscrit of Brahmegupta and Bhascara*, trans. H. T. Colebrooke, John Murray (London, 1817).

18. For a picture of this abacus, see *Euclid and Jesus*, cited above.

19. For detailed documentation of this claim of transmission of calculus from India to Europe, see C. K. Raju, *Cultural Foundations of Mathematics: The Nature of Mathematical Proof and the Transmission of the Calculus from India to Europe* (Pearson Longman, 2007).

20. For a complete tabulation of the sine values and errors involved, see Tables 3.1 and 3.2 in ch. 3, "Infinite series and π" in *Cultural Foundations of Mathematics*, cited above.

21. R. Descartes, *The Geometry*, trans. David Eugene and Marcia L. Latham, Encyclopaedia Britannica (Chicago, 1996), Book 2, 544.

22. For a short account of Galileo's letters to Cavalieri, see Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (Oxford University Press, Oxford, 1996), 118–22.

23. George Berkeley, *The Analyst or a Discourse Addressed to an Infidel Mathematician*, 1734, ed. D. R. Wilkins, http://www.maths.tcd.ie/pub/HistMath/People/Berkeley/Analyst/Analyst.html.

24. C. K. Raju, "Towards Equity in Math Education 2. The Indian Rope Trick," *Bharatiya Samajik Chintan* (New Series) 7, no. 4 (2009): 265–69.

25. I. Newton, *The Mathematical Principles of Natural Philosophy*, A. Motte's translation revised by Florian Cajori, Encyclopedia Britannica, Chicago, 1996, "Absolute, true, and mathematical time . . . flows equably without relation to anything external." For a detailed analysis of how Newton made time metaphysical, see C. K. Raju, "Time: What Is It That It Can Be Measured," *Science & Education* 15, no. 6 (2006): 537–51.

26. Sriharsa, *KhandanaKhandaKhadya*. IV.142. For a discussion in the context of McTaggart's paradox, see "Philosophical Time," ch. 1 in *Time: Towards a Consistent Theory* (Kluwer Academic, 1994).

27. Raju, *Time: Towards a Consistent Theory*. For a quick summary, see, C. K. Raju, "Retarded Gravitation Theory," in *Sixth International School on Field Theory and Gravitation*, eds. Waldyr Rodrigues, Jr., Richard Kerner, Gentil O. Pires, and Carlos Pinheiro (New York: American Institute of Physics, 2012), 260–76, http://ckraju.net/papers/retarded_gravitation_theory-rio.pdf.

28. For the construction of formal real numbers using "Dedekind cuts," see any standard text on mathematical analysis, e.g. W. Rudin, *Principles of Mathematical Analysis* (McGraw Hill, New York, 1964). Real numbers are an uncountable infinity.

29. C. K. Raju, "*Atman*, Quasi-Recurrence, and *paticca samuppada*," in Self, *Science and Society, Theoretical and Historical Perspectives*, ed. D. P. Chattopadhyaya, and A. K. Sengupta, PHISPC (New Delhi, 2005), 196–206, http://ckraju.net/papers/Atman-quasi-recurrence-and-paticca-samuppada.pdf.

30. C. K. Raju, *The Eleven Pictures of Time*, cited above.

31. Ioannes Philoponus, *De aeternitate mundi contra Proclum*, (Leipzig: B. G. Teubner, 1899).

32. The church-state alliance meant that the state ruled. This was achieved by the post-Nicene change in church beliefs. "Rulers rule by driving terror in the hearts of the ruled. But what weapon did the priest have to perpetuate his rule? All he had was a doctrine of universal love which frightened no one. Therefore, the Christian doctrine itself was refashioned into a weapon. . . . The new doctrine enabled the priest to strike superstitious terror in the hearts of simple people who believed what they were told." C. K. Raju, *Euclid and Jesus* (Multiversity, 2012), 79.

33. An example computer program is given in C. K. Raju, "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhâsâ," *Philosophy East and West*, 51, no. 3 (2001): 325–62, http://ckraju.net/papers/Hawaii.pdf. Similarly, a computer can never do Peano arithmetic. The theory for both floats and ints is explained in my classroom notes on computer programming, put up at http://ckraju.net/hps2-aiu/ints.pdf and http://ckraju.net/hps2-aiu/floats.pdf.

34. C. K. Raju, "Probability in Ancient India," ch. 37 in *Handbook of the Philosophy of Science*, vol. 7, *Philosophy of Statistics*, ed. Prasanta S. Bandyopadhyay and Malcolm R. Forster. General Editors: Dov M. Gabbay, Paul Thagard, and John Woods (Elsevier, 2011), 1175–96.

35. An elementary construction of the non-Archimedean field of rational functions is given in E. Moise, *Elementary Geometry from an Advanced Standpoint* (Reading, Mass.: Addison Wesley, 1963).

36. *Cultural Foundations of Mathematics*, cited earlier.

37. Aryabhata did solve differential equations using what is today called Euler's method. (Euler studied Indian texts.) Specifically, his method of obtaining sine differences cannot be understood as an algebraic equation. *Cultural Foundations of Mathematics*, cited earlier, ch. 3.

38. This is along the lines that teaching Aquinas's irrefutable belief in "laws of nature" through "Newton's laws," as the first lesson in science, can and has been used for religious propaganda against Islam. See C. K. Raju, "Islam and Science," keynote address at International Conference on Islam and Multiculturalism, University of Malaya, in *Islam and Multiculturalism: Islam, Modern Science, and Technology*, ed. Asia-Europe Institute, University of Malaya, and Organization for Islamic Area Studies, Waseda University, (Japan, 2013), 1–14, http://ckraju.net/hps-aiu/Islam-and-Science-kl-paper.pdf.

39. The continuum is a culturally biased axiom set as pointed out by Naqib al Atas, for Islamic philosophers preferred an atomic number system. See, further, C. K. Raju, "Teaching Mathematics with a Different Philosophy. 1: Formal Mathematics as Biased Metaphysics," *Science and Culture* 77, no. 7-8 (2011): 275–80, http://www.scienceandculture-isna.org/July-aug-2011/03%20C%20K%20Raju.pdf.

40. See Raju, "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhâsâ." The theory for ints is explained in my classroom notes on computer programming, at http://ckraju.net/hps2-aiu/ints.pdf.

41. C. K. Raju, "Distributional Matter Tensors in Relativity," in *Proceedings of the 5th Marcel Grossmann Meeting on General Relativity*, ed. D. Blair and M. J. Buckingham, R. Ruffini (series ed.) (Singapore: World Scientific, 1989), 421–23. arxiv: 0804.1998.

42. A detailed analysis of Stephen Hawking's singularity theory and its linkages to Christian theology may be found in *The Eleven Pictures of Time*, cited earlier.

43. C. K. Raju, "Products and Compositions with the Dirac Delta Function," *J. Phys. A: Math. Gen.* 15 (1982): 381–96.

44. C. K. Raju, "Junction Conditions in General Relativity," *J. Phys. A: Math. Gen.* 15 (1982): 1785–97. See also "Distributional Matter Tensors in Relativity" cited above for new shock conditions. For a more recent exposition of a link between renormalization theory and a modification of Maxwell's equations, see C. K. Raju, "Functional Differential Equations. 3: Radiative Damping," Physics Education (India), 30, no. 3, article 7, Sept. 2014, http://www.physedu.in/uploads/publication/15/263/7.-Functional-differential-equations.pdf.

45. C. K. Raju, "On the Square of x^{-n}," *J. Phys. A: Math. Gen*. 16 (1983): 3739–53. C. K. Raju, "Renormalization, Extended Particles, and Non-Locality," *Hadronic J. Suppl.* 1 (1985): 352–70.

46. "Renormalization and Shocks," appendix to *Cultural Foundations of Mathematics*, cited above.