Proofs and Refutations in Mathematics and Physics: an Indian Perspective

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1 Introduction

Mathematics, today, is regarded as characterized by proof. This at least is the opinion of those who are engaged in teaching and research in universities and other centres of higher learning, and who would regard an ordinary carpenter, say, as an illiterate person not socially authorized to give an opinion on the nature of mathematics. The socially recognized mathematician would neither seek to nor be able to convince the carpenter that mathematics actually relates to proof and not to practical calculation. The opinion of the carpenter would simply be discarded as valueless, for, in this perception of mathematics, the nature of mathematics cannot be decided in any way that is democratic or equitable—the nature of mathematics can only be decided by those socially regarded as knowledgeable *according to the currently socially dominant definition of mathematics*!

Physics, on the other hand, is regarded as characterized by refutation, or refutability, according to Popper's well-known criterion, better known by the word falsifiability, the initial use of which word Popper later put down to his inadequate knowledge of the English language. At any rate this criterion of refutability aims to demarcate physics from non-physics or metaphysics, and, incidentally, mathematics. As we will see later on, ironically, this criterion of refutability assumes a key Western belief about mathematics, namely that *mathematical proof is infallible* and incorporates necessary truth, *just because* mathematical proof does not depend upon the empirical.

1.1 "Aristotle" on the difference between mathematics and physics

Such a distinction between mathematics and physics has been customary in Western thoughts ever since the earliest recorded 11th c. CE Western [actually Arabic/Byzantine Greek] texts attributed to Aristotle. There is, of course, no independent evidence, apart from these late texts, to say whether this idea was part of the thinking of early Greeks—whether the "Aristotle" in question refers to the individual who received a few of the large number of books looted by Alexander from Egyptians, Persians and Babylonians, or to a group of people who later translated those books, or even whether these ideas attributed to Aristotle originated much later, as seems likely, in Islamic rational theology. Some of the articulations in the text itself suggest that these parts of the text reflect a state of knowledge appropriate to the Arabic world of 11th c. CE, and completely inappropriate to the Athens at the time of Aristotle, but this dating or location of the idea is not critical on account of the considerations below, which identify the real beginnings of Western thought about physics and mathematics in 13th c. Christian rational theology. At any rate, according to this generic "Aristotle", or the translators or author(s) of the text called *Physics*:

> The next point to consider is how the mathematician differs from the physicist. Obviously physical bodies contain surfaces and volumes, lines and points, and these are the subject matter of mathematics. Further, is astronomy different from physics or a department of it? It seems absurd that the physicist should be supposed to know the nature of sun or moon, but not to know any of their essential attributes, particularly as the writers on physics obviously do discuss their shape also and whether the earth and the world are spherical or not.[1]. . . Now the mathematician, though he too treats of these things. . . does [not] consider the attributes indicated as the attributes of physical bodies. That is why he separates them, for in thought they are separable

[1] This is an example (a mere example) of a statement which is more appropriate to 11th c. Arabs than to Athens, where a death penalty was sought for Socrates against the allegation (denied by Socrates) that he taught that the sun and moon were not gods, but that the moon was merely a clod of earth. Thus, Aristotle's reference to earlier writers on physics is not consistent with Plato's *Apology*, which provides a more realistic account of an Athens from which Aristotle himself was forced to flee for the same reasons.

from motion. . . How far then must the physicist know the form or essence? Up to the point, perhaps, that the doctor must know sinew or the smith bronze (i.e., until he understands the purpose of each); and the physicist is concerned only with things whose forms are separable indeed, but do not exist apart from matter.[2]

1.2 The schoolmen's limited understanding of mathematics

The schoolmen like Aquinas were trained only in geometry, but were ignorant of calculations requiring arithmetic, or trigonometry, which were not part of their curriculum (until the 16th c. CE reform of the Jesuit mathematical syllabus by Clavius). The best known demonstration of this ignorance of elementary arithmetic is found in the case of Gerbert (became Pope Sylvester II in 999 CE) who wrote an authoritative book on the abacus, called "Rules for Computations with Numbers", but completely failed to understand the Indian numerals, imagining that all that was needed was that the symbols for the Indian numerals should be put on the counters used in the abacus!

> Yet it would be wrong to see in the *apices* nothing more than a trivial innovation introduced by Gerbert. The truth is that he did adumbrate the use of the new numerals; he had heard marvelous things about the new computation which they made possible but which he, and perhaps also his informants, did not essentially understand.[3]

Since the schoolmen's education did not equip them to understand even elementary arithmetic calculations, and since they did learn the *Elements* it was natural that they conflated mathematics with geometry (and, in fact, conflated mathematics with an abstract model of theological argument). The priest's job was to argue and persuade people to believe in absolutely fantastic things that no person in his senses could possibly believe—therefore, the schoolmen naturally and systematically ran down the evidence of the senses, placing metaphysics above physics, and valuing there own books and rules

[2] *Physics*, Great Books of the Western World, vol 7. *Aristotle: 1*, trans. R. P. Hardie and R. K. Gaye, Encyclopaedia Britannica, Chicago, 1990, pp. 270–71.

[3] Karl Menninger, *Number Words and Number Symbols: A Cultural History of Numbers*, trans. Paul Broneer, MIT Press, Cambridge, Mass., p. 325.

above the evidence of the senses. There was no other way their political objective could be achieved.

Accordingly, the schoolmen interpreted this passage from "Aristotle" ("in thought...separable from motion") to mean that mathematics related to motionless space, the object of geometry, while physics related to motion. At least, this was the interpretation on which there was a consensus among Western philosophers at the beginning of the twentieth century, especially among influential writers on the foundations of geometry like Russell and Hilbert.

1.3 The present-day Western philosophy of mathematics

In fact, both these worthies began on the foundations of mathematics by writing extensive tomes on the foundations of geometry.[4] The entire dispute was about the 4th proposition in the *Elements*. The proposition asserts that if two triangles have two sides and included angles equal then the two triangles must be *equal*. This is known today to school children[5] as the Side-Angle-Side (SAS) *postulate*, with the further change that *equality* is replaced by *congruence*. The transition from theorem to postulate came about because the original proof in the *Elements* involved picking one of the triangles, carrying it, placing it on top of the other triangle, and rotating it in space so that the two triangles manifestly coincided.

This process was objected to by Kant on the grounds that it involved the empirical:

> an empirical proposition cannot possess the qualities of necessity and absolute universality, which, nevertheless, are the characteristics of all geometrical propositions. [6]

Similarly, Schopenhauer criticised the axiom related to coincidence, [7] which supports SAS:

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[4]David Hilbert, *The Foundations of Geometry*, trans. E. J. Townsend, Chicago, Open Court, 1902; Bertrand Russell, *An Essay on the Foundations of Geometry*, [1897], London, Routledge, 1996.

[5]School Mathematics Study Group, *Geometry*, Yale University Press, 1961. NCERT, *A Textbook for Secondary Schools, Class IX*, Ninth Reprint Edition (sic), 1998.

[6]Immanuel Kant, *The Critique of Pure Reason*, trans. J. M. D. Meiklejohn, Chicago, Encyclopaedia Britannica, 1990, p. 31.

[7]"Theonine" Axiom 8 corresponding to the "Heiberg" Common Notion 4, T. L. Heath,

> ... *coincidence* is either mere tautology, or something entirely empirical, which belongs not to pure intuition, but to external sensuous experience. It presupposes in fact the mobility of figures; but that which is movable in space is matter and nothing else. Thus, this appeal to coincidence means leaving pure space, the sole element of geometry, in order to pass over to the material and empirical.[8]

Hence, both Russell and Hilbert concluded that mathematics could not, by its very nature, involve the empirical, and they defined the notion of mathematical proof in terms that ensure that mathematical proof makes no reference whatsoever to the empirical.[9] It is those latter ideas of the formal nature of mathematical proof that have led directly to the present-day socially dominant notions of mathematics and physics, according to which physics concerns empirical facts, and mathematics qua geometry concerns formal logical relationships in which empirical facts play not the slightest role whatsoever.

2 The historical development of the Western notion of mathematical proof

One may enquire how this definition of mathematics and mathematical proof came about historically.

2.1 The mythological narrative about mathematical proof

First let us look at the Western mythology surrounding the notion of mathematical proof. In this Western mythology, the first move is to discard the mathematical traditions of all non-Western cultures as not-quite-mathematics, in exactly the way one discards the opinion of the carpenter on mathematics, on the simple ground that these are not socially dominant, and *hence* must

The Thirteen Books of Euclid's Elements, Dover Publications, New York, [1908] 1956, vol. I.

[8]Schopenhauer, *Die Welt als Wille*, 2nd ed, 1844, p 130, cited in Heath, p 227.

[9]For the currently accepted definition of mathematical proof see e.g. the undergraduate text by E. Mendelson, *Introduction to Mathematical Logic*, van Nostrand, 1964.

be intellectually inferior. Nothing much needs to be said about this idea that the present social or world order is so utopian that truth can be decided on this basis, or the idea that a historically contingent military superiority can be used to decide the nature of truth—or that "truth flows from the warhead of a missile". If this is the ultimate foundation of Western mathematics philosophy, no serious philosopher needs to bother about it any more than one needs to bother about the society pages of newspapers.

The second step, in this Western mythology of mathematical proof, is to relate Western culture to an allegedly "Greek" tradition. Of course, even this mythology concedes that all the allegedly "Greek" works on mathematics and science can be related only to Alexandria, located in a different continent—Africa.

2.2 The origin of the books in the library of Alexandria

I have no intention of entering here into a debate on the quibble about "Greek" vs "Hellenic", which I regard as merely a distracting red-herring. The point here is about the epistemological roots of the knowledge that physically accumulated through books in the library of Aristotle—the first "man" to have a library according to Strabo—and in the Great Library of Alexandria. The number of books—estimated at half a million—is far too large to have been written by Greeks or Yugoslavians (Macedonians), even if we suppose that the all the soldiers in Alexander's army, most of whom were illiterate, did nothing but write books! The books obviously represented an accumulation of the past Egyptian knowledge of 2000 years, as also books looted from Persia and Babylon, and books confiscated by later Ptolemies from traders (including Indian traders) visiting Alexandria for trade. The point here is about the multicultural epistemological roots of this knowledge, and how this multicultural origin had a decisive influence on the subsequent growth of this knowledge. I hope this point will become clear as we go along. For those understand epistemology and history in terms of the color of the skin one might put things as follows: anything that came from Alexandria (or from the library of "Aristotle" for that matter) is far more likely to have been of black African origin than of white "Greek" origin, even if this had similarities to Platonic philosophy, which itself could have hardly avoided deep Egyptian influence. The Greeks under Alexander looted knowledge from the rest of the world, but whether they added anything of their own to it is a point on which we have no definite evidence.

The reason we have no evidence of the actual Greek contribution to this "Greek" knowledge in Alexandria is that *our* knowledge of this alleged "Greek" knowledge comes from sources from a thousand year or more later, typically in 10th and 11th c. CE Arabic or later Byzantine Greek manuscripts. As regards sources of *scientific* texts, it would have been very unreasonable for these scientific texts *not* to have been regularly updated with numerous anonymous contributions, reflecting the growth of knowledge, and indeed these texts reflect a knowledge appropriate to the time period in which they are found, and inappropriate to Alexandria. In fact, it is fairly clear that these composite sources patently involve extensive inputs from Arabs, including the inputs in mathematics and astronomy that that reached Arabs via India and the translations of the Bayt al-Hikma (House of Wisdom) in Baghdad. Western mythology seems to have addressed itself only to a very credulous audience easily carried away by narrative, hence even while masquerading as historical scholarship it is so very uncritical as to resemble a dream-sequence. Thus, no one has stopped to question the commonly repeated fantastic assumption that this knowledge suddenly appeared, immaculately conceived, in isolated individuals such as a "Claudius Ptolemy", and then disappeared without a trace, until it reappeared among the Arabs, who merely translated these earlier texts without adding anything to them.

As an example closer to our immediate concerns, as seen above, the present-day notion of mathematical proof (as given by Hilbert or Russell) may be traced to the the study of the foundations of geometry by these two,[10] using a text of the *Elements*, attributed to a certain Euclid of Alexandria. So many narratives have been spun around this "Euclid", that it is perhaps a bit of a shock to realize that the SOLE reliable evidence[11] for even the existence of this "Euclid" is a single, vague remark in a manuscript from at least fifteen centuries later: probably begun by Proclus of Alexandria, who comes eight centuries after the purported date of Euclid! Even the earliest known version of the *Elements* of 888 CE, is a composite text, probably being a Greek translation from an Arabic text. The single remark in the Proculs (Monacensis) text is completely out of tune with the rest of the text, since neither this "Euclid" nor his philosophy is ever again mentioned in the text, although the text discussed Plato's philosophy, and its criticism in great detail. Since there is no subsequent reference to "Euclid" in the text, the

[10]D. Hilbert, *Foundations of Geometry*, cited earlier.

[11]Heath, cited above, p. 75

single out-of-tune remark about Euclid is probably a later-day interpolation by a priest—priests were rightly notorious for their numerous forgeries and interpolations. The remark itself asserts that no historian of geometry prior to the author of the remark has mentioned this "Euclid"—thus suggesting an apologetic attempt to cover up the invention of a "Euclid" not mentioned by anyone earlier, by attributing this "Euclid" to the authority of Proclus. Even if, by a stretch of the imagination, we suppose that the remark is genuinely due to Proclus, who comes some 4 centuries before the remark and some 8 centuries after "Euclid", then, since Euclid is discussed no further (and was mentioned by no one earlier, if the remark is valid), the remark can only be regarded as having a symbolic rather than a literal meaning. Admittedly such symbolic statement were common among Neoplatonists, and the vicious religious politics then prevalent in the Roman empire meant that Proclus was putting his life at risk in writing on mathematics from his religious viewpoint contrary to the religious beliefs that were then being sought to be enforced. I have discussed this symbolic meaning elsewhere.[12]

Given the linkages between the history and philosophy of mathematics and science, a bad history of mathematics and science can easily misguide the philosophy of mathematics and science. In fact, if we use either Birkhoff's metric interpretation, the entire book of *Elements* gets trivialised. (On the other hand, Hilbert's synthetic interpretation of the *Elements* does not fit the entire *Elements*, since the areas in proposition 35 are equal but non-congruent.)[13] Alternatively, if one simply accepts the empirical in mathematics as was done in Indian tradition, the *Elements* again gets trivialised, since proposition 47 can be established in just one empirical step, as in the *Yuktibhāṣā* proof of it. Hence the *Elements* was not translated into Sanskrit from Persian, until Jaisingh, in ca. 1720, since it was regarded as more of an Islamic or Sūfī religious text than a book on mathematics.

If the *Elements* is regarded as a religious text rather than a book on mathematics, as happened in Indian tradition, then, of course, the justification for founding a philosophy of mathematics on that text is not very credible. Thus it is clear that the Western historical grand narrative about "Greeks" and "Euclid" has played a key role in the development of the present-day philosophy of mathematics, and continues to provide support to it. (Otherwise the

[12] C. K. Raju, "How should 'Euclidean' geometry be taught", in Nagarjuna G. ed., *History and Philosophy of Science: Implications for Science Education*, Homi Bhabha Centre, Bombay 2001, pp. 241–260.

[13] C. K. Raju, "How should Euclidean geometry be taught?", cited earlier.

issue of the empirical in mathematics needs to be examined afresh.)

But what is the evidence for this so-called history? A single vague and suspicious remark in a text some 1200 years after the date of the alleged "Euclid" is a very shabby standard of evidence on which to base historical beliefs—especially beliefs of such magnitude which have led Western philosophers like Kant, Russell, and Hilbert, to name only a few, to philosophical assertions concerning the nature of universal and necessary truth, the sort of truth. Nevertheless, this shabby standard of evidence has virtually characterized the entire Western grand narrative; it is what one has come to expect at every step from Western scholarship.

This evidence for the existence of an actual "Euclid" is credible only to the racist imagination, and it is easier for me to believe instead the story which attributes the *Elements* to Apollonius the carpenter, who wrote the *Elements* to demonstrate the equity of all men—hence all propositions in the *Elements* are about equality. This was the political sense of the Neoplatonic statement that "there is no royal road to geometry"—a sense intimately connected with the believed function of geometry, to help realize one's soul, and hence lead to a realization of the oneness of all life. This was also the political and religious sense in which equality in the *Elements* was interpreted in later-day Islamic rational theology.

3 The religious and political implications of the *Elements*

The political and religious implications of mathematics, as understood in Athens, Alexandria or medieval Europe, were not incidental. In fact, historically speaking, the Western understanding of mathematics was not always divorced from the empirical, as we have seen in the case of the proof of SAS in the *Elements*. However, Western notions of mathematics always were connected closely to religious beliefs.

3.1 Plato

Thus, Plato in his *Republic* advocated the teaching of mathematics for its beneficial effects on the soul, and explicitly decried its practical applications.

> If it [geometry] only forces the changeful and the perishing upon

our notice, it does not concern us.[14]

3.2 Socrates

Socrates questions the slave boy, to establish the slave boy's innate knowledge of geometry *only to prove that the soul exists.*

> *Socrates* ... if he has acquired the knowledge he could not have acquired it in this life, unless he has been taught geometry.... Now has anyone ever taught him all this? You must known about him if, as you say, he was born and bred in your house.
>
> *Meno* And I am certain that no one ever did teach him.
>
> *Socrates* And yet he has the knowledge?
>
> *Meno* That fact, Socrates, is undeniable.
>
> *Socrates* But if he did not acquire the knowledge in this life, then he must have had and learned it at some other time?
>
> *Meno* Clearly he must.
>
> ...
>
> *Socrates* And if the truth of all things always existed in the soul, then the soul is immortal.... [15]

Note also the subtle reference to equity. All souls are eventually equal, so the slave boy's soul is equal to that of his master or a learned philosopher.

3.3 Proclus

Similarly, for example, Proclus states that the essential nature of mathematics is that it leads to a discovery of the soul, hence the realization of oneness, as in Advaita Vedanta.

[14] Plato, *Republic*, Book VII, 526, (trans.) J. L. Davies and D. J. Vaughan, Wordsworth, Hertfordshire, 1997, p 240. Jowett's translation reads "if geometry compels us to view being it concerns us; if becoming only, it does not concern us". *The Dialogues of Plato*, (trans.) B. Jowett, Encyclopaedia Britannica Inc., Chicago, 1996, p 394.

[15] Plato, *Meno*, 86-87. *The Dialogues of Plato*, trans. B. Jowett, vol. 7 of Great Books of the Western World, R. M. Hutchins, ed. in Chief, Encyclopaedia Britannica Press, Chicago, p. 182–83.

> This, then, is what learning ($\mu\alpha'\theta\epsilon\sigma\iota\zeta$ [mathesiz]) is, recollection of the eternal ideas in the soul; and this is why the study that especially brings us the recollection of these ideas is called the science concerned with learning ($\mu\alpha\theta\epsilon\mu\alpha\tau\iota\kappa\eta'$ [mathematike]). Its name thus makes clear what sort of function this science performs. It arouses our innate knowledge. . . takes away the forgetfulness and ignorance [of our former existence] that we have from birth,. . . fills everything with divine reason, moves our souls towards Nous,. . . and through the discovery of pure Nous leads us to the blessed life.[16]

In fact, Proclus explains that the whole object of his exposition is to bring out this dimension of mathematics which people commonly lose sight of. To give an analogy today *yoga* is identified with a system of exercises and complex postures. However, the term *yoga* clearly means union, and relates to the union of *ātman* with Brhman. The object of *haṭha yoga* is to prepare the body to induce such a meditative state. For Proclus, the object of mathematics is likewise to turn the mind inward and induce such a meditative state.

However, Proclus' religious beliefs did not require the enforced supremacy of metaphysics. Hence, Proclus *does* permit an appeal to the empirical in the proof of proposition 1.1 or proposition 1.4 (Side-Angle-Side *theorem*, later converted to a postulate by Hilbert and Russell).

3.4 Equity, equality, and religious doctrine

This understanding of the soul as bringing out the unity of all humanity, and indeed all life, was prevalent also in the early Christianity of Origen of Alexandria. Origen also explicitly advocated equity, he justified it using his belief in the *karma* doctrine and reincarnation, and his Neoplatonic philosophy was similar to Proclus, who came from the same school.

However, equity had become a key bone of contention in Christianity after Constantine. For after Christianity had allied with the state, it could no longer accept non-Christians as equal to Christians. Accordingly, later day Christian theology and notions of the soul may be entirely characterized through its rejection of equity. Augustine's notions of heaven and hell

[16]Proclus, *A Commentary on the First Book of Euclid's Elements*, trans. Glenn R. Morrow, Princeton University Press, 1992, 47, p 38.

reflected this clear separation—heaven was where good and obedient Christians went, and hell accommodated all the rest![17] Accordingly, when the *Elements* came into the theological curriculum, the political equity aspect of the *Elements* came into conflict with the prevailing theology against equity, and so this aspect of the *Elements* came to be regarded as somehow superfluous and inessential, and in fact politically embarrassing. Hence, when the this political component was neutralised by replacing the notion of equality was replaced by the notion of congruence, by Hilbert, no one objected.

3.5 The impact of Islamic theology on mathematics and physics

In fact, the connection of theology with mathematics changed the nature of mathematics, when the West borrowed mathematics for the second time from Islamic rational theology. (The first time it borrowed mathematics was from Egypt, as Herodotus recounts in his History.[18]) Thus, in the hands of Christian rational theology, mathematical proofs became no more than a tool of argument used to convince those heathens who did not believe in the Christian scripture, and did not accept citations from the Christian scriptures as a valid mode of argument. Hence also proof *had* to be detached from the empirical, for the acceptance of the empirical in proof would have demolished the belief system that was sought to be imposed on people.

When the West borrowed rational theology from Islamic rational theology the connection between mathematics and theology was also deeply influenced by the connection between mathematics and theology in Islam, particularly Averröes (Ibn Rushd) whose books remained as texts in Universities like Paris and Oxford for some three centuries. A key concern for Ibn Rushd, in

[17] C. K. Raju, "The curse on cyclic time", *The Eleven Pictures of Time*, Sage, 2003

[18] Herodotus, *History, Euterpe*, 109, p. 70 recounts as follows. "Sesostris also, they declared, made a division of the soil of Egypt among the inhabitants, assigning square plots of ground of equal size to all, and obtaining his chief revenue from the rent which the holders were required to pay him year by year. If the river carried away any portion of a man's lot, he appeared before the king, and related what had happened; upon which the king sent persons to examine, and determine by measurement the exact extent of the loss; and thenceforth only such a rent was demanded of him as was proportionate to the reduced size of his land. From this practice, I think, geometry first came to be known in Egypt, whence it passed into Greece.". vol. 5 in *Great Books of the Western World*, Encyclopaedia Britannica, Chiacago, 1990.

his *Tahafut at Tahafut*[19] was to refute the arguments of al Ghazali which the latter had put forward in his *Tahafut al Falasifa*.[20]

The contemporary Western understanding of the relationship of mathematics and physics today derives from these arguments of al Ghazali.

Al Ghazali himself was not interested in the relationship of mathematics and physics, and only wanted to promote the meaning of Islam as total surrender to God. To this end he argued against the ideas of physicists and philosophers—he argued that there are *no* necessary causal relationships in the world. He believed that God continually created the world afresh at each instant, and therefore felt that causal relationships would impede the powers of God to create a world according to his liking. He argued that observation could only demonstrate simultaneity, and not any necessary causal relation. Accordingly, he argued that God was not bound by any causal laws, and could create any sort of world that he wanted. However, so strong was the influence of Neoplatonic philosophers like Plotinus and Proclus in Islam, that al Ghazali conceded that God was bound by the logic.

Thus, al Ghazali allowed that God was bound by logical relationships, but not by causal relationships, and in the contemporary understanding, mathematics concerns logical relationships, while physics concerns causal relationships. And in the contemporary understanding, too, the causal relations of physics are accorded a lower status than the logical relations of physics.

The arguments of al Ghazali about logic and causation are correct from the contemporary Western perspective, with an appropriate change of terminology—as seen in Hume, for instance. Thus in contemporary terminology, al Ghazali's point of view could be restated as follows: deductive inference is binding even on God (since God is bound by the laws of logic) but inductive inference is not binding on God (since God is not bound by any laws of cause and effect). According to current beliefs about induction (discussed in more details below) a million observations that fire causes burning do not establish that this will happen on the millionth-and-one occasion! The responses of Ibn Rushd to al Ghazali's arguments are quite scattered and unconvincing—all he has to say is that these arguments involve sophistry, but he fails to pinpoint the alleged sophistry.

[19]Ibn Rushd, *Averroes' Tahafut al Tahafut* trans. S van den Bergh, Cambridge, 1901.

[20]*Al Ghazali's Tahafut al Tahafut* trans. S. A. Kamali, Pakistan Philosophical Congress, Lahore, 1963.

3.6 The misunderstanding of al Ghazali by the schoolmen

However, al Ghazali's arguments were further completely misunderstood by Aquinas and the schoolmen. The reason for the misunderstanding was that post-Constantine Christianity had changed also the nature of God. Origen's God was immanent, Augustine's was transcendent. Proclus' desire to turn the mind's attention inwards was because he saw that as the path to Nous. Al Ghazali's too was a Sufi, and hence naturally believed in an immanent God. This is further demonstrated by his remark about al Hajjaj who was crucified (in reality not merely narrative) for asserting 'Ana'l haq' ("I am the truth"). [Al Ghazali's remark was that al Hajjaj should not have stated this publicly.]

Now, it is perfectly possible to reconcile the idea that all creativity resides in God with the idea that living beings are creative agents, provided one allows that this God resides within living beings and exercises creativity only through them.

But the schoolmen viewed God in transcendent terms. When combined with the notion of a transcendent God, al Ghazali's arguments endowed far too much power in God. If an apple could change at any moment into a man, then God would have to be a quite an awful sort of tyrant and sadist to torture this man eternally in hell, especially since such punishment had no future objective of reforming that person, and indeed the very concept of the person became suspect. The schoolmen who routinely committed numerous blunders of translation (e.g. the word "sine" or the word "surd") did not notice this change of meaning. But this change in the perceived meaning of the word "God" was the cause of much discomfort with al Ghazali's arguments in the West. While al Ghazali's point of view was maintained also within Christian rational theology, by John Duns, his followers, known as Dunsmen, came eventually to be known as Dunces, and it came to be believed that after the initial moment of creation of the world, God operated the world on the basis of rigid "laws of nature". Hence they argued that the Bible was the word of God while Nature was the work of God. It was in this context that Galileo stated that the book of Nature was written in the language of mathematics.

3.7 The "laws" of physics

Newton thought that it was these laws with which God controlled nature that had been prophetically revealed to him,[21] hence he referred to his model of motion as the "laws" of motion, and such religious and theological beliefs[22], are still the only reason why people continue to refer to them as Newton's "laws of motion". Of course, Newton himself was not able to prove the stability of the planetary system so he thought that God nevertheless made some rare providential intervention, like a master clockmaker may occasionally intervene with his clock to set things right. However, this problem of stability was set right by Laplace (since which time philosophers have been busy trying to exorcise Laplace's demon). Thus, Western theology came to believe not only that God was bound by the laws of logic, but that God operated in this world solely through the "laws" of nature, or laws of causation.

3.8 Creation in theology and physics

However, Christian rational theologians clearly preferred one-time creation to al Ghazali's continuous creation (which they called providential intervention), they agreed with al Ghazali that God was not bound in any way by causal laws *at the moment of creation*, and could have created a world entirely of his liking. In present-day science, this belief is incorporated in the work of Stephen Hawking and G. F. R. Ellis, who conclude their treatise on singularity theory[23] with the line:

> ... the actual point of **creation**, the singularity, is outside the scope of the presently known laws of physics.

The exact theological significance of this is made explicit by Hawking himself in his more popular writings.

> At the big bang and other singularities, all the laws would have broken down, so God would still have had complete freedom to choose what happened and how the universe began.[24]

[21] C. K. Raju, "Newton's Secret", chap. 4 in *The Eleven Pictures of Time*, Sage, 2003.

[22] Incidentally, Newton's laws of motion, by themselves, are *not* falsifiable. C. K. Raju, "Newton's Time", *Physics Education* **8** (1991) 15–25.

[23] S. W. Hawking and G. F. R. Ellis, *The large Scale Structure of Spacetime*, Cambridge University Press, 1974, last line of main text, p. 364, emphasis mine.

[24] Stephen Hawking, *A Brief History of Time*, Bantam, New York, 1988, pp. 183–84.

Note that according to Hawking, this result is practically an irrefutable one just because it has been mathematically proved according to the general theory of relativity:

> [my] arguments showed that... [our] universe must have begun with a singularity.... The final result... at last proved that there must have been a big bang singularity provided only that general relativity is correct....[25]

Note that Hawking believes that the truth of this theologically correct state of affairs has been established since it has been mathematically *proved* from the postulates of general relativity. Setting aside the question of the validity of this belief,[26], the point is that Hawking claims that this conclusion is *necessary* and irrefutable unless the general theory of relativity itself is refuted.

This incidentally shows how deeply theological ideas have infiltrated both present-day mathematics and physics.

4 Necessity and contingency in the philosophy of science

This idea of al Ghazali that God is bound by the laws of logic, but not by any empirical facts can be expressed somewhat differently as follows, in the language of Wittgenstein and Tarski, or, more recently Lewis. A logical deduction represents *necessary* truth, i.e., something true in *all* possible worlds. An empirical fact, however, represents contingent truth, i.e., truth in only *some* possible worlds. That is, mathematical proof concerns necessary truth, while empirical facts concern contingent truths. This idea of the empirical world as contingent is absolutely essential from the theological perspective, quite irrespective of Plato, since otherwise God would have no significant role left in the creation of the world.

[25] Stephen Hawking, *A Brief History of Time*, pp. 52–54.

[26] On which question see C. K. Raju, "Junction Conditions in General Relativity", *J. Phys. A*, **15** (1982) 1785–97; C. K. Raju, "Distributional Matter Tensors in Relativity", *Proc. MG5*, D. Blair et al. eds, Singapore, Wiley Eastern, 1989. C. J. S. Clarke, *The Analysis of Spacetime Singularities*, Cambridge University Press, 1993. A more non-technical account may be found in C. K. Raju, *The Eleven Pictures of Time*, Sage, 2003.

This distinction between necessary truth of mathematics and the contingent truth of physics is not only part of present-day theology, mathematics, and physics, but it has also motivated Popper's criterion of refutability.

4.1 Refutation vs verification

To begin with, Popper thought[27] that refutation was a more economical process than verification. The reason being that he felt that any number of experiments did not verify a theory, but a single critical experiment could refute it. The idea here is that any number of empirical outcomes in favour of a proposition do not establish it. If we toss a coin a hundred times, and the outcome is heads on all those 100 occasions it does not follow that the coin has heads on both sides. If, however, we find tails on the first toss of the coin, then the theory that the coin is two-headed stands refuted.

The fallacy in Popper's propositional thinking is, of course, quite transparent. If no proposition A can be conclusively *verified* through experiment, then neither can any proposition C be conclusively *refuted* empirically (for if it could be conclusively refuted, then the proposition $\sim C$ would have been conclusively verified). Therefore, also, it is pointless to introduce an intermediate *modus tollens*, and claim that since $A \Rightarrow C$, the refutation of C refutes A. At the time of the initial experiments on Bell's inequalities, there was a stage when two experiments showed a violation of the inequality while two experiments showed no violation. Subsequently the score changed, but the point is that there could always remain some philosophical doubt as to the actual outcome of the experiment. Settling this doubt requires repeating the experiment (indefinitely).

4.2 Popper's "solution" of the problem of induction

Popper's claim[28] to have settled the problem of induction is equally fallacious. What Popper is trying to refute is that a series of experiments in favour of an outcome somehow increases the probability of the outcome. Popper concedes that the frequentist account of probability is fallacious but feels that probabilities represent objective "propensities" rather than degrees of subjective belief. However, setting this aside, and using the Kolmogorov

[27]K. R. Popper,*Realism and the Aim of Science*, Postscript to LScD, vol. 1
[28]K. R. Popper, *Postscript*, vol. 3

account of probabilities, Popper argues that the probability of an outcome does not change on account of a favourable outcome. This is quite right, but again trivially fallacious. That is because empirical observations can never give us the probabilities, but can only provide *estimates* of probabilities. And estimates of probabilities, as any one who has performed a sample survey knows, can keep changing with time. In particular, in supposing that 8 experiments for and 2 experiments against Bell's inequalities better establish the violation of Bell's inequalities, while 2 experiments for and 2 experiments against do not, we are using *estimates* of probabilities regarding the outcome of the experiment.

4.3 What is refuted: hypothesis or deduction?

Having set aside these preliminaries, we can now come to Popper's substantive point, which is the following. In the actual historical development of scientific theories (read physics), what has happened is that starting from a (usually abstract) hypothesis A, one has drawn a chain of consequences, $A \Rightarrow A_1 \Rightarrow A_2 \cdots A_n \Rightarrow C$, such that the final consequence C may be tested by means of refutation. Rejection of C has then led, by *modus tollens* to $\sim C \Rightarrow \sim A_n \cdots \Rightarrow \sim A_1 \Rightarrow \sim A$, hence to the rejection of A.

Quite specifically, the belief here is that the deduction $A \Rightarrow A_1 \Rightarrow A_2 \cdots A_n \Rightarrow C$ represents *necessary* truth, which connects the hypothesis A with the conclusion C. Hence the rejection of C should be used to reject A rather than, say, the deduction itself. That is, from $\sim C$ one infers $\sim A$ but *not* $\sim (A \Rightarrow A_1 \Rightarrow A_2 \cdots A_n \Rightarrow C)$ (unless there is an error in the deduction).

4.4 Deduction as cultural truth

But, as I have pointed out in various other contexts,[29] deduction does not represent necessary truth but only represents cultural truth. Thus, deduction depends upon the logic used: a deduction such $A \wedge \sim A \Rightarrow B$ is valid in two-valued logic but fails in 3-valued Jaina logic or in non truth-functional

[29]e.g. C. K. Raju, "Mathematics and Culture", in: in *History, Culture and Truth: Essays Presented to D. P. Chattopadhyaya*, ed. Daya Krishna and K. Satchidananda Murthy, Kalki Prakash, New Delhi, 1999, pp. 179–193. Reprinted in *Philosophy of Mathematics Education* **11** (1999). Available online at `http://www.ex.ac.uk/~PErnest/pome11/art18.htm`

logics like Buddhist logic. If the nature of logic is to be decided empirically, this can only be done inductively, hence deduction would have to be more fallible than induction. Further, if the empirical is permitted in mathematics then refutation of C may only refute the deduction of C from A and not actually refute A. On the other hand, if the empirical world has no place in mathematics, which is pure metaphysics, then empirical facts cannot also be used to decide the nature of the logic underlying mathematics. Which logic one uses for mathematics then becomes purely a matter of cultural preference like which music one prefers to listen to. But under these circumstances the consequences of deduction—mathematical theorems—can only be regarded as cultural consequences of the axioms, not as necessary consequences. Far from being binding on God these consequences would only be binding on people who share a certain culture or faith. In this case, too, since deduction rests on mere cultural authority, it is bound to be more fallible than induction. Western philosophers may keep insisting like Kant does that the deduction has some sort of universal validity or applicability, but this cannot be anything other than amusing from another cultural perspective.

To summarise, Popper's criterion of refutability assumes that mathematical theorems, are necessary truths, and hence that physics is refutable but mathematical proof is not, but this belief involves cultural presumptions about the nature of logic.

4.5 Prediction and prophecy

In fact, the vulgar form of the belief in refutability is the belief in prediction: the common belief among physicists is that a physical theory must make good predictions. This belief appears to derive from the cultural value attached to the notion of prophecy. This belief in prophecy came into Greece from Egypt, in the form of oracles, as Herodotus recounts in his History, and subsequently became a fundamental part of Western Christian religious dogmas: nothing much would be left of them without the prophecy of the apocalypse.

Although these religious beliefs in prophecy deeply influence the formulation of present-day physics, since Newton, I, also reject the naive idea of physicists that the only those theories are valuable which make predictions (prophecies).

4.6 Testability of mathematics as an auxiliary physical theory

On the other hand, as must be evident by now, I am quite sympathetic to the Popperian enterprise of rejecting metaphysics masquerading as physics. But I think this can be best achieved by admitting also the empirical in mathematics, so that mathematics itself becomes refutable, and becomes something like an auxiliary physical theory that enters into another physical theory. For reasons of theoretical "convenience" (in the sense of Poincaré) one may choose at times to reject the main theory instead of rejecting the auxiliary physical theory. But there may arise situations where the auxiliary theory itself must be rejected.

To fix ideas, let us consider a concrete contemporary situation such as the renormalization problem of quantum field theory—it is well known that quantum gravity is not renormalizable. Now, if, say, the Lagrangian corresponding to the Fermi five point interaction is not renormalizable what should we say? That the Fermi theory is not valid? Or that the mathematics of renormalization is defective? On the grounds of "convenience" we might want to say that the auxiliary theory of products of distributions that enters into the S-matrix expansion[30] should also be the auxiliary theory of products of distributions that enters into the theory of relativistic shocks[31] or classical Navier-Stokes shocks, and this requires empirical inputs.

4.7 The tilt in the arrow of time

My own position, incidentally, is that the nature of both logic and cause are dependent upon the empirical notion of time. If there is a tilt in the arrow of time (the most general case after relativity[32]) then logic cannot

[30] C. K. Raju, "On the square of x^{-n}" *J. Phys. A: Math. Gen.* **16** (1983) 3739–53. "Renormalisation, Extended Particles and Non-Locality." *Hadronic J. Suppl.* **1**, 1985, pp. 352–70.

[31] C. K. Raju, "Distributional matter tensors in general relativity", Proc. MG5, cited earlier

[32] C. K. Raju, "Electromagnetic time", chp. 5b in *Time: Towards a Consistent Theory*, Kluwer Academic, 1994.

be two-valued[33] , and there must exist uncaused events.[34]. In al Ghazali's terminology, I would deny that God is bound by any laws of logic that can be determined in advance of the facts, but I would agree that there might be occasional uncaused events, like a convergent ripple in Popper's pond, which, as Popper correctly points out would be "miraculous"—but only in the sense they would admit no explanation, in principle, from past causes.

5 Proof and refutation in Indian tradition

In contrast to all the theological considerations that are so intertwined with the Western notion of mathematics, Indian mathematics is refreshingly practical and down-to-earth.

5.1 Proofs in Indian mathematics

As for the notion of proof, there is nothing particularly special about mathematics, since mathematics was seen as relating primarily to calculation rather than proof. Calculation is the real basis for the practical and social value of mathematics, and I see no reason to deprecate it, or run it down. On the contrary, I quite see the point of Swift's satire of a world ruled by pure mathematicians who always stay up in the air, and have no contact with the real world.

In any case, the notion of proof in mathematics in India is not particularly different from the usual notion of *pramāṇa* that applies to all things from religion to physics. No one, as far as I know seems to have singled out for comments mathematics as something intrinsically different, and requiring a different method of proof.

Mathematical proofs involving the empirical obviously are permitted, since the *pratyakṣa* or empirically manifest is the first *pramāṇa*. We have seen the example of the proof in the *Yuktibhāṣā*, which dates from ca. 1550 CE. But this obviously was exactly the geometrical method used in the *śulba*

[33]C. K. Raju, "Quantum Mechanical Time", chp. 6b in *Time: Towards a Consistent Theory*, Kluwer Academic, 1994.

[34]C. K. Raju, "Time travel and the Reality of Spontaneity", `http://philsci-archive.pitt.edu/archive/00002416/01/Time_Travel_and_the_Reality_of_Spontaneity.pdf`; for an even more non-technical account, see chp. 7, Time Travel", in *The Eleven Pictures of Time*, cited earlier.

sūtra a couple of thousand years earlier. The very name *śulba*, meaning rope, refers to the rope used to *measure* lengths—a procedure that is manifestly both *empirical* and also *metric*, acceptance of either of which immediately trivialises the *Elements*. The references of Western scholars to the *śulba sūtra* as "ritual" geometry cannot be regarded as anything but the wildest of propaganda. The appellation "ritual" is better applied to the geometry of the *Elements* because of (a) its explicit esoteric significance, underlined by Proclus, and (b) the fact, already noted, that the *Elements* gets trivialised by accepting either empirical procedures or even Birkhoff's metric postulates (which we ought to do since Hilbert's synthetic interpretation does not extend beyond the 35th proposition of the *Elements*).

It is incidental that the *śulba sūtra* are preserved in the context of building Vedic *citi*-s, the text is concerned solely with the practical aspect of building the brick structure, and the exact use to which that structure would be put is not critical. Thus *rajju* (=rope) was also part of the elementary school syllabus, from as far back as we know, the rope is also used for constructions used in preparation for astronomical observations (such as the "fish" figure, corresponding to the first proposition of the *Elements*), and there is no doubt that the same method was also used to construct the *stūpa* at Sanchi, a brick structure used for other purposes.

From the mathematical point of view, the interesting thing about the *stūpa*—either the big one or the numerous small one's—is that it represents the first known brick structure which has a domelike appearance, which is a curved surface. (This precedes the dome of the church of Sophia at Istanbul by some eight centuries.)

The point here is about how the empirical procedures (using the rope) provided inputs to mathematics. Thus a rope can be stretched to draw a straight line, and it can be used not only to draw a circle, but it can also be used to measure a curved line, such as the circumference of a circle. Hence, the use of a rope to measure lengths makes it easily meaningful to talk of the length of curved lines, and hence the circumference of a circle. In contrast, the traditional Western geometry instruments are a straight edge and collapsible compass for synthetic geometry, while metric geometry (as used by navigators) permits the use of a ruler and compass. (The straight edge is unmarked and does not permit lengths to be measured; the collapsible compass collapses when lifted from paper, so that it does not permit one to pick and carry a length from one point to another. Length measurements are, however, permissible in metric geometry—which trivializes the *Elements*) On neither of these two sets of instruments is it obvious that curved lines can at all be measured. Accordingly, Descartes boldly asserted in his *Geometry*

> geometry should not include lines that are like strings. . . the ratios between straight and curved lines are not known, and I believe cannot be discovered by human minds, and therefore no conclusion based upon such ratios can be accepted as rigorous and exact.[35]

However, the *śulba sūtra* gives an approximate value of π as 3.1. By the 5th c. Āryabhaṭa is able to specify this approximate (*asanna*) value to 4 decimal places. (However, the 5th c. CE Bible gives the value of the ratio of circumference to radius as 3.) By the early 9th c. CE Govindasvamin is already attempting third sexagesimal minute precision in trigonometric values, which requires a value of π accurate to about 10 decimal places. While the efforts of Govindasvamin failed, it is to be presumed that he had already attained the 2nd sexagesimal place precision found also in the later text of Vaṭeṣvara (904 CE). While the 13th c. CE Arabic texts, nowadays extravagantly attributed to Archimedes, naturally give a reasonably good approximation, it is nowhere near this level of precision. It is this process of calculating the length of the chords and the circumference which led to the infinite series calculus.

Similar examples can be found in the *Āryabhaṭīya* which declares that perpendicularity must be tested using a plumb line, and flatness by a water level. This resort to empirical methods in Indian mathematics also means that the Western distinction between mathematics and physics has no relevance to Indian tradition.

The only possible differences between the notion of *pramāṇa* in mathematics and the notion of *pramāṇa* in other spheres is that analogy (*upamaña* seems not to have been *much* used, unlike the case of astronomy where models were certainly used. Also there is no instance of anyone having used authority or *śabda* as a means of *pramāṇa* in mathematics. On the contrary, Varahamihira clearly rejects the *śabda pramāṇa* of the *Vedānga Jyotiṣa*, on the grounds of the *pratykṣa* as the first *pramāṇa*.

This is a fundamental reason why there never was a need to talk of either harmony or conflict between science and religion in Indian tradition, since

[35]René Descartes, *The Geometry*, trans. David Eugene and Marcia L. Latham, Chicago, Encyclopaedia Britannica, 1990, Book 2, p. 544.

both science and religion had similar methods of proof[36]

This non-use of *upamāna* and *śabda* in mathematics also meant that there were no differences between Nayyayikā's on the one hand and Buddhists and Jains on the other regarding the nature of mathematics. Accordingly, it is entirely mischievous to speak, as numerous Western scholars and their imitators in India have done, about "Hindu" mathematics or "Jaina" mathematics. As clear from what was argued above, the close relation between mathematics and religion is confined to the West, and it is indeed possible to distinguish a Neoplatonist mathematics from a Western Christian mathematics.

While I have no certain information on how the Lokāyata viewed mathematics, my impression is that the Lokayata had no objection to the use of deduction or inference *anumāna*) in the practical sphere. That is, when the husband left the house, they did not expect the wife to live the life of a widow, but only to carry on with her normal life, while accepting that her inference that the husband was alive was fallible. Since mathematics in India represented a practical activity, there is no doubt that Lokāyata would have no objection to the use of inference for the purposes of mathematics. On the other hand, it is quite likely that Cārvaka would have objected to the metaphysical way in which deduction is used in Western mathematics, and would have certainly not have accepted that deduction can be used to arrive at certain truth.

The other aspect of proof in Indian tradition is that in the absence of any claims of divine infallibility or eternal forms or perfection attached to mathematics in India, "approximation" was perfectly acceptable in mathematics, right from the time of the *śulba sūtra*. Thus, Baudhayana (2.12) uses the term *sa viśesa*—this and a small remainder—for the value the of $\sqrt{2}$. Similarly, Apastamba (3.2) uses the term *sa anitya* for $\sqrt{2}$—meaning that the value stated is *not* a permanent or eternal value, and is subject to revision. Likewise, Āryabhṭa (*Gaṇita* 10) uses the term *āsanna* or "approximate" for his value of π. As Nīlakaṇtha points out in his *Bhāṣya* on *Gaṇita* 10, the near (*āsanna*) value is given, and the real *vāstavik*) number is left out, since the "real value of the number" cannot be stated.

This is very interesting in the Buddhist context of *śunyavāda*, for it accepts that numbers may be non-representable from an intrinsic perspective, and that representations are used only for practical purposes. Thus there is

[36] C. K. Raju, "Science:Religion::Reason:Faith?", talk presented at the Seminar on Indic Traditions, New Delhi, 2003. Abstract at:

no point in asking "what is a number?" any more than there is any point in asking "what is a seed?". Just as the seed has no permanent identity, and in fact no "essential" feature stable across two instants of time, and just as one merely continues to refer to it as "the seed" for reasons of practical convenience, due to paucity of names, so also one gives the name $\sqrt{2}$ to the number corresponding to the diagonal of a unit square, since no number can ever be found which correctly represents it. Note that this is quite different from saying that there is such a number, symbolically given the name $\sqrt{3}$, and that we use an "approximate" or "erroneous" representation of it, such as 1.414 for practical purposes. What is being denied here is the real existence of the number, like the denial of the real existence of a soul. (Dedekind's formalisation of what are today called real numbers does nothing whatsoever to solve this problem—all that it enables us to do is to assert the "existence" and "uniqueness" of a real number corresponding to $\sqrt{2}$, where that existence is guaranteed only according to certain formal notions of proof, which do not in any way help us to represent or specify this number. This inability to represent numbers is made manifest in computers, which cannot be deceived into believing that they "understand", or can operate with, an intrinsically woolly concept; accordingly, computers use only floating point numbers.)

5.2 Refutations

There are numerous interesting refutations that can be found in Indian tradition. Aryabhata asserts in *Gola* 9 that as in a boat moving in one direction one sees objects on the river bank moving in the opposite direction, so also the the fixed (*acal*) celestial sphere seems to be revolving westward because the earth is rotating eastwards. He goes on in *Gola* 10 to play on words to characterize the rotation of the celestial sphere as an illusion (*bhrama*.

Numerous people after Aryabhata, including his followers have sought to refute this. Varahamihira gives the following arguments, If the earth rotates, then there should be a terrific wind in the other direction, and this would be seen in the movement of clothes on a clothesline. Addressing the implicit doubt that the wind somehow participates in the motion of the earth, Varahamihira asks about the falcon which rises high in the sky. He asks, how can this falcon find its way back home? Similar arguments are given by numerous other people such as Brahmagupta, Bhaskara, Vatesvara etc.

5.3 Proof and refutation in the Michelson-Morley experiment

From a historical perspective, there are two interesting aspects to these arguments of Varhamihira et al. seeking to refute Aryabhata. First, it is noticeable that the 11th c. CE Arabic text attributed to Claudius Ptolemy summarises these very same common arguments in Indian tradition, without however, naming Aryabhata or otherwise identifying the "some people" who argue in favour of the rotation of the earth, as would be natural for an 11th c. CE Arabic text, but would have been quite unnatural for a Roman in the 2nd c. CE. (Ptolemy is dated on the basis of a passage, in which the only reliable predecessor he acknowledges is Hipparchus.)

The other interesting aspect of this refutation relates to the Michelson-Morley experiment. Contrary to the myths that are propagated in numerous physics texts today, that the experiment sought to measure the speed of light, the experiment was actually performed, following a posthumously published suggestion by Maxwell, to distinguish between the theories of Stokes and Fresnel.[37] We recall that according to Fresnel, the aether could be regarded as fixed in space, and streaming through bodies (this corresponds to the case of an aether wind due to the motion of the earth). On the other hand, according to the theory of Stokes, the aether was dragged along by the motion of the earth, so that it was at rest with respect to the earth. Michelson concluded that his experiment favoured the Stokes theory.

Unfortunately, what the Stokes theory suggested was seen as having been proved to be mathematically impossible. (Stokes required the motion of the aether to be irrotational, so that plane waves would remain plane. The aether was assumed to be incompressible. But the irrotational flow of an incompressible fluid must be a potential flow. The potential equation (Laplace's equation) admits a unique solution if the normal derivative is specified at the boundary (Neuman problem). Therefore, if the normal derivative were zero, as would happen if the aether was at rest with respect to the earth, then a possible solution of the potential equation (aether at rest everywhere), which satisfied the prescribed boundary conditions, would be the unique solution.

Thus, there was a conflict between a theory (Fresnel's) seen to have been

[37]for the details and references, and the additional background to these theories in the problem of stellar aberration, see C. K. Raju, "The Michelson-Morley experiment", *Physics Education*, **8** (1991) 193–200, or chp. 3 in *Time: Towards a Consistent Theory*, Kluwer, 1994.

experimentally refuted, and a theory (Stokes) seen as having been mathematically proved to be incorrect! There was little doubt of what physicists would choose, for they regarded mathematical proof as clearly superior to experimental refutation. Thus Planck suggested that the problem could be avoided by making the aether compressible, but Lorentz suggested that the Stokes theory should be abandoned altogether. Hence, he sought to explain the outcome of the experiment by proposing the length contraction, first that the spaces between the molecules got compressed in the direction of motion, and then that the molecules themselves got shortened in the direction of motion. It was left to the subtle Poincaré to clear up the mess with his elegant proposal of the special theory of relativity, and to Einstein to grab the credit for it, although Einstein never fully understood the theory of relativity, and never abandoned the notion of aether or its origin in the related notion of action by contact.

5.4 Refutation of the popular beliefs

In Indian tradition, definitely from the time of the *Sūrya Siddhānta* and Aryabhata, and probably from long before that, the earth was regarded as a sphere. As Aryabhata describes it (*Āryabhaṭīya, Gola* 6-7):

> The globe of the Earth stands supportless in space... Just as the [spherical] bulb of a Kadamba flower is covered all around by blossoms, just so is the globe of the Earth surrounded by all creatures, terrestrial as well as aquatic.

While Aryabhata does not feel the need to defend the idea of a round earth, later writers like Lalla (748 CE) do. Lalla, in the twentieth chapter of his *Sishyadhivrddhida*[38] examines various false notions, and states that some people have the following false notions about the earth.

> (20.6) Some think that the earth is infinite; others that it is plane like a mirror. Again, others say that it extends to many yojanas and floats on water like a boat.

[38]Lalla, "False Notions", chp. 20 of *Sisyadhivrddhida Tantra of Lalla*, with the commentary of Mallikarjuna Suri, ed. and trans. Bina Chatterjee, INSA, New Delhi, Part II, p. 269.

(20.7) Some say that the earth is supported by a tortoise, a serpent, a boar, an elephant or by mountain ranges…

He then refutes the belief that the earth is plane through a variety of arguments, some of which are the following.[39]

(20.31) The eclipse, the conjunction and rising of planets, the cusps of the Moon, and the length of the shadow (of the gnomon) at any time—the calculation of all these five depends upon the measurement of the earth, and agrees with the observed result.

(20.35) Mathematicians say that one hundredth of the circumference of the earth appears to be plane.

(20.36) If the earth is level, why cannot tall trees like the date palm, alas, be seen by man, though at a very great distance from the observer.

He separately refutes the belief that the earth is supported:[40]

(20.39) Clay is destroyed by water, so it is not possible for the earth [made of clay] to remain in water or to float on it like a boat.

(20.40) If the heavy sphere of the earth can remain on water, which water stands supportless in space, why can the earth not remain in space?

(20.41) If the earth is supported by a tortoise or other things, by whom are *they* supported in space? If they can remain in space [unsupported] what prevents the earth from remaining thus [unsupported]?

This idea is elaborated by Vaṭeṣvara in his book also called *Gola*(meaning round or spherical, since this too deals with the same subject of spherics).[41]

[39]Lalla, cited above, p. 274–75.

[40]Lalla, cited above, p. 276.

[41] *Vaṭeṣvara Siddhānta, and Gola of Vaṭeṣvara*, ed. and trans. K. S. Shukla, pt. II, English translation and commentary, Indian National Science Academy, New Delhi, 1985, pp. 638–639. Emphasis added. Vaṭeṣvara was well known as a critic of Brahmagupta. Vaṭeṣvara's book (*Siddhānta*) was written in 904 CE, and is referred to by subsequent scholars like al Bīrūnī (b. 973 CE) and Sripati (1039 CE).

> (V.2) Just as an iron ball surrounded by pieces of magnet does not fall through standing (supportless) in the sky, in the same way this Earth though supportless does not fall...

> (V.5) If the earth is supported by Sesa [serpent], tortoise, mountains, and elephants etc. how do *they* stand supportless (in space)? If they are believed to be endowed with some power [to stand supportless], why is not the same power assigned to the Earth?

He also refutes the idea that the earth would fall down, on the grounds that "up" and "down" are decided by reference to the centre of the earth.

> (V.3) If you are inclined to believe that it falls down, say what is up and down for an object standing in space. The globe of the Earth...in what direction should it then fall?

> (V.7) As here in our locality a flame of fire goes aloft in the sky and a heavy mass falls towards the Earth, so is the case in every locality around the Earth. As there does not exist a lower surface (for the Earth to fall upon), where should it fall?

He goes on to comfort people who are afraid they might fall off the earth.

> (V.8) Just as a house lizard runs about on the surface of a pitcher [pot] lying in open space, so do the human beings move about comfortably all around the Earth.

Writers who precede Lalla and Vaṭeṣvara, e.g. writers like Aryabhata, or Bhaskara, or Brahmagupta, all invariably state that the earth is spherical, they state its dimensions etc., but they do not refute any such beliefs in a flat earth. This suggests that the view was not seriously contested in their time.

5.5 Refutation of the demonic theory of eclipses

The demonic theory of eclipses has often been used to demonise Indian traditions. The following refutation of the demonic theory within Indian tradition also serves to refute the demonisation of Indian tradition.

First the background. Western historians have often quoted al Biruni on India, particularly famous is his "pearls and dung quote", and this is is especially favoured with those historians who want to write about Indian mathematics and astronomy, but are unfamiliar with the original sources, and are misguided by Western authorities like Pingree. The context of the quote is a comparison between Greek and Indian science, especially mathematics and astronomy

> The Greeks...had philosophers who...discovered and worked out from them the elements of science, not of superstition. . . .
>
> Think of Socrates when he opposed the crowd of his nation as to their idolatry and did not want to call the stars gods. At once eleven of the twelve judges of the Athenians agreed on a sentence of death, and Socrates died faithful to the truth.
>
> The Hindus had no men of this stamp both capable and willing to bring sciences to a classical perfection.[42]

So far as al Biruni's political formula for conquering and ruling India is concerned, if all he wanted to do was to make a general observation to the effect that the Indian intelligentsia by-and-large lacks a spine, he was perhaps right, and this probably remains true today. But as a peice of history this is all muddled: Socrates was a martyr all right, but proposing him as a martyr against idolatry is a bit thick, and a matter fit for consumption only by ignorant and cruel kings like Mahmood of Ghazni.

First of all, Socrates was not at all concerned about the physical or mathematical sciences. Plato's Socrates, at any rate, emphatically denied during his trial that he had anything to do with sciences: ". . . the simple truth is, O Athenians, that I have nothing to do with physical speculations". [43]. He went on to say that his accusers, believing the audience to be illiterate (like al Beruni, or perhaps Sachau), had mixed him up with Anaxagoras.

> [Socrates:] Do you [Meletus] mean that I do not believe in the godhead of the sun or moon, like other men?

[42]Al Bīrūnī, *Kitāb al Hind*, trans. E. C. Sachau, Alberuni's India, reprint, Munshiram Manoharlal, New Delhi, 1992, vol 1, p. 25

[43]Plato, *Apology*, trans. Benjamin Jowett, Encyclopaedia Britannica, Chicago, 1990. 201

> [Meletus:] I assure you, judges, that he does not: for he says that the sun is stone, and the moon earth.
>
> [Socrates:] Friend Meletus, you think that you are accusing Anaxagoras, and you have but a bad opinion of the judges, if you fancy them illiterate to such a degree....[44]

Confounding Socrates with Anaxagoras is, of course, fatal to al Biruni's argument, since Anaxagoras (like Aristotle) did not die a martyr, but ran away instead! This is similar to the case of Galileo vs Giordono Bruno: Galileo apologised to the church, Bruno chose to be burned at the stake. Scientists do not die for their beliefs, religious people do.

At any rate, at his trial, Socrates went on to swear by Zeus, thereby denying that he is an atheist, and he argues that since he is accused of believing in demi-gods, he must, therefore, also believe in the existence of gods:

> But this is what I call a facetious riddle invented by you...you say first that I do not believe in gods, and then again that I do believe in gods that is, if I believe in demigods. For if the demigods are the illegitimate sons of gods...what human being will believe that there are no gods if there are the sons of gods? You might as well assert the existence of mules, and deny that of horses and asses.[45]

The fact that a death penalty could be demanded for Socrates' alleged belief about the moon shows that Athenian society was terribly superstitious. This is corroborated by the extensive Greek belief in oracles.

There are other details that do not gel. The number of jurors, at several hundred, was lot more than the twelve jurors mentioned in Sachau's translation: the figure twelve for the number of jurors was arrived at after much statistical research into the Poisson probability distribution. Also, Socrates expresses surprise that the juror's votes were "so nearly equally divided" [46], so 11 out of 12 cannot even be taken as a valid metaphor, especially coming from one, like al Biruni, who is himself a mathematician. All this goes to show how inaccurate is the account provided by the current text.

[44]Plato, *Apology*, p. 204
[45]Plato, *Apology*, p. 205
[46]Plato *Apology*, p. 209

As a representation of Indian tradition, al Biruni is equally incorrect. India admittedly had very few martyrs like Carvaka (about whom al Biruni evidently had never heard), but this was rather because criticism was freely permitted and practised: people attacked each others theories, not the persons holding those theories. Carvaka was a special case, who criticised a victorious king for lack of ethics immediately after a major and painful war. Having said that, it nevertheless seems to me, further, that martyrdom and truth are unrelated, except as an aspect of Christian and perhaps Islamic belief. The fact that Bruno died, and Galileo did not can hardly be taken as an indication that Bruno was right and Galileo was wrong.

Similar cultural presuppositions are reflected in al Biruni's further observations as follows:

> Therefore you find that even the so-called scientific theorems [sic] of the Hindus are in a state of utter confusion, devoid of any logical order, and in the last instance always mixed up with the silly notions of the crowd, e.g. immense numbers, enormous spaces of time, and all kinds of religious dogmas, which the vulgar belief does not admit of being called into question. Therefore, it is a prevailing practice among the Hindus jurare in verba magistri. . . .[47]

The accusations of lack of logical order merely reflect the cultural presuppositions. One can understand al Biruni's frustration: the *Āryabhaṭīya* is not the *Elements*—it is neither a religious, nor a pedagogic, nor an elementary text. The *Ārabṭīya* is a specialist text, written purely from the viewpoint of practical applications, which is bound to present difficulties to those who lack the requisite background, such as some present-day historians.

Unlike a pedagogical text, a specialist text assumes the reader to be knowledgeable, and does not bother to develop the subject step by step. So there was a clash of cultural expectations between the writer of the book who expected the reader to be knowledgeable, and the reader (in this case al Biruni) who expected the writer to explain himself in the step-by-step hand-holding manner to which he (al Biruni) was accustomed. Under these circumstance, to blame the writers of the books in one cultural setting for not living up to the expectations of a person from another cultural milieu is an unfair political act of blamesmanship. There is a perfectly well-defined

[47]al Beruni, I.25. This continues into the "pearls and dung" quote.

order in the *Āryabhaṭīya*, but this is not a pedagogical order, nor is it the order of the *Elements*.

The large numbers are obviously due to the requirements of higher precision. To refer to the requirements of higher precision as unscientific only reflects on al Biruni's own lack of scientific knowledge.

Against this background, let us consider the oft-quoted claim that Indian tradition was unscientific because of the belief that Rahu and Ketu are the causes of eclipses.

This is what Lalla has to say about it in his chapter on "False Notions":

> 18. ...when a demon, an enemy of the gods, was drinking the nectar, his head was chopped off by his enemy Hari. But the head did not die. Some say this is Rahu. The Sun (and the Moon) are devoured by it.
>
> 22. If you are of the opinion that an artful demon is always the cause of an eclipse by swallowing (the Sun or Moon), then how is it that an eclipse can be determined by means of calculation? Moreover, why is there not an eclipse on a day other than the day of New or Full Moon.
>
> 25. An eclipse cannot be cause by Rahu, because the sides of the discs of the Sun and Moon, which are first to be eclipsed, are not the same; nor are the portions eclipsed the same; and nor even are the durations the same.
>
> 26. In a solar eclipse, people at different part (of the earth) see different portions of the Sun eclipsed. Some do not see (the eclipse) at all. Knowing this, who can maintain that an eclipse is caused by Rahu?
>
> 27. Because of the great authority of Brahmā, at the time of eclipse, the Sun is near Rahu. So in the Vedas, Smrtis, and Samhitas is has come to be known that Rahu is the cause of the eclipses.[48]

Note that the Rahu in the last stanza is an astronomical term, a reference to the ascending node of the Moon, and has nothing to do with the legendary demon who tried to steal the nectar from the gods! Thus, quite possibly

[48] Lalla, cited above, pp. 272–273

al Biruni confused the two meanings, just as Vasco da Gama confused the Malayalam *kau* (meaning pole star) to mean *kau* (meaning teeth), since the celestial navigator ("pilot") who navigated him from Africa to India held his instrument between his teeth, and accordingly recorded in his diary that the pilot was telling the distance by his teeth.