

Developing an alternative math curriculum at school level: geometry

A proposal to develop a textbook to teach geometry at about the 8th std level using techniques similar to the *sulba sutra*

C. K. Raju

ckr@ckraju.net

Abstract. We explain what is wrong with current methods of teaching geometry to (1) teach reasoning, (2) synthetic geometry, and (3) compass-box geometry. We also explain why the alternative of string geometry is superior. This project will (a) develop this alternative math curriculum, (b) test it through teaching experiments in village school, and (c) create a textbook which can be used by teachers to learn and teach it.

Preamble: geometry and deductive proof

A century ago, Bertrand Russell wrote: “it is...a scandal that Euclid is still being taught to boys in England.”¹ He said there were “countless errors” in the proofs of the first eight propositions of the *Elements*. In what sense are these errors? Those proofs are not pure deductive proofs from some axioms, but involve empirical elements. For example, the first proposition of the *Elements* is about constructing an equilateral triangle on a given base. It requires us to draw two intersecting arcs. The arcs are *seen* to intersect (an empirical proof) but the *Elements* has no axiom from which that intersection can be deduced. The arcs may not actually intersect: for example, if the arcs are drawn on a computer screen consisting of discrete pixels, for the two may have no pixel in common.

These errors are fatal to the claim that the *Elements* has deductive proofs, for they affect the whole book: the proof of the Pythagorean theorem (the penultimate theorem) in the *Elements* depends upon the 4th proposition. Consequently, the proof of the “Pythagorean theorem” in the *Elements* is just a prolix empirical proof and nothing special as had wrongly been claimed for centuries. Indeed, there was never any purely deductive proof of the Pythagorean theorem or any other theorem in the *Elements*. But the contrary myth is so widespread, that though this fact was admitted at the turn of the 20th c., it is still not widely known.

These errors demolish another myth: that deductive proofs are infallible. Human reasoning is *more* fallible than human perception and may easily mistake an invalid proof for a valid one. ALL the best minds in the West wrongly believed the invalid proofs in the *Elements* were valid deductive proofs. This collective error persisted for 8 centuries (from the 12th c., when the *Elements* first came to Europe, until the 20th c.) Such persistent errors are possible only with deductive proofs, not empirical proofs, so deductive proofs are more fallible than empirical proofs.

Further, if the best Western minds lacked a clear understanding of deductive proof for 8 centuries, how can the notion of deductive proof be taught even to our school teachers (leave alone students)? And *why* should it be taught? For those 8 centuries, the absence of a valid deductive proof made not the slightest difference to any practical application of geometry. So, why is deductive proof important? And why use geometry to teach it?

Deductive proofs make simple things extremely complicated. For example, the deductive proof of $1+1=2$ took Whitehead and Russell 378 pages in their *Principia*. Few, even among expert

1 Bertrand Russell, “Mathematics and the metaphysicians”, chp. 5 in *Mysticism and Logic*, George Allen and Unwin, 1917.

mathematicians, would be able to explain just what is there in those 378 pages. Russell himself believed it was a triumph that “modern mathematics” had been reduced to symbolic logic, and added, “symbolism is useful because it makes things difficult”. Actually, this complexity arises not because of symbolism but because “modern mathematics” prohibits the natural empirical way to understand integers by pointing to two apples, two dogs etc. To define an integer axiomatically one must take into account that there are an infinity of integers: so one needs to define the whole infinity of integers at once. That is, the formal definition of a simple thing like the integer 2 hence involves a complicated metaphysics of infinity. It is important to note that this complicated metaphysics of infinity adds nil to the practical value of integer arithmetic which was done for thousands of years before Peano and Russell, and continues to be used by common people today who never heard of Peano and Russell.

That metaphysics of infinity is the base not only of integers but of all present-day formal mathematics through set theory. Axiomatic set theory is so complicated that even most professional mathematicians skip learning it, and cannot define a set beyond parroting the NCERT “definition” that “a set is a collection of objects”. Even the head of an IIT math department could not define what a set is, when challenged to do so. That is, the demand for deductive proofs results in structures too complicated even for professional mathematicians. And, if even most expert mathematicians skip deductive proofs, at the base of mathematics, as too complicated to learn, what right do they have to advocate that geometry should be taught with a view to teach deductive proofs?

This metaphysics of infinity adds nothing to the practical value of any kind of mathematics, but is passed off as a “superior” way to do things. In fact, this metaphysics of infinity is NOT universal but is closely tied to church dogmas about eternity.² Even “expert” mathematicians, since they skip axiomatic set theory, and rarely delve into church theology, do not understand that this claim of “superiority” is calculated to induce propagation of church dogmas. The metaphysics of infinity tied to its dogmas of eternity may have been important for the church. But why is that important for an Indian student today?

Historically speaking, it was the church which first used geometry to teach a particular way of reasoning to its priests, from about the 12th c. In using geometry to teach reasoning (or deductive proof) we are imitating that religious curriculum, called quadrivium. Recall that church theology was changed at that same time by Aquinas and his schoolmen, to the new Christian theology of reason which glorified “reason”. This new theology was copied and adapted from the Islamic theology of reason, with a view to persuade Muslims to convert, for they could not be converted by force, or by appealing to the Christian scriptures, but did accept “reason” or aql (not the same as mechanistic reasoning). Apart from the “Aristotelian” syllogism, the only book then known to the church which provided a model of such reasoning was the *Elements*. Hence, that geometry was taught as part of the religious curriculum to teach “reasoning”. “Euclid” was concocted as the author of the *Elements* so that the church could claim this “reason” as its legacy.

This doctrine of deductive proof was put to political use by church/racist/colonial historians: they incessantly boasted for centuries that the Greeks did something “superior” and special in mathematics claiming that only Greeks understood and used pure deductive proofs, As we have seen that claim is historical bunkum: the *Elements* did not in fact have valid deductive proofs. Indeed, the long-standing error of 8 centuries was further compounded by Russell who himself erred in imagining that Euclid made an error in executing his intention of deductive proofs: for there is nil serious evidence for the

2 C. K. Raju, “Eternity and Infinity: the Western misunderstanding of Indian mathematics and its consequences for science today.” *American Philosophical Association Newsletter on Asian and Asian American Philosophers and Philosophies* 14(2) (2015) pp. 27-33. Draft at <http://ckraju.net/papers/Eternity-and-infinity.pdf>.

existence of Euclid and even less for his purported intention of deductive proof. On the contrary, there is plenty of solid counter evidence that the book *Elements* was a religious book intended to “arouse” the soul, on the Platonic and Neoplatonic view of geometry.³ As Proclus explained, reason helped to arouse the soul since it turned the mind inward (like yogic meditation). This involved a “pagan” notion of soul, long cursed by the church, so the book *Elements* was reinterpreted to disconnect reasoning from the soul. Further, reasoning too was reinterpreted as purely a tool to persuade others, a tool manifestly useful for missionaries.

Setting aside history: why exactly are deductive proofs “superior”? This sort of reasoning greatly suited the church for a little known reason: **anything at all, absolutely any nonsensical proposition whatsoever (such as “there exists a rabbit with three horns”), can be deductively proved as a mathematical theorem** starting from some equivalent propositions as postulates. That fact itself shows that a deductive proof is *not* valid knowledge and indeed provides nil indication whether the theorem proved is valid knowledge.

Even formalists admit that a deductively proved theorem is NOT valid knowledge, and claim only that it is “relative truth”, truth relative to postulates. As Russell put it, “mathematics consists entirely of assertions to the effect that, if such and such a proposition is true..., then such and such another proposition is true...” For example, the “Pythagorean theorem” (even if deductively proved) is NOT valid knowledge and could not be applied to navigational triangles to solve the practical problem of determining longitude, for it does not apply to triangles drawn on the curved surface of the earth.⁴

Further, there is another deep seated trick here. Since formal mathematics prohibits any reference to the empirical, the postulates are entirely metaphysical. As such, *there is no way to verify or refute the truth of any mathematical postulates*. As Russell put it, “It is essential **not** to discuss whether the first proposition [the postulate] is really true...”! For example, we cannot say there is any place in the real world where Euclidean geometry exactly applies. So, there is no way ever to know whether formal mathematical theorems are valid knowledge. As Russell put it, “in mathematics, we never know whether what we are saying is true” (and don't care).

Naturally, such a method of reasoning, totally disconnected from anything empirical, which enabled one to arrive at any pre-desired conclusion whatsoever, greatly suited the church. One could go on endlessly discoursing in a “superior” way on, say, virgin birth and its consequences for the doctrine of sin, all the time maintaining like Russell that it is “essential” *not* to consider whether the postulate of virgin birth is really true and applies to the real world, either exactly or even approximately. This philosophical trick can be combined with additional psychological tricks. One could slip in a plausible postulate unnoticed in the middle of a long and complex argument, or just brazenly postulate that virgin birth was a miracle. *Hence*, geometry was used to teach this way of reasoning to priests, for it helped the priest to talk any nonsense and claim that it was supported by Divine Reason (as Aquinas called it).

But why should we teach this sort of reasoning to our students? What is instead required is that, as in science, the postulates or their consequences *must* be verified empirically as the critical step.

3 C. K. Raju, *Euclid and Jesus: How and why the church changed mathematics and Christianity across two religious wars*, Multiversity, 2012.

4 Incidentally, even the story of non-Euclidean geometry is a myth: as noted by Brahmagupta, a thousand years before the official advent of non-Euclidean geometry, without precise knowledge of the radius of the earth, longitude calculations (with such triangles) are futile. ब्राह्मस्फुटसिद्धान्त 11.15-16. “भ्रूव्यासस्याग्यानाद् व्यर्थं देशान्तरं” (“ignorance of the radius of the earth makes longitude calculations futile”).

Mathematics, like science, should be taught for its practical use, and it is useful only in so far as it relates to the empirical, which is regrettably excluded from formal mathematics.

As a postscript, it is not even true that the theorems of formal mathematics are truths relative to the postulates. Reasoning depends upon logic. Present day mathematics exclusively uses 2-valued logic. But this logic was NOT universal as Aquinas, Kant, and Russell wrongly and naively believed. They were simply ignorant of the existence of multiple systems of logic, as in pre-Buddhist Indian tradition, Buddhist *catuskoti* and Jain *syadavada*.⁵ So the tall claims of certitude in logical proofs amount to nothing. Logical proofs are NOT certain even relative to the postulates. How can they be when logic itself is not certain? In fact logic itself must be determined either by fallible empirical proofs or on even more fallible cultural grounds. The certitude of logical proofs is just a false myth deeply internalised by the colonised mind.

Colonial education and the church

Anyway, the sorry fact is that the Indian education system has mimicked the church education system ever since colonialism. Though the colonially educated are loathe to admit it, and do their best to hide the connection of colonial education to church education, the fact is that Western education was 100% church education, when it first came to India, in a big way in 1857, and long before that through mission schools. That education system was manifestly brought to India by missionaries. This church education, since the first Western universities in Bologna, Paris, Cambridge, Oxford, was of the church for the church and by the church. It was totally controlled by the church and designed for church purposes to produce a missionary mindset or the captive mind of a missionary. Keeping the mind captive helped to contain revolts—the reason why Macaulay recommended that church education for India.⁶

Some key characteristics of the missionary mindset are the following. (1) An abundance of myths (such as the myths of Euclid, Pythagoras, and other mythical Greeks), acquired through childhood indoctrination (as today taught through NCERT school texts⁷), hence without ever cross-checking the evidence (when challenged, NCERT could not point to the primary historical evidence for these myths). (2) An ignorance of the basics (such as the definition of a set) so that the educated were forever unsure of their knowledge, and learn to proceed on guesswork, guided by myths. (3) A great faith in and reverence for church authorities, glorified as saints. (Colonialism replaced this by great faith in and reverence for Western authorities glorified as heroes.) When guesswork fails, the colonised/missionary mind appeals to authority, and follows the Wikipedia principle of trusting only Western authority. (4) The total faith in the West is complemented by extreme distrust of the non-West, resulting in extreme insularity. Typically, any critique of the West, such as this one, is generically met by privately denouncing and slandering the critic without ever publicly engaging with the critique.

It should be emphasized that the myths are not only about people. Those myths are implanted with a view to guide the guesswork-thinking of the ignorant, and make them reject commonsense. For

5 C. K. Raju, “Logic”, article in Springer *Encyclopedia of Non-Western Science, Technology, and Medicine*, 2008, 2014. <http://ckraju.net/papers/Nonwestern-logic.pdf>.

6 C. K. Raju, “Education as counter-revolution” online comment in *Frontier Weekly*, edited and republished as an article “Education and Church: Decolonising the hard sciences” in *Frontier Weekly* 46(7) 25-31 Aug 2013. <http://ckraju.net/papers/Education-and-counter-revolution.pdf>.

7 The result of this childhood indoctrination is that Russell did not cross-check the claim of Euclid and his purported intentions, and unwittingly furthered the church agenda, despite being an atheist.

example, consider the myth that there is beauty or aesthetics in math. Without getting into the origins of this myth in the Platonic religious belief that mathematics arouses the soul, like music, we have to only observe the fact that millions of students today love music but hate math. The reaction of students is commonsense proof that there is no aesthetics in present-day math. Indeed, formal math is fundamentally different from Platonic math; it is soul-less, both literally and metaphorically, so why should one expect any beauty in it? But the colonised mind rejects commonsense, and just blindly accepts whatever some myths and authorities say.

The rejection of commonsense is easily explained away by facile apologias. To save the myth of beauty in formal math, the mass experience of students is often explained away by blaming the collectivity of students and teachers. This is a trivial and irrefutable way to escape from the harsh fact of the ugliness of formal math, which originates in the Western religious curriculum.

The solution is to change the very mathematics that is taught, not only the *way* mathematics is taught. Geometry should be taught for its practical use, and not to indoctrinate children into a particular way of reasoning, divorced from the empirical, which suited the church.

Synthetic geometry

There is a second problem with the teaching of geometry in schools today. Even if we allow the dubious teaching of formal geometry, there are many different sets of postulates one could begin from. In principle, we are free to use whatever postulates we want. As Russell (cited earlier) put it, “We ...take any hypothesis that seems amusing, and deduce its consequences”.

In practice, however, school education is *not* a matter of amusement. We are NOT free to begin with any postulates we like. The unwritten rule is that the postulates we use to teach mathematics in schools must have the approval of authoritative Western mathematicians. For example, even formal calculus could be better taught using a formal arithmetic which is non-Archimedean.⁸ But all universities across the world use only formal real numbers (with the Archimedean property) as approved by the West.

However, as regards the formalisation of “Euclidean” geometry, there are at least two choices, both originating from authoritative Western mathematicians. One may use Birkhoff's metric axiom set⁹ or one may use Hilbert's synthetic axiom set.¹⁰ Axiomatic metric geometry should not be confounded with empirical length measurements, but it has a similar effect: it makes the proof of the Pythagorean theorem very easy. In effect it trivialises the book *Elements* (if regarded as a book about deductive proof). This is fatal to the myth of “Euclid” and his “superior” proofs: for it means that the book *Elements* was needlessly prolix, and made a simple thing enormously complicated. On the other hand, Hilbert's synthetic geometry is designed to make the proof of the “Pythagorean theorem” more difficult. “Synthetic” means length measurements (whether axiomatic or empirical) are disallowed. So, which geometry should we choose?

Now myths are important for the West, for it has long invested in the claim of superiority based on those myths. The church too is deeply invested into the theology of reason. Therefore, a need is felt to save those myths from refutation. The clear discrepancy between the actual book *Elements* (which has

8 See, for example, my talk at MIT, “Calculus: the real story”, [abstract](#), [video](#), [presentation](#), [blog](#). Or see the Appendix to my *Cultural Foundations of Mathematics*, Pearson Longman, 2007.

9 G. D. Birkhoff, “A set of postulates for plane geometry (based on scale and protractor)”, *Ann. Math.* **33** (1932).

10 D. Hilbert, *The Foundations of Geometry*, trans. E. J. Townsend, Open Court, La Salle, 1950.

no deductive proofs) and the church myth about it (that it was written to demonstrate deductive proofs) could have been resolved differently: by just dismissing the myth as absurd and patently contrary to facts. However, that did not happen. Despite preaching the merits of scepticism, Russell did not display the slightest scepticism about the myth of Euclid or his intentions. He did not cross check the evidence. Instead, both Russell and Hilbert wrote books on the foundations of geometry with a view to rewrite the *Elements* to force the (rewritten) book to fit the myth about it. Eventually, in 1961, the Yale School Mathematics Study Group¹¹ recommended the teaching of Hilbert's synthetic geometry, for it was more useful to save the myth of “Euclid” than Birkhoff's metric geometry.

The cardinal dharma of the colonised is blind imitation of the West, so this recommendation was adopted in Indian school texts since about the 1970's. However, is that OUR motive for teaching geometry to students? Just to save the crumbling and false myth of Euclid? Are students informed that an easier alternative is available (even as regards formal geometry) but our educators rejected it for reasons that they were unable to state clearly for the past 50 years.

The teaching of synthetic geometry creates various problems. The compass-box (which has a ruler) is NOT compatible with synthetic geometry which disallows a ruler whether real or axiomatic. It is never explained to students that they are taught two distinct and opposing systems of geometry: non-empirical, and synthetic geometry, and the empirical and metric geometry of the compass box.

The confusion between the different systems of geometry is manifest from the term “congruence”. Hilbert replaced the original term “equality” in the *Elements* by the new term “congruence”. Equality of lengths is an empirical and metric matter: it typically involves moving a ruler about in space to compare two lengths. Since it is empirical it is disallowed in formal geometry. Further, since even axiomatic metric geometry kills the myth of Euclid, it is disallowed with Hilbert's synthetic postulates.

Most (if not all) teachers and students would be hard put to differentiate between the two notions of equality and congruence: they would be unable to explain congruence without appealing to equality. Probably even most teachers don't understand the difference. Teaching “Euclidean” geometry was no doubt a scandal, but teaching synthetic geometry today is a bigger scandal. And it is a still bigger scandal to teach it along with the use of the compass box without explaining the difference.

A final note: the actual situation with Hilbert's synthetic geometry is even more hilarious than appears at first sight. The “Pythagorean theorem”, as stated in the *Elements*, is a proposition about the equality of two *incongruent* areas. Hence, to be able to prove it, Hilbert needs to define area even though he does not define length! Are students informed about the mental gymnastics involved in this way of doing geometry by defining area but not length? Can teachers explain it to students? Do they even know about it? And what exactly is *our* motivation for teaching this stuff, invented to hide a Western blunder?

Compass-box geometry

To summarise, in teaching geometry to teach reasoning, we are blindly indoctrinating children into a church method of using metaphysical reasoning, which provides the ability to “rigorously” prove any nonsense proposition, even if contrary to commonsense. In teaching Hilbert's synthetic geometry (and congruence) we are teaching a way to preserve the myth of “Euclid” and his supposed intention of

11 *School Mathematics Study Group: Geometry*, Yale University Press, 1961.

deductive proof, a myth for which there is not the slightest evidence, and plenty of counter-evidence.

However, since geometry cannot be all myth and metaphysics, school geometry today also teaches a third kind of geometry: viz the geometry of the compass box. The ritualistic elements of the compass box geometry are manifest from its redundant features: the set squares which are never used, or even the divider which is used only to make holes. Once again this demonstrates the sorry way in which our educationists have been blindly copying the West without ever daring to apply commonsense even as regards such a simple matter.

Worse, the practice of teaching a combination of two incompatible types of geometry (synthetic geometry plus compass box geometry) creates conceptual confusion about basic geometric notions such as angles. An angle is still defined (in NCERT texts) in the way of the *Elements* as something related to two straight lines (or rays): “when two straight lines meet at a point they are said to form an angle”. This is a bad definition for many reasons.

When we measure an angle, what exactly is it that is measured? The two-straight-lines definition of angle leads to all sorts of subtle doubts which may or may not be articulated by students. When a degree is defined as the 90th part of a right angle what exactly is it that is divided into 90 equal parts: the space between the two lines!? If so, how does one divide infinite space equally? Given the paradoxes of infinity, could not there be various ways of doing that? Why select a particular metaphysics of infinity?

The actual operational way in which school students are today taught to measure an angle is to use the protractor in the geometry box. The protractor is taken as a given and students neither construct one nor is any explanation given for its construction. If they do try to construct it themselves, several questions arise. Why is the protractor semicircular in shape? If we measure one and the same angle with two protractors of different sizes, will we get different answers? Ask any student: would a larger-size protractor lead to a different angle measure? If not, why not?

As is clear from the last question, explaining that the angle measured does not change with the size of the protractor requires a (metric) property of the *circle*, viz. that the lengths of its circumference and radius are in constant proportion. Let us set aside the question of how it could possibly be established in synthetic geometry which permits no empirical verification. It cannot even be established empirically in compass box geometry because the compass box has a key defect: it has no instrument in it for measuring the length of a curved line. Indeed, in the usual way of teaching math, the length of a “rectifiable” arc is first defined only in a calculus course. All the known ways of defining the length of a curved line axiomatically involve a complicated metaphysics of infinity. Thus, there is no way on existing school geometry that we can even explain to a student what an angle really is.

Even more confusing is the radian measure of angles, since no instrument is provided in the compass box to measure angles in radians. The conversion factor π cannot be expressed exactly, but students are taught the myth that mathematics is exact, and the further myth that this exactitude is possible only through a particular metaphysics.

The alternative: string geometry

So, what is the alternative? Clearly the simple alternative to formal math is normal math! $1+1=2$ can be understood in a simple empirical way as children actually learn by pointing to tangible things.

Acceptance of the empirical as a superior means of proof is the first step to replace formal math by normal math. Equality of two triangles is something very easily understood empirically by putting one triangle on top of another and inspecting the result. There is no need to teach congruence.

This empirical way of doing arithmetic or geometry is no longer treated as mere pedagogical simplification to be first taught to children and later denounced as erroneous and substituted by the “rigorous” 378 page proof. Rather, it is the claim of “rigour” in formal math, or the claim of “superior” deductive proof, which is declared erroneous and permanently rejected. Deduction is not being outlawed, it may, of course, continue to be used, but *strictly in combination with empirical proof*, as in science, with the clear understanding that experiment and empirical proof is the ultimate means of proof.

We should not teach church metaphysics or its consequence to children, but should focus instead on the practical applications of numbers and geometry. All practical applications involve the empirical, so no practical application will ever suffer because of an empirical approach. Practical applications need metric geometry. Synthetic geometry was just a retrospective superposition of metaphysics, unfit for young minds who do not understand its complex political motives. Empirical geometry is a clear alternative to both deductive and synthetic geometry.

There is an alternative also to compass box geometry. Traditionally, Indians worked with curved lines¹² as in the *sulba sutra*. Practical manuals for masons, the *sulba sutra*-s used a *sulba* or flexible string to measure curved lines, and straightened it to measure straight lines. This elegant and practical method of doing geometry was not unique to India, and is found also in Egypt where a cord was used, along with a good value of π to calculate the area of a circular field or the volume of a cylindrical granary. However, the *sulba sutra* is the most detailed surviving description of that geometry we have. Unlike the instruments in a compass box, a string or cord is a practical instrument, which can actually be and is used to measure the area of an agricultural field.

To reiterate, the string is a very versatile instrument, which can be used in various ways.

- (1) Since it is flexible, it can be used to measure the length of curved lines, impossible with any instrument in the geometry box. Further, unlike the geometry box, which is suited only for geometry done on paper, a cord or rope can be used even in the field, thus allowing the study of geometry to be related to real-life problems, as is done even in the Rhind papyrus.
- (2) By keeping it taut, a string can be used to measure straight lines.
- (3) By keeping it taut and keeping one end fixed it can be used to draw a circle.
- (4) By drawing two intersecting arcs (as in the traditional fish figure or the first proposition of the *Elements*) one can draw right angles. (In general, by virtue of (2) and (3) one can construct anything which can be constructed with straight edge and compass on formalist theory.)
- (5) By keeping two points fixed, a string can be used to draw an ellipse impossible to do with any instruments in a geometry box. Ellipses are important for astronomy.
- (6) Lengths can be standardised by using knots or even a printed measuring tape.
- (7) Precise measurement (more precise than the scale in a compass box) is possible with a pair of knotted strings (or a pair of specially printed measuring tapes) by using the two scale principle (wrongly called the Vernier principle).

12 C. K. Raju, “Towards Equity in Math Education 2. The Indian Rope Trick” *Bharatiya Samajik Chintan* (New series) 7 (4) (2009) 265–269. <http://ckraju.net/papers/MathEducation2RopeTrick.pdf>.

- (8) “String technology” can even be used to measure real life angles, and not merely angles drawn on paper. (Recall that this was used in the traditional navigational instrument¹³ called the *kamal* or *rapalagai* by the navigator who brought Vasco da Gama from Africa to India. Recall also that it achieved high accuracy through the two scale principle for two harmonic scales. Recall that this ability to measure real-life angles is important also for celestial astronomy, linked to the Indian calendar needed to determine the monsoons on which Indian agriculture still depends.)
- (9) This ability to measure real-life angles, when combined with precise sine values, allows us to solve simple practical problems like measuring the radius of the earth,¹⁴ and determine both local latitude and longitude (and localise the calendar for the local place).
- (10) By far the biggest advantage of string geometry is that it leads to great conceptual clarity about basic geometric concepts, such as angle, π , sine values etc.

In particular to return to the basic geometric concept of an angle. An angle is actually the length of a curved arc. A string provides a way to measure the length of curved arcs, so an angle is naturally defined as the length of an arc of the circle. The related property of a circle—that its diameter and circumference are in constant proportion—can be verified empirically using a string to measure the circumference. Hence, also, both degree and radian measure are very easily understood as the length of an arc, in units of the circumference or the radius.

Incidentally, that was how angles were historically defined, as the length of a curved arc. That historical legacy still persists in the curved arc commonly used to depict an angle, but which symbolism is rarely explained. That historical legacy also persists in the definition of a degree as the arc traced out by a star (or the celestial sphere), and the measure of 360 degrees relates to the year, the calendar, and agriculture as is rarely taught.

There is another big advantage of string geometry which is little noticed, because the discourse is clouded by the constant chest-thumping about Western superiority, and its reiteration by faithful colonised followers. Thus, on the Western story of geometry, the great achievement of Western geometry was the *proof* of the Pythagorean theorem (though nothing is known about Pythagoras, or what proof he gave). However, while metaphysical proof was important to the church, which was only interested in persuasion, **real mathematics is about practically useful calculation**, something which children immediately understand.

A mere proof (real or imagined) of the Pythagorean theorem is of little value without an accompanying method of *calculation*. Specifically, one needs a method of calculating (1) the diagonal from a knowledge of the two sides or (2) the two sides from a knowledge of the diagonal and one angle. (I say diagonal and not hypotenuse, because in all early non-Western traditions—Iraqi, Egyptian, Indian—from long before the purported time of “Pythagoras”, the proposition is stated in relation to the diagonal of a rectangle.) The calculation (1) requires a way to *calculate* (not merely define!) square roots. The calculation (2) requires a way to calculate (not merely define) sine values for a variety of angles. All three ancient traditions provide precise values of $\sqrt{2}$. Indeed, the *Manava sulba sutra* (10.10) explicitly states the “Pythagorean proposition” using square *roots* (and not merely squares). Indian tradition certainly had precise sine values from before Aryabhata, and very precise sine values from his time (5th c.) onwards.

When the *Elements* first came to Europe it was impossible for the West to state the Pythagorean

13 C. K. Raju, “Kamal or Rapalagai”, In: *Cultural Foundations of Mathematics*, cited above, chp. 5.

14 *Cultural Foundations of Mathematics*, cited above, appendix to chp. 5.

proposition in this way. Indeed, until the 12th c., the West did not understand even basic arithmetic algorithms for addition, subtraction, multiplication, division, for the West was still tied to the abacus, used with Roman numerals. The abacus provides no way to express or calculate $\sqrt{2}$. Therefore, neither square roots nor sine values were known to the West until the 12th c. This is clear from the very words in current use, which relate to the Toledo mass translations of Arabic texts into Latin, organized by the church the 12th c.

Thus, the term for $\sqrt{2}$, still used in school, is surd. The word “surd” derives from the Latin *surdus* meaning deaf. That was a 12th c. Latin translation from Arabic from Sanskrit. Deaf comes from “bad ear”, a wrong translation of “bad *karna*”, for while the Sanskrit word *karna* means both diagonal and ear, in the context of $\sqrt{2}$ the intended meaning was diagonal not ear! Similar remarks apply to the term sine, from the Latin *sinus* meaning fold which is a translation of the Arabic *jaib* (pocket), a misreading of *jiba* from the Sanskrit *jiva*.

These linguistic errors were accompanied by conceptual errors. For example, as the very term *jya* or *jiva* shows, the concept of sine relates to a chord, hence to the circle and its measurement, and not merely to triangles as currently taught. The conceptual error is clear from the very word “trigonometry” (measurement of triangles). The usual trigonometric definition of sine (opposite side upon hypotenuse) is inferior for it gives us not the slightest clue about how to calculate sin 1. It is also 1500 years behind the times, for Aryabhata had developed a method to do that. Indeed, having imported this knowledge and not developed it, the West was unequipped for the underlying philosophical problems which do not arise on the primitive abacus.

The philosophical problems are immediately manifest, for if we try to calculate $\sqrt{2}$ using an algorithm (earliest found in the *Aryabhatiya*, and the same as the one taught in schools today) we find a continued fraction or an infinite decimal series expansion: $\sqrt{2} = 1.414... =$

$$1 + \frac{4}{10} + \frac{1}{100} + \frac{4}{1000} + \dots$$

If we stop after a finite number of terms (the only practical possibility)

and square it back we get only 1.999... and never 2. That is, we can compute only an *inexact* square root of 2. This situation is described in the *sulba sutra* as सविशेष¹⁵ (*savisesa*, with a remainder). A similar problem arises with squaring the circle or the value of $\pi = 3.1415...$. In the *sulba sutra* this is described as अनित्य¹⁶ (*anitya*, impermanent). Likewise, Aryabhata describes his value of π as आसन्न¹⁷ (*asanna*, near value). This acceptance of inexactitude in calculations is part of normal mathematics. This is captured and made explicit by the realistic philosophy of zeroism.¹⁸ We need to teach in schools that inexactitude (as in computer calculations) is inevitable in all mathematical calculations, instead of teaching the religious belief that mathematics is exact and eternal truth.

Relation to future courses

A geometry course does not exist in isolation. We also need to take into account how the course will relate to the future teaching of mathematics. In fact, the new philosophy of zeroism has already been used in various pedagogical experiments to teach calculus and “trigonometry”. The two courses, string geometry and calculus, are directly linked. (Recall that calculus developed in India as an outgrowth of

15 Baudhayana *sulba sutra* 2.12

16 Apastamba *sulba sutra*, 3.2

17 गणित 10.

18 C. K. Raju, *Cultural Foundations of Mathematics*, Pearson Longman, 2007. Also, “Zeroism”, article for *Encyclopedia of Non-Western Science, Technology, and Medicine*, 2014, <http://ckraju.net/papers/Springer/zeroism-springer-f.pdf>.

string geometry and Aryabhata's method of calculating 24 sine values by numerically solving a differential equation.) This new philosophy makes calculus so easy that the contents of the typical fat calculus text of 1300 odd pages (in double column and small type) can be taught in 5 days.¹⁹ Further, because it makes calculus so easy, it enables students to solve harder problems.²⁰ As such the proposal to teach string geometry with zeroism will help the student acquire a better conceptual and practical grasp of trigonometry and calculus.

As for applications to science, this way of doing calculus very well fits into the way all practical applications of the calculus are actually done today on a computer, using floating point numbers instead of real numbers. The only difference is that instead of declaring the practical inexact calculations as erroneous, inexactitude is accepted as inevitable in zeroism, and it is the metaphysical real numbers and claims of certitude and exactitude which are rejected as erroneous.

There is another reason for rejecting “real” numbers. Calculus actually developed in India with a non-Archimedean arithmetic, and a method of discarding infinitesimals. That allows calculus with this new philosophy to be applied in situations where university calculus with real number fails (e.g. when discontinuities such as shock waves arise with nonlinear partial differential equations, especially in general relativity, where there is no statistical mechanics to fall back upon). Because of this failure of university calculus, alternatives to the university calculus such as the Schwartz theory of distributions have been proposed, which, too, fail, e.g. in the renormalization problem of quantum field theory. Calculus with zeroism and non-Archimedean arithmetic can be readily applied to this situation as well.²¹

Incidentally, correcting Newton's misunderstanding of calculus, leads also to a better theory of gravitation.²² That is, the agenda is not merely an easier way of doing calculus, but a better way of doing mathematics, better not only in the sense of greater conceptual clarity, but also in the sense that it leads to a better science. String geometry is the first step in this chain which leads naturally to trigonometry and calculus.

Aim

What is required as a first step is a text-book on string geometry to enable this model to be replicated and taught across diverse groups. The aim of this proposal is to prepare such a text at about the 8th standard level, where the students already have some exposure to area calculations, square roots and π .

Timeline

The preparation of the text would be done in three stages followed by field trials. In the first stage,

19 C. K. Raju, “Teaching mathematics with a different philosophy. Part 1: Formal mathematics as biased metaphysics.” *Science and Culture* 77 (7-8) (2011) pp. 274–279. <http://www.scienceandculture-isna.org/July-aug-2011/03%20C%20K%20Raju.pdf>, arxiv:1312.2099. “Teaching mathematics with a different philosophy. Part 2: Calculus without limits”, *Science and Culture* 77 (7-8) (2011) pp. 280–85. <http://www.scienceandculture-isna.org/July-aug-2011/04%20C%20K%20Raju2.pdf>, arxiv:1312.2100.

20 For an actual school project along these lines in 12th std, see Suvrat Raju, “Pendulum project”, <http://ckraju.net/11picsoftime/pendulum.pdf>. For the anecdotal details listing the bad science teaching with the first serious scientific experiment even in our best schools see, C. K. Raju, “Time: what is it that it can be measured?” *Science & Education*, 15(6) (2006) pp. 537–551. Draft available from http://ckraju.net/papers/ckr_pendu_1_paper.pdf.

21 See MIT talk, already cited, and references cited therein.

22 C. K. Raju, “Functional Differential Equations. 4: Retarded gravitation”, *Physics Education* (India) 31(2) April-June, 2015, [http://www.physedu.in/uploads/publication/19/309/1-Functional-differential-equations-4-Retarded-gravitation-\(2\).pdf](http://www.physedu.in/uploads/publication/19/309/1-Functional-differential-equations-4-Retarded-gravitation-(2).pdf).