Executive summary

Developing an alternative math curriculum at school level: geometry

C. K. Raju
ckr@ckraju.net

Geometry is currently taught in our schools as (1) synthetic geometry, and (2) compass-box geometry. While the very word geometry (geo + metron) means measurement of the earth, Hilbert's synthetic geometry, peculiarly enough, disallows length measurement. Paradoxically, it nevertheless, allows area measurement. The (metaphorical) instruments of synthetic geometry are an unmarked straight-edge (different from a ruler), and "collapsible" compasses (compasses which are loose). That is, while synthetic geometry permits straight lines and circles to be "drawn", distances can neither be measured nor compared by picking and carrying from one place to another.

The hallmark of synthetic geometry is the term "congruence" (typically of two triangles), found in our school geometry texts. Congruence differs from the equality of two triangles (a metric notion), though most school teachers cannot explain the subtle difference. Synthetic geometry is, *on principle*, divorced from the real world and is hence of nil practical value to the village students to whom it is taught.

We teach this unreasonable and useless synthetic geometry under pressure of colonisation. We blindly copy our syllabus from the West and are intolerant of any critical re-examination. The West had different motives: geometry was for long a part of the church curriculum, to teach a special kind of reasoning, in principle divorced from the real world, hence very useful for the church. A huge bunch of myths accumulated around that religious geometry curriculum. Synthetic geometry was invented by D. Hilbert to try to "save" those myths from rejection, when they started being exposed as false at the turn of the 20^{th} c. Saving false myths from exposure cannot be our motive for teaching geometry. Hence rejecting synthetic geometry would make a statement of cultural independence: that we are competent to decide on our own what we teach our children, and we will teach only what is useful to them and not things glorified in the West because they were useful to the church.

The compass-box geometry too is blindly copied from the West. The proof of blind copying is that the compass-box ritualistically includes set squares which are never used in school, and are not needed. Compass-box geometry is incompatible with synthetic geometry since its instruments (ruler, protractor, compass) are different, and it can actually be applied to the real world to draw visible figures etc.

The current practice of teaching of two incompatible forms of geometry (synthetic geometry plus compass-box geometry) creates confusion among students and teachers about basic geometric notions such as an angle. Thus, an angle is often defined, as in synthetic geometry, as something involving two straight lines (NCERT definition: "when two straight lines meet at a point, they are said to form an angle"). However, an angle is actually measured using the protractor provided by the compass-box.

Does the measure of an angle depend upon the size of a protractor? It does not, but to explain why not we need a key property of the *circle*: that the length of its circumference is in constant proportion to its diameter. How is this property to be established? It cannot be established through synthetic geometry which disallows length measurement even of straight lines. Nor can it be established through compass box geometry, for the compass box has a key defect: it has no instrument in it for measuring a curved line, such as the circumference of a circle. Naturally, students and teachers are left confused.

What is the alternative? The geometry of *sulba sutra*-s provides a simple alternative: for it uses a flexible string to measure curved lines. The string can be straightened to measure straight lines. This way of doing geometry, using a string or cord, was not unique to India but is found also in Egypt, though the documentation survives only in Indian tradition. This geometry was practically applied in brick constructions: the *sulba sutra-s* are practical manuals for masons. Unlike the compass box a string can actually be used for geo-metry: to calculate the areas of agricultural fields, as in the Rhind papyrus. It also enables students to do much bigger problems like measuring the radius of the earth.

A string is eco friendly and local, unlike the plastic and steel of a compass box. It is also low cost, hence the ideal way to teach geometry in village schools.

Because this string geometry relates to the real world it provides a very easy (empirical) proof of the "Pythagorean theorem". It also leads to a superior, and practical reformulation of the "Pythagorean theorem". Thus, to put the "Pythagorean theorem" to actual practical use we need square roots (to calculate the diagonal of a rectangle from a knowledge of its two sides). This form of the "Pythagorean theorem" using square roots is explicitly found in the Manava *sulba sutra*.

The "Pythagorean theorem" is also commonly applied to determine local latitude and longitude (by calculating the two sides of the rectangle from a knowledge of the diagonal and the angle it makes with one of the sides). Knowledge of latitude and longitude was required for navigation but also for the Indian calendar, useful for Indian monsoon-driven agriculture. Doing this calculation requires sine values. The word sine derives from the Sanskrit *jya* or *jiva*, via the Arabic *jiba* misread as *jaib* (or pocket or fold, hence the Latin *sinus*). Apart from a linguistic confusion, this involved a conceptual confusion: in schools today sine is defined in trigonometry, using a triangle, but the concept of sine relates naturally to a circle, as the term *jya* (meaning chord) shows.

These calculations (of square roots, sines) related to the "Pythagorean theorem" also tell us that mathematical calculations are never exact. If we calculate $\sqrt{2}$ and square the result, we never get back 2. In the *sulba sutra*, $\sqrt{2}$ is hence described as *savisesa* (with a remainder). This practical philosophy of mathematics as involving empirical proof and inexact calculation has been reformulated in contemporary terms as zeroism. This is better than the religious Western belief in math as exact and eternal truth. The latter philosophy puzzles students no end, for a geometric point on Western philosophy is NOT a dot on a piece of paper, and, in fact, corresponds to nothing real.

This philosophy of practical mathematics interfaces very well with the more advanced applications of mathematics taught at a higher level. As my survey of applications for C-DAC showed, all key practical applications of mathematics involve the calculus and inexact numerical solutions of differential equations calculated using computers. The little-known fact is that the calculus developed in India in this way (as an inexact way to numerically solve differential equations) just because Indians had a clear concept of the length of a curved line, arising from string geometry. Though the Indian calculus was transmitted to Europe, they did not understand its infinite series for which they developed a complicated metaphysics of infinity. That was declared "superior" and returned to us through colonial education. String geometry not only integrates with an easier "trigonometry" but also with an easier calculus as already demonstrated in courses taught to 8 groups in 5 universities in 3 countries. However, all this is little known to experts, leave alone school teachers.

The present project will develop this alternative math curriculum of string geometry at about the level of 8^{th} standard. The outcome will be a school text suited to teach this curriculum. The text will be tested in teaching trials in remote village schools. A more detailed proposal is attached.