Raju, C. K. 'Towards Equity in Math Education. 1: Goodbye Euclid!' Bharatiya Samajik Chintan (New Series) 7, no. 4 (2009): 235–64. Paper presented at 31st Indian Social Science Congress, SNDT Univ., 27-31 Dec 2007.

Towards Equity in Mathematics Education 1. Good-Bye Euclid!

C. K. Raju
Centre for Studies in Civilizations,
36, Tughlakabad Institutional Area, New Delhi 110 067
c_k_raju@vsnl.net

Racist history has been an instrument of inequity, and is still uncritically propagated in current Indian school texts in mathematics. As a first step towards equitable mathematics education, we need to do away with this. Critics have argued that the thrust for social justice in the mathematics classroom handicaps students. We argue to the contrary that the difficulties in teaching or learning mathematics arise because inequity and a brand of "theological correctness" are already embedded into the history and philosophy of current formal (theorem-proving) mathematics. The philosophy of current formal mathematics derives from an analysis of the *Elements* by Hilbert et al. That analysis proceeded from a historical narrative about Euclid and his method of proof. However, in the absence of any serious evidence for the historical "Euclid" this narrative must be rejected as a racist fantasy. The real philosophy of the *Elements*, and its religious significance for Greeks, is brought out by Proclus in his *Commentary*—virtually refuting point-by-point the inequitable post-Nicene (Augustinian) Christian theology with which he had to contend. This linkage of mathematics and religion persisted in Islamic rational theology (*aql-ī-kalām*) which too used the *Elements* to promote equity and justice. However, during the Crusades, history was Hellenized at Toledo. The Inquisition enforced theological correctness, and the *Elements* was reinterpreted to align it with the prevailing Christian theology. Current school texts use Hilbert's synthetic reinterpretation, which substituted "equality" by "congruence", and eliminated also the empirical, thus completing the process of making the *Elements* theologically correct. However, synthetic geometry (apart from being an invalid interpretation of the *Elements*) is harder to understand, and counter-intuitive, compared to metric or empirical or traditional geometry, and certainly does not add any practical value. This applies not only to geometry but to all formal mathematics: it is this "theologification" that has made mathematics difficult to learn or teach. The remedy is to "de-theologify" or secularize mathematics and teach it in the cultural and practical context in which it developed.

History is a well known instrument of soft power, and racist history was used for this purpose during colonisation: the best slave is one who is mentally subjugated. Interestingly, given the all-but-forgotten connection of mathematics to religion, the case of mathematics education also illustrates how racist inequity originated in religious doctrines of inequity.

The post-colonial attempt to undo racist history in mathematics education by teaching "multicultural mathematics" has led to a sharp reaction. For over two decades, a war has been raging in the United States over the teaching of mathematics in (K-12) schools. The critiques of multicultural mathematics are summarised in the widely cited article, entitled "Good-Bye Pythagoras?"[1]. That article answers the question raised in its title as follows.

> But even the most ardent professors of ethnomathematics say they are not trying to replace the great Greek and other European thinkers who have shaped modern mathematics. Instead, they say, they are blending European ideas with African, Asian, Native American, and other mathematical innovations, teaching both European and non-European practices.

The key criticism articulated by e.g. Klein[2], founder of "Mathematically Correct", is that in being politically correct, the teachers of multicultural mathematics are being mathematically incorrect, hence handicapping students.

This article outlines a new answer to such criticism. The answer embodied in the title has been articulated in more detail in my earlier publications and recent book *Cultural Foundations of Mathematics*.[3]

[1] Elizabeth Greene, *The Chronicle of Higher Education*, 6 October 2000

[2] D. Klein, "A quarter century of US 'math wars' and political partisanship", http://www.csun.edu/~vcmth00m/bshm.html preprint of article in British Society of History of Mathematics, 2006

[3] C. K. Raju, *Cultural Foundations of Mathematics*, Pearson Longman, New Delhi 2007. (Project of History of Indian Science, Philosophy and Culture, vol X, part 4.)

What the Critics Assume

The critics assume that (1) mathematics originated with the Greeks, (2) that it is universal and secular, and (3) that the mathematics of theorem-proving is what is valuable today, so this is the kind of mathematics that ought to be taught.

This article will focus on assumption 1, although my other publications address all the assumptions.

The racist formulation of assumption 1 is explicitly stated by Rouse Ball in his celebrated *History of Mathematics*, still being reprinted as a "classic":[4]

> The history of mathematics cannot with certainty be traced back to any school or period before that of the Greeks.... though all early races knew something of numeration... and though the majority were also acquainted with the elements of land-surveying, yet the rules which they possessed... were neither deduced from nor did they form part of any science.

In other words, geometry, proper, began with the Greeks, what others did may have been land-surveying or something like that. This is Rouse Ball's answer to Herodotus' account that the Greeks aped all the practices of Black Egyptians and also learnt geometry from them.

Earlier historians such as Rouse-Ball were concerned with the Greek *race* rather than culture. But how do we know Euclid's race? After all, Euclid could well have been Black, for historical authorities currently maintain that Euclid was from Alexandria, which is located in the African continent. In fact, some Arab sources (e.g. al Qifti) tell us that Euclid, though a Greek national, was from Tyre[5] a place where Alexander made 30000 slaves. Similarly, some Arab sources tell us that Archimedes was a short black man. Heath discounts these sources on the grounds of "the Arab tendency to romance" etc. Heath's attempt to brand all (inconvenient) Arab sources as unreliable is clearly racist, and such racist remarks no longer carry any conviction, so how is the claim of Arab sources to be refuted?

Rouse Ball and Heath are hardly isolated cases. Martin Bernal (son of the historian J. D. Bernal) argued in *Black Athena* that racist historians of the previous two centuries systematically appropriated African culture by falsely claiming key parts of it to be of "Greek" origin. Although Bernal does not touch mathematics and science, the situation here is not fundamentally different. Further, racist "fabrication of ancient Greece" did not really stop in 1985.

Thus, take the recent 9th standard mathematics text used throughout India by schools affiliated to the Central Board of Secondary Education. This is a recent text, created and approved by the National Council of Educational Research and Training (the apex Indian body for K-12 school education). A key aim of this text is to counter the "saffronization" of history that took place during the previous government, under the influence of what are called Hindutva forces—also referred to as Hindu-Nationalists. This new text[6] has what look like photographs of Greek mathematicians such as Euclid. The pictures in the text include those of Pythagoras (p. 5), Archimedes (p. 13), Thales (p. 79), Heron (p. 199), and, of course, Euclid (p. 80). After looking at this text, which he was compelled to study, my son asked: why do all Greek mathematicians look alike?

Now, when a sixth standard student from the US asked me for a photograph of the famous 5th c. Indian mathematician Āryabhaṭa[7] I felt obliged to tell her that photography did not exist in Āryabhaṭa's time. Therefore, one naturally wonders from where the pictures in my son's school text were sourced.

The lead author of this school text has admitted his ignorance of history[8] so one can understand that these pictures were taken from a secondary source such as the MacTutor website on the history of mathematics (which has similar pictures). So what is the real source of these pictures which present so concrete and vivid an image of Greek mathematicians to impressionable young minds? Did the contemporaries of these worthies make statues of them which were later photographed? Not at all. No such concrete historical information is available about Greek mathematicians, and these pictures are based on what is usually called "the artist's imagination". This understanding of the source makes it possible to answer my son's question: the artist's imagination was racist, and portrayed some "generic" Caucasian features. The images look alike because they project a stereotype. So, without a single word being said, the question about Euclid's race has been settled, along with the race of a number of other Greek names associated with the history of mathematics!

The fact that this starkly racist belief can be distributed as fact to millions of impressionable young Indian school children, today, shows the level of confidence with which the question about Euclid's genetic history is regarded as settled. The psychological trick involved here is well known:

---

[4] W. W. Rouse Ball, *A Short Account of the History of Mathematics*, Dover, New York, 1960, pp. 1–2.

[5] T. L. Heath, *The Thirteen Books of Euclid's Elements*, vol. I, Dover, New York, [1908] 1956, p. 75.

[6] *Mathematics: Textbook for Class IX*, (J. V. Narlikar, P. Sinclair, et al.), NCERT, New Delhi, 2005.

[7] This name continues to be mis-spelt in the new NCERT text as Āryabhaṭṭa. The point of this mis-spelling is that it changes his caste: "Bhaṭa" refers to a slave, whereas Bhaṭṭa is a title used by a high-caste brahmin. The false impression generated by the wrong spelling is reinforced by an image (in another NCERT text for class X) which again suggests that Āryabhaṭa was a brahmin. If we examine the sources, Indian mathematicians who follow Āryabhaṭa often refer to him simply as "bhaṭa", and to those who followed his system as "bhaṭa's disciples". In 5th c. India, the Buddhists were very influential, especially in the vicinity of Patna, where Āryabhaṭa lived, and the caste system was not particularly strong.

[8] J. V. Narlikar, "Four questions that history might answer", in: *Science, Philosophy and Culture: Multi-disciplinary Explorations* ed. D. P. Chattopadhyaya and Ravinder Kumar, PHISPC, New Delhi, 1997. That paper was referred to this author who, as adjudicator, suggested that the paper could be re-titled "Four questions that the library might answer", but recommended it be published to enable a later discussion on it. The other two referees were the late David Pingree and the late K. V. Sarma. All three referee reports were later published in the inaugural issue of *Sandhan*.

children do not question the first story they are told. But if they are subsequently told that Euclid was Black, they ask for evidence. So, the racist history of mathematics has not disappeared, it has merely assumed more covert forms. This is troubling, since the only difference between (a) assumption 1, above, used by current critics of multicultural mathematics, and (b) the stand of earlier historians like Rouse Ball and Heath, is that the word "Greek" is *implicitly* understood to relate to "culture" rather than "race".

What are our sources for Euclid?

Nevertheless, let us help Heath along a bit. Let us ask, what are al Qifti's sources? Since al Qifti came so long after Euclid, if we do not know his sources of information about Euclid, we could very well suppose that he invented that detail about Euclid being from Tyre. However, a non-racist would address exactly the same question also to Heath. How do we know that Euclid was from Alexandria?

It is hardly obvious that Euclid was from Alexandria, since for some five *centuries* Western historians believed that Euclid was from Megara. This earlier-accepted history is today regarded not only as mistaken, but as a completely baseless myth. However, if such myths could be propagated as history for five centuries, that says something about the way the Western history of science has developed—we cannot trust a narrative merely because it is centuries old for it may be myth, not history. For history, we need valid sources of information.

The key valid source of information about Euclid, that Heath acknowledges, is a remark by Proclus in his *Commentary* on the *Elements*. Why, one wonders, does Heath need this roundabout route? Don't we have copies of the *Elements* which state Euclid to be the author? The answer to this innocuous question is "No". As Heath admits—"All our Greek texts of the *Elements* up to a century ago... purport in their titles to be either 'from the edition of Theon'... or 'from the lectures of Theon'."[9] As Heath further admits, Euclid's name does not appear even in the commentaries which "commonly speak of the writer of the *Elements* instead of using his name."[10]

So, on this information, the *Elements* could well have been authored by Theon (as he states) or his daughter Hypatia. Since she preceded Proclus in the same tradition, that would nicely explain Proclus' interest in writing a commentary on that text. It could also explain why he used the phrase "the author of the Elements" when he mentions so many others by name. However, today, what is officially called the "primary source" of the *Elements* is a single manuscript found in the Vatican which has been valued solely for the curious property that, unlike all other known texts, it does *not* state to have been derived from Theon!

Anyway, that one remark about Euclid, which is our key source of information about Euclid, is very vague and speculative, and obliges us to ask what Proclus' sources of information were—for Proclus comes over 7 centuries after the date ascribed to Euclid. Evidently Proclus (or whoever authored that remark) had no particular prior sources for he himself admits that historians of geometry before him have *not* mentioned Euclid: "All those who have written histories [of geometry] bring to this point their account of the development of this science. Not long after these men [pupils of Plato] came Euclid...."[11]

That Euclid was little known prior to Proclus is substantiated by the archaeological evidence. In attributing the *Elements* to an early Greek called Euclid, we are supposing that there was a fixed text which was repeatedly copied out without any significant change by subsequent scribes. But the available papyri on geometry from Alexandria do not correspond to the received text, and do not show any such evidence of the existence of a fixed, early text.[12] On this evidence we could well suppose that the *Elements* was but the Greek version of an ancient Egyptian mystery tradition related to geometry. On the other hand, if there really had been a "Euclid" who remained so little-known for over seven centuries after his death, it is hard to understand how his books survived—in the days of papyri, for a book to survive, it would have had to be repeatedly copied out, and it is hard to imagine why many different people would have wanted to fund the copying of books by a little-known author.

To complete our enquiry, we need to ask what are our sources about Proclus? In fact, our source for Proclus is a manuscript called "Monacensis 427". Since this manuscript is on paper, and paper made a late entry into Europe (after the demand for books grew in Europe after the Toledo translations), it probably comes from after the 13th c., though it has been optimistically dated to as early as the 10th c. CE. Anyway, that earliest date is still five centuries after Proclus, and there is no continuous tradition linking that text to Proclus. So, why should we believe that every remark in this book is due to Proclus?

On the contrary, we have good reason to regard this remark as an interpolation. The remark says, "This man [Euclid] must have lived in the time of the first Ptolemy; for Archimedes, who followed closely the first [Ptolemy? book?] makes mention of Euclid". The author of the remark seems to be estimating the date of "Euclid" based on the belief that "Euclid" was mentioned by Archimedes. However, the only known reference[13] to the *Elements* (not "Euclid") in the works attributed to Archimedes has been regarded as not genuine,[14] since it was not the custom in Archimedes' times to make such references (in the style of Christian theology), and there were many more places where such a reference

---

[9] Sir Thomas Heath, *A History of Greek Mathematics*, Dover, New York, 1981, p. 360.

[10] Heath, *Greek Mathematics*, p. 357.

[11] Proclus, *A Commentary on the First Book of Euclid's Elements*, Glen R. Morrow, Princeton University Press, Princeton, New Jersey, 1970, p. 56. Heath suppresses this remark.

[12] David Fowler, *The Mathematics of Plato's Academy*, Clarendon Press, Oxford, 1999, 2nd ed., p. 216.

[13] *On the Sphere and the Cylinder* I, Proposition 6, in *The Works of Archimedes*, trans. T. L. Heath, Great Books of the Western World, vol. 10, Encyclopaedia Britannica, Chicago, 1996, p. 407.

[14] J Hjelmslev, " 'Uber Archimedes' Grössenlehre", *Danske Vid. Selsk. Mat.-Fys. Medd. 25* (15) 1950.

could have been made. (Making references in the modern style could not have been the custom in Archimedes time for the simple reason that standardized editions of books did not exist prior to printing and mass production of books.) But if the Archimedes remark is an interpolation, and the author of the "Proclus" remark knew of *that* interpolation, then the author of the "Proclus" remark must come after the author of the "Archimedes" remark. Therefore, the "Proclus remark" (our key source of information about "Euclid") must be a 16th c. interpolation. (Doubtless, ways could be found to meet all the above objections for it is well known that *any* facts can be made to conform to *any* theory with the help of enough additional hypotheses.)

Another key reason to suspect "Proclus' " remark is that it articulates a philosophy (of "irrefragable demonstration") which is completely at variance with the Neo-Platonic philosophy expounded in the rest of Proclus' *Commentary*. If we discount this remark as a 16th c. interpolation, then that philosophy of the *Elements*, articulated in the rest of Proclus' *Commentary*, needs to be taken more seriously.

Mathematics and Religion

Indeed, Proclus states that the point of writing his *Commentary* is to bring out the *religious* dimension of mathematics. Proclus derives mathematics from *mathesiz*—meaning learning—thus characterizing mathematics as "the science of learning". Proclus understood "learning" as a process by which the soul remembered its past lives—for he thought, like Plato, that "learning is recollection" (of eternal ideas or memories acquired by the soul in its previous lives). The underlying picture is that of "cyclic" time (more properly, quasi-cyclic time): in which the cosmos goes through a series of cycles in which people and events approximately repeat. (This is described by saying that the soul is reborn in successive cycles of the cosmos.)

Learning, regarded as a process by which the soul remembered its previous lives, hence made people virtuous, since learning led to the realization of the soul. The function of mathematics, the science of learning, was to facilitate this recollection. This is the point of Socrates' demonstration with the slave boy in *Meno*: the untutored slave-boy's intrinsic knowledge of mathematics, which Socrates brings out, is regarded by Socrates as proof of the existence of the soul, and hence of its previous lives. But this belief in past lives, called the "doctrine of pre-existence" was banned by the post-Nicene church.[15]

Proclus, however, continued to regard mathematics, like *haṭha yoga*, as an instrument which facilitates the realization of the soul, and "leads us to the blessed life". How? By putting one into an introspective state, and helping to stir the soul from its slumber, since then "the soul… is set free of the hindrances that arise from sensation". It is beyond the scope of this article to examine Proclus' philosophy in more detail.[16]

However, even a quick examination of that philosophy brings out the *Elements* as a step-by-step refutation of the post-Nicene Christian doctrines (as distinct from the ante-Nicene beliefs of Origen which were similar to those of Proclus).[17] Let us recall that after the state and church came together in the Roman empire in the 4th c. CE, there was widespread persecution of non-Christians in the Roman empire: by Proclus' time, fanatic Christian mobs had smashed every single "pagan" temple across the Roman empire[18] and vast numbers of non-Christian books were burnt on the orders of Roman emperors. Prominent non-Christians like Hypatia were lynched by Christian mobs. Considering that Proclus succeeded Hypatia, he had every reason to want to write such a defence of the philosophy of geometry she explicated along with her father Theon in the tradition coming down from Plato and Pythagoras (who brought it from Egypt, according to Herodotus).

For example, the use of "images" was a key issue of contention slightly before Proclus—"pagan" temples had images of gods which images were derided by Christians, and used as justification for destroying them. While Proclus' predecessor Porphyry wrote a book *On Images* to defend this practice, Christian kings ordered Porphyry's books to be burnt. However, numerous theorems of the *Elements* are illustrated with the aid of figures. Proclus remarks that these figures (like images of gods) serve to move the soul (i.e., help learning), and points out[19] that Socrates had hence used a similar argument (drew a figure) in his conversation on geometry with the slave boy.[20]

Equity was another point of conflict. The original *Elements* used the word "equality" (not "congruence"). In a well-known mystery story, a king asked a geometer whether there was no shorter road to learning geometry. The reply was that there is "no royal road to geometry". The real meaning is that geometry assists in realization of the soul, and all souls are equal, since they are all equally part of one immanent Nous. This provided the basis for the belief in political equity for all are equally part of one God. Hence, also, Socrates chose a slave boy for a dialog on mathematics. But equity became a key point of contention with the post-Nicene Christian church erecting a transcendent God. A key aspect of the post-Nicene Christian story was that there would soon be a Day of Judgement when this transcendent God would consign all non-Christians to hell, as so morbidly described by Dante. Hence, the church regarded non-Christians as fun-

[15] C. K. Raju, "The curse on 'cyclic' time", *The Eleven Pictures of Time*, Sage, New Delhi, 2003, chp. 2.

[16] This is outlined in C. K. Raju, *Philosophy East & West*, 51(3) pp. 325–62.

[17] The distinction between pre-Nicene and post-Nicene Christianity is explained in C. K. Raju *The Eleven Pictures of Time*, cited above.

[18] E. Gibbon, *Decline and Fall of the Roman Empire*, Encyclopaedia Britannica, Chicago, 1996, p. 460.

[19] Proclus, p. 37.

[20] Plato, *Meno*, 81–83. *The Dialogues of Plato*, trans. B. Jowett, *Great Books of the Western World*, vol. 7, R. M. Hutchins, ed. in Chief, Encyclopaedia Britannica, Chicago, p. 180.

damentally and eternally unequal to Christians—paving the way for the future doctrine of racism.[21]

A third issue concerned creation. Proclus believed that mathematics, since it embodied eternal truths, demonstrated the eternity of the cosmos: the cosmos had existed and would continue to exist forever. This belief about the cosmos directly contradicted the post-Nicene Christian doctrines of creation and apocalypse. This conclusion was important enough for a Christian theologian of the time (John Philoponus) to write a book-length refutation of Proclus who was declared a heretic by Justinian, when he proclaimed a death penalty for all heretics in his kingdom.

In any case, the contentious issues naturally important to Proclus, in his social and philosophical circumstances, are far removed from the "irrefragable demonstration" mentioned in the remark used as evidence also for Euclid's philosophy. This "irrefragable demonstration" incidentally is later stated to be based on "causes and signs"!

In fact, this idea of mathematics as proof or "irrefragable demonstration" is foreign to Proclus' philosophy of mathematics. Mathematics as the science of learning is intended to stir the soul by presenting it with eternal truths. Proof was important, no doubt, but Proclus states that "proof varies with the kinds of being". Proclus characterizes mathematics as an intermediate state of being, connecting a lower state of being to a higher state of being. Therefore, he asserts that methods of proof may vary *within* mathematics just as they may vary between mathematics and mechanics which deal with different sorts of being.

> Proofs must vary. . . and be differentiated according to kinds of being concerned, since mathematics is a texture of all these strands and adapts its discourse to the whole range of things.[22]

The proof of the side-angle-side (SAS) theorem (*Elements* 1.4) uses an empirical technique that is not subsequently used: for its subsequent use would trivialise the *Elements*. This "inconsistency" was not a mistake, as Hilbert and Russel, and current school texts take it to be: for Proclus it was a demonstration of how proof may vary within mathematics. On the contrary, Proclus would have regarded the formalist approach to mathematics as a mistake, for it does little to stir the soul!

After Proclus, the *Elements* was adopted by Islamic rational theology (*aql-ī-kalām*), the details of which are beyond the scope of this article.[23] Europe first learnt about the *Elements* through translations from the Arabic books in the Toledo library during the Crusades against Islam.

The Hellenization of History

At that time, Europe was in its "Dark Age", at the beginning of which the church had burnt a vast number of books.[24] Arabs, on the other hand, had been building vast libraries for centuries. Hence, the knowledge in Arabic books available even in the library at Toledo, a small fraction of the Cordoba Caliphate, was far far ahead, of anything available in Christian Europe.

However, the Crusades were also a time of intense Christian religious fervour against Islam. This made it unacceptable for the church to admit to learning wholesale from the Islamic enemy by the mass translations at Toledo—financed and managed by the church. It was also unacceptable to the church to acknowledge its late Greek opponents like Theon, Hypatia, and Proclus as the source of this knowledge.

An early church historian, Eusebius, is believed to have advocated the use of history, falsified as convenient, as an instrument of religious propaganda. The same Eusebius, also regarded the early Greeks as "friendly" to Christianity, since there was evidently no possibility of conflict with them. Thus, the convenient story was given out that all the (secular) knowledge available in the Arab books at Toledo was of early Greek origin—history was Hellenized. The extraordinary story went that during the centuries of the "Golden Age of Islam", when the Arabs patronised knowledge, and developed a vast library system from Samarkand to Cordoba, they merely made and preserved literal translations of early Greek works, and all those Arabs in all those centuries adding nothing of substance to that "Greek" knowledge. Christian Europe was not learning from Islamic Arabs, it was only getting back its inheritance from the early Greeks!

Though this pathetic story was acceptable to a Europe which was then largely illiterate (and still remains ignorant of other cultures), the story is quite contrary to what is known. The Baghdad House of Wisdom was started by Khalifa al Mamun to encourage the *aql-ī-kalām*, whose adherents believed that *aql* (creative intelligence) must be used to interpret passages in the Koran whose meaning was not evident. The one thing these philosophers most utterly despised was the opposite of *aql*, called *naql*, meaning mindless copying

[21] Racism is usually associated with the color of the skin, so the relation of religious inequity to racist inequity may not be obvious. In fact, the moral and legal justification for enslaving blacks derived from certain 16th c. CE papal bulls, Romanus Pontifex etc., collectively known as the "doctrine of Christian discovery", which justified the killing and enslavement of non-Christians. However, when black slaves in US turned Christian, this "moral" justification for slavery started floundering, and the system of slavery was justified by taking the color of the skin as an index to discriminate new Christians from old. (The Inquisition had earlier used similar quick visual indicators, such as dress, as an index of the orthodoxy of religious beliefs.) For more details about the doctrine of Christian discovery, and its current legal acceptance in US law, see Steve Newcomb, "Five Hundred Years of Injustice: The Legacy of Fifteenth Century Religious Prejudice", web article, based on article with the same title, *Shaman's Drum*, Fall 1992, pp. 18-20. See the website of the Indigenous Law Institute, `http://ili.nativeweb.org/sdrm_art.html`. For its consequences on history, see, C. K. Raju, *Cultural Foundations of Mathematics*, Pearson Longman, New Delhi 2007.

[22] Proclus, p. 29.

[23] For a quick account, see, C. K. Raju, "Religious roots of mathematics", *Theory, Culture and Society*, **23**, pp. 95–97.

[24] Clarence A. Forbes "Books for the burning" *Transactions of the American Philological Society* **67** (1936) pp. 114–25.

(or blind adherence to tradition). For example, in the case of al Khwarizmi's translation of Indian arithmetic ("Algorismus") texts at Baghdad, it has not been possible so far to identify any single Indian text of which it was a literal translation. Another well known case is that of the "Arabian Nights", "translated" from Pahalvi, but which acquired characters like Khalifa Haroun al Rashid. Clearly, what transpired in the Baghdad House of Wisdom was creative reworking, rather than literal translation, and this was not confined to "Greek" knowledge. This negates from the very beginning the extraordinary claim that the Arabs simply translated and carried forward an earlier tradition of knowledge that was exclusively Greek.

On the other hand, the Toledo translations are characterized by an extreme literalness. Where Latin equivalents were not available, Arabic terms (including *al kāyeda*!) were transliterated. The reason was that the Toledo translators did not know Arabic very well, and typically used Mozarab and Jewish intermediaries who translated from Arabic into an intermediate Romance language from which the "official" translators, such as Gerard of Cremona, translated into Latin. In this process there were some interesting translation errors. For example, as the OED tells us, the term "sine" derives from sinus meaning fold, from the Arabic *jaib*, meaning fold for a pocket. This was written as "jb" omitting the vowels, but was intended to be read as *jībā*, from the Indian term *jīvā* corresponding to the earlier Sanskrit *jyā* used for the chord. Possibly, the name "Euclid" was inspired by a similar translation error made at Toledo regarding the term *uclides* which has been rendered by some Arabic authors as *ucli* (key) + *des* (direction, space). So, *uclides*, meaning "the key to geometry", was possibly misinterpreted as a Greek name Euclides.

What, after all, is the evidence, nine centuries after Toledo, that a Greek named Euclid wrote the *Elements*? When I raised a similar query about "Euclid" on earlier occasions, it aroused angry responses and once a query: what is known about Euclid at the present moment? The late David Fowler answered succinctly: "nothing".[25]

Nevertheless, racist history has invested centuries of effort into that name, "Euclid" about whom this nothing is known, for it is this name which enables the extraordinary claim that mathematics is of Greek origin, whether as a race or culture. Further, the Greek-sounding name also makes it permissible to attach to it images of Caucasian features that can then be mass-marketed to the gullible through the Internet, and eventually permeate into school texts.

Thus, by introducing multicultural history in the classroom, one is secularizing it as is the only proper thing to do.

Making "Euclid" Theologically Correct

In the days of intense religious fanaticism in Europe, institutions like the Inquisition ensured that only the theologically correct survived physically: whether people or books. Merely attributing authorship of the text to an early Greek was inadequate. For example, many texts attributed to "Aristotle" were placed on the proscribed list by the Inquisition, on the grounds that they might spread heresy. Thus, in addition to attributing texts to a theologically correct author, the text itself had to be made theologically correct.

Aquinas and the schoolmen showed that the *Elements* could serve an important theological function. At that time, as the church hoped to expand among the wealthy Arabs, a key concern of the church was how to convert Muslims to Christianity. It was not clear how to do this, since the Muslims rejected the Christian scriptures which were the priest's main tool. However, as Adelard of Bath, one of the first translators of the *Elements*, remarked, Muslims accepted reason, while authority prevailed in Christian Europe. Adelard, who spent many years spying in the disguise of a Muslim student, gave a typical *aql-ī-kalām* argument: the mental faculty was given to man to be used. Even the traditionalist Muslims like al Ghazālī who staunchly opposed the *falāsifā*, accepted reason.

Al Ghazālī's books were among those translated at Toledo, and read by Aquinas, and it is well known that books by his opponent Ibn Rushd (Averroes) were the key texts for centuries in the first European universities. Seeing this consensus on reason, Aquinas and the schoolmen developed the Christian version of rational theology. In this version, reason was not the window to the soul (as Proclus thought) but was rather a universal means of persuasion—a weapon to be used by the theologian against the heathen, who accepted reason. Thus, the *Elements* came to acquire a stellar role in Christian theology. In this reinterpretation, all those things that were important to Proclus—figures, equity, eternal truths, learning as recollection, etc.—were rejected as inconsequential. The only thing of value in the *Elements* was taken to be deduction, for this was the only thing of value to the theologian. In this manner, "irrefragable demonstration" came to be associated with "Euclid", providing the perceptions which motivated the subsequent interpolation.

It was also this process of making the *Elements* theologically correct which led to mathematics being divorced from the empirical. The theological argument concerned creation. Thus, al Ghazālī allowed that Allah was bound by reason, but not by causes, so that although Allah could not create an illogical world, he could create a world of his choice at every instant, regardless of what had happened in the past. Hence, logic which bound Allah came to be perceived as stronger than empirical facts that did not bind God. (In present-day terminology, we would say that logical connections are necessary, while empirical connections are contingent.) Although the Christian doctrine of creation was a bit different, this also required God to create the world, so these perceptions of the relative strengths of logic and empirical facts persisted in Christian rational theology, as indeed they persist to this day in Western philosophy. Hence, it came to be believed that introducing empirical methods into a mathematical proof weakened it. This prepared the ground for the eventual rejection of the proof of the SAS theorem by

[25] `http://mathforum.org/kb/thread.jspa?threadID=381990&messageID=1175734`

Hilbert and Russell who changed it into a postulate as in current school texts. Equality was eliminated and replaced by "congruence" on the grounds that equality brought in the idea of superposition, bringing in the taboo feature of motion in space. As stated by Schopenhauer, motion is the subject matter of physics, while mathematics—and geometry, in particular—deals with motionless space. And, it was thought that this physical proof introduced contingent empirical features which weakened mathematical proof.

Although Western philosophy has long supposed this, there is, in fact, nothing universal about downgrading the empirical world, and elevating metaphysics above physics as the basis of knowledge: all Indian systems of philosophy, for instance, start from the opposite viewpoint taking the empirically manifest as the first means of proof. The insurmountable difficulty with the Western "metaphysics-first" approach is manifest: the 2-valued logic assumed to be universal in the West is neither culturally universal nor empirically necessary.

Pedagogical consequences

It is this process of aligning mathematics with theology that has made mathematics so difficult to understand.

The notion of "equal triangles" is easy enough to understand by a process of superposition. However, in Hilbert's synthetic geometry, not only is "equality" replaced by "congruence",[26] but superposition is disallowed, since it requires us to move a triangle in space. (Hence the geometry is called synthetic: for distances too cannot be picked and carried, so that no measurement is possible.) This is the system followed by school texts since the 1960's, following the recommendations of the US School Mathematics Study Group. However, it is extremely hard to explain to a child why there is something wrong about this natural process of superposition, and measurement, and why it should not be applied.

The problem arises from the inappropriateness of the theological view of the practical value of mathematics as an inferior appendage. For, if mathematics does have some practical value, why should one be so afraid of contamination by the empirical? However, the burden of the underlying theological difficulties has been passed on to the school child.

Currently, there is a half-hearted compromise: empirical methods of mathematical proof are restricted to pedagogy at the school level. That is, school children are taught mathematics in a way that is considered intrinsically wrong from the perspective of "higher" mathematics, just because that perspective simply cannot be imparted at the school level. This is a recipe for mass illiteracy in mathematics, for most children will never study that "higher" mathematics. What actually transpires is a bit worse, for school texts tend to be written by people regarded as experts in this "higher" mathematics—who permit empirical methods for pedagogical reasons, but keep trying to indicate the "incorrectness" of such "lower" mathematics, thus introducing a variety of subtle obscurities in the mind of the student. This albatross of theological correctness is too heavy a load for school children to bear.

Plato recommended the teaching of mathematics, like music, for the good of the soul. The Procluvian approach to geometry, as in the "Theonine" texts of the *Elements* like those of Todhunter, that used to be current in schools until the 1960's, had a certain intuitive charm, like music. This aspect of mathematics, however, has now been declared to be valueless: the mathematician is trained to mistrust intuition, for formal mathematics is all about the rigors of persuading others, and not about the joys of communing with oneself. The "value" attached to a formal mathematical theorem very often depends upon how counter-intuitive it is. The "thinking" of a computer of today represents the ideal of formal mathematical thought, and the difficulty of programming in low-level languages on the one hand, or the difficulty of making intelligent computers on the other hand, is a concrete indication of the gulf between formal mathematics and natural human thought-processes. Thus, the burden of theological correctness placed on mathematics by an inequitable theology has robbed mathematics of its practical value as well as its intuitive appeal.

The alternative is to look at the way mathematics developed in other cultures, as something practical and useful—as primarily a means of computation rather than persuasion. Once we have discarded the combined weight of an inequitable theology and a concocted history, it is easier to see that the exactitude that has been claimed of mathematics is as much of a chimera as its alleged certainty. For example, one may see inexactitude in mathematics from a fresh perspective such as that of *śūnyavāda*. This realistic Buddhist philosophy takes as its starting point the difficulty of representing a thing—anything. It is beyond the scope of this article to go into more details on Buddhist thought, but a couple of examples should illustrate what is meant. The problem of representation is made manifest by the difficulty of representing numbers (whether integers or real numbers) on a computer, for only finitely many numbers can actually be represented on any actual computer. Hence, computer arithmetic (or, indeed, any practical process of arithmetic) can never agree with formal arithmetic. From the point of view of *śūnyavāda*, the resulting "peculiarities" of computer arithmetic are not "errors of computation" but a natural state of affairs that cannot be avoided. To give another example, from this realistic perspective the dot on a piece of paper is real, it is the notion of an ideal geometrical point that is erroneous and empty.

The time has come to welcome such multicultural mathematics and firmly say good bye to Euclid.

[26] Hilbert's synthetic interpretation does not fit the actual *Elements*, where the notion of equality is applied to non-congruent ares after proposition 1.34. See, C. K. Raju, "How Should 'Euclidean' Geometry be Taught", in Nagarjuna G. (ed.) *History and Philosophy of Science: Implications for Science Education*, Homi Bhabha Centre, Bombay, 2001, pp. 241–260.