Book Review

# Cultural Foundations of Mathematics

**It has been common understanding that mathematical proof based on deduction is universal and is the ultimate proof and also that mathematical truths are eternal universal truths. C K Raju argues that this is a narrow European view of mathematics and the Indian view was very different and empirical. Thus he has raised the important issue of cultural foundation of mathematics. We present here a summary of his startling new book.**

British scholars have known since 1832 that traditional Indian mathematicians had developed a way to handle infinite series, a key component of the calculus. However, Western historians have denied that this amounted to the calculus proper, and many aspects of this fascinating Indian contribution to science have remained unclear for the last 175 years.

In this book, Raju asks four questions that have not been asked before. (1) How were infinite series useful to Indian society? (2) Did the Indian infinite series amount to the calculus? (3) Was this Indian mathematics transmitted to Europe before Newton and Leibniz? (4) Does the traditional Indian approach to mathematics have any practical applications today? Raju's answers are as follows:

1) The main source of wealth in India is agriculture which depends on the monsoons. The monsoons are "erratic", so a good calendar is indispensable to Indian agriculture. The traditional Indian calendar identifies the months of Sawan and Bhadon as the rainy season, unlike the common Gregorian calendar which has no rainy season. Traditional Indian festivals like Rakhi and Holi do not occur on "fixed" days of the Gregorian calendar (such as 25 December or 15 August), and are related to agriculture. Constructing this specialised Indian calendar required complex planetary models. Calibrating this calendar across the length and breadth of India required precise knowledge of the size of the earth, and of ways of determining latitude and longitude of any place and all this required precise trigonometric values. (This knowledge was useful also for navigation and overseas trade with Alexandria, Arabs, Africa, and China was also a key source of wealth in India.) The required trigonometric values were developed in India since the Surya Siddhanta (3rd c.) and Aryabhata (5th c.). Over the next thousand years these trigonometric values were gradually made more precise, and that led to the development of the Indian infinite series. Thus, Raju concludes that the social utility for agriculture and navigation drove the development of the Indian infinite series.

2) Western scholars have dubbed the Indian infinite series as "pre-calculus", claiming that the calculus proper emerged with the "fundamental theorem of calculus" which was absent in India. Raju responds to this criticism in various ways. (a) First, he questions the premise that mathematics means theorem-proving rather than calculation. This requires a re-examination of all Western history and philosophy. Raju argues that "Euclid" is a historical concoction, and that the Elements, attributed to "Euclid", is actually a Neo-Platonic religious book. It was radically reinterpreted by Christian rational theologians after the 12th c. CE, to support their agenda of converting Arabs, during the Crusades. To this end they declared reason (and mathematics) to be universal. However, since Buddhists and Jains have used a different logic from that used in mathematical proof today, this notion of proof can never be universal. Different principles of proof or pramana were used in Indian tradition, where mathematics is not singled out as requiring a special sort of proof. Raju concludes that

the current Western conception of mathematics as theorem-proving is loaded with religious beliefs unlike calculation which is secular. Hence, the alleged superiority of present-day formal mathematics ultimately rests on religious prejudices which, though deep seated, must be rejected. In particular, Raju reaches the radical conclusion that deduction is more fallible than induction:

a conclusion which stands most of Western philosophy on its head. (b) Raju completes the "missing links" in past studies of Indian infinite series to show that there actually was valid pramana for the derivation of the Indian infinite series. This pramana differed from formal mathematical proof, but it was not inferior for that reason.

(c) Finally, Raju argues that Aryabhata had already developed a neat technique, similar to what is today known as Euler's method of solving ordinary differential equations, and that this calculation technique is a superior substitute for the fundamental theorem of calculus even today. Thus, Raju concludes that Indians did have the calculus, and that denying this merely amounts to an imposition of Western religious beliefs.

3) Raju points out that information often flows towards the military aggressor. Examples are (a) Alexander's loot of books, (b) Hulegu and the Samarkand observatory, (c) the Latin translations of Arabic books at Toledo, during the Crusades, and (d) the British colonialists. This happens because the military aggressor is often in a lower state of development. (Toynbee calls these "barbarian incursions"). Specifically, Raju points to Vasco da Gama's and Columbus' ignorance of celestial navigation: Vasco was brought to India from Africa by an Indian navigator whose instrument the befuddled Vasco carried back to Europe to study. Europe then dreamt of wealth through overseas trade, so several European nations instituted huge prizes for a good technique of navigation. In 16th c. Europe, precise trigonometric values were critical to navigation: and the problems of determining latitude, longitude and loxodromes. Catholic missionaries were in Cochin, since 1500, and had started a college for the local Syrian Christians. The Portuguese shared a common patron in the Raja of Cochin with the authors of key Indian texts on the Indian infinite series, such the Yuktidipika of Sankara Variyar. Later Jesuits got numerous Indian texts translated and despatched them to Europe on the model followed earlier at Toledo. Jesuits like Matteo Ricci specifically looked for Indian astronomy texts, to assist with the Gregorian calendar reform. Thus, Europeans had ample opportunity and motivation to obtain the relevant Indian mathematical texts. A trail of circumstantial evidence is visible as the contents of these Indian texts start appearing implicitly or explicitly in European astronomical and mathematical works from the mid-16th c: Mercator's chart, Tycho Brahe's planetary model, Clavius' trigonometric tables, Kepler's planetary orbits, Cavalieri's indivisibles, Fermat's challenge problem, Pascal's quadrature, "Newton's" sine series.

Why were the Indian texts not acknowledged? To understand this, Raju first points to the Hellenisation of history that took place at Toledo. During the religious fanaticism of the crusades, all secular world knowledge in Arab libraries up to the 11th c. was appropriated to the West by attributing it to the theologically correct "Greeks". The Arabs, against whom the crusades were on, were declared to be mere intermediaries who helped to restore this "European inheritance" to Europe. (The arrival of Byzantine Greek texts in the 15th c. further confounded matters.)

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This claim of transmission of hypothetical "Greek" knowledge to Arabs and all others, involves double standards of evidence. To expose this, Raju considers, as an example, the Almagest. The text dates from post-9th c., but is attributed to a "Claudius Ptolemy" from the 2nd c. However, Raju argues, the Almagest is an accretive text which was repeatedly updated with inputs from Indian astronomy and mathematics at both Jundishapur (6th c.) and Baghdad (early 9th c.) where Indian texts on astronomy are known to have been imported and translated into Pahlavi and Arabic. Thus, transmission of trigonometric values took place from Indian texts to the Almagest, rather than the other way round as is usually claimed by stock histories today, without any evidence. In support of this claim, Raju contrasts the relative sophistication of the Almagest text with the non-textual evidence of the crudeness of the Greek and

Roman calendars, which could not get the length of the year right, despite repeated attempts at calendar reform in the 4th through 6th c. CE. This could hardly have happened if the Almagest had then existed in its present form.

This European tradition of suppressing non-Christian sources was continued during the Inquisition, when it was dangerous to acknowledge anything theologically incorrect. For example, a key navigational breakthrough in Europe was Mercator's chart (common "map of the world") which shows loxodromes as straight lines. This required precise trigonometric values and a technique equivalent to the fundamental theorem of calculus. Mercator, however, was arrested by the Inquisition, and his sources remain a mystery to this day. Similarly, Clavius, a high religious official, could hardly be expected to acknowledge his debt to non-Christians for trigonometric values, or the Gregorian calendar reform. This habitual nonacknowledgment of non-Christians led to the proliferation of claims of "independent rediscovery" by Europeans. Raju also points to the papal "Doctrine of Christian discovery", promulgated in that period, according to which only Christians could be "discoverers" - hence it was said that Vasco da Gama "discovered" India or that Columbus "discovered" America, since this "discovery" implied ownership, and the existing inhabitants did not count.

To correct this long-standing racist bias, and given the difficulty of obtaining suppressed documents, Raju hence proposes new standards of evidence to decide transmission. This includes the consideration of opportunity, motivation, and circumstantial evidence, as above. It also includes the common-sense "epistemic test": when two students turn in identical answer sheets, the one who does not understand what he claims to have authored is the one who has copied. Thus, lack of understanding is proof of transmission. Hence, proof of transmission of the calculus comes from the fact that for centuries it remained half-understood in Europe: given the notorious difficulties with Newton's fluxions, and Leibniz's infinitesimals both of which were eventually abandoned. While Clavius published elaborate trigonometric tables, he did not know the elementary trigonometry needed to use them to determine the size of the earth. (This was then badly needed for navigation, especially since Columbus had underestimated the size of the earth by 40 percent to support his project of reaching East by going West.)

Similarly, there is the issue of "epistemic discontinuity": the Indian infinite series developed gradually over a thousand years; but in Europe they appeared almost overnight. From Cavalieri to Newton it hardly took 50 years, and this whole development started just 50 years after Europe first learnt of decimal arithmetic. Interestingly, in an appendix on the "transmission of the transmission thesis", Raju applies this "epistemic test" to prove the transmission of his own ideas to Europe, in a remarkable case of history repeating itself! Thus Raju concludes that the calculus was transmitted from India to Europe, and that similar processes of transmission of information are going on to this very day.

4) Raju points out applications of traditional Indian mathematics to mathematics education, computers, and frontier areas of physics today.

The European difficulties in understanding arithmetic and the calculus, both imported from India, are today replayed in the classroom, in "fast forward" mode, and this is what makes it difficult for students to understand mathematics. The solution is to go back to the understanding within which that mathematics originated.

For example, Indians used a flexible rope (= sulba) to measure curved lines since the days of the sulba sutras. However, exactly measuring curved lines was declared to be "beyond the capacity of the human mind" by Descartes, since Western geometry was based on the straight line. Hence, Galileo left it to his student Cavalieri to articulate the calculus. These conceptual difficulties can be avoided, Raju argues, by switching back to the rope as a superior substitute to the compass box today used in schools.

Raju explains how the Indian understanding of number, in the context of sunyavada, is related to the representation of numbers on present-day computers, and why this should be taught in preference to the impractical and formal notion of number taught today in schools.

Raju also explains how to apply this understanding to tackle unsolved problems related to the infinities and infinitesimals that arise in the extensions of the calculus today used in frontier areas of physics, in the theory of shock waves, and the renormalisation problem of quantum field theory.

Thus, in the process of answering these four questions, Raju has challenged the entire Western tradition of mathematics, both its history and its philosophy. Especially interesting is the conclusion that the believed certainty of mathematical proof and the infallibility of deduction are based on religious dogma which is remedied by allowing the empirical in mathematics as in the alternative philosophy of mathematics Raju proposes. The formal approach to mathematics, Raju argues, is based on the wrong belief that logic is a metaphysical and metamathematical matter on which it is possible to impose a "universal" social consensus. Given the social disagreement over logic, even logic ought to be decided empirically, he maintains.

This alternative philosophy follows realistic sunyavada thinking on representability. (In Buddhist thought, the problem of representability arose because of the denial of the soul, and the consequent problem of representing an "individual" when nothing "essential" stays constant for even two instants.) Raju interprets sunya as the non-representable, "something" which is neglected in a calculation. For example, the number today called $\pi$ can never be fully specified or distinguished from a potential infinity of other nearby numbers. In any actual calculation, one can only write down its decimal expansion up to a trillion digits, say, beyond which one does not care what happens. This problem of representability is today made manifest by the finitary thinking of computers, increasingly used for complex mathematical tasks: surprisingly, even integers cannot be "correctly" represented on a computer. This, argues Raju, is not any sort of limitation or "error" of representation, but is in the nature of things. The error, to the contrary, is in the idealistic approach to mathematics based on a wrong belief in the possibility of upertasks (an infinite series of tasks): the mere name $\pi$ does not represent a unique number any more than the name of a person represents a unique individual. Given the paucity of names, a person's name is just a practical device by means of which we refer to a whole procession of individuals, from birth to death, who differ from each other so "slightly" that we "don't care" about the difference. The "slight" difference may vary with the context, and may even be manifest as the difference between a child and the (same) child as an adult. The same thing applies to representations of numbers on a computer (or in any practical rule-based calculation).

Apart from being very useful for computation, the mere existence of such an alternative philosophy of mathematics poses a challenge to Western thought, which has never conceived of the possibility that mathematics might be ultimately based on physical hypotheses about logic, and that it might be realistic, fallible and less than perfect.

**Summary of : "Cultural Foundations of Mathematics: the Nature of Mathematical Proof and the Transmission of the Calculus from India to Europe in the 16th c. CE (Pearson Longman, 2007).** ■

***Dr C K Raju is a mathematician, historian and philosopher and has made important contribution through many articles and monographs towards combating Eurocentric rendition of the history of Mathemtaics. He has particularly brought out the impact of theological controversies in Europe on revisionist history of mathematics.***