

# A proposed experiment to test retarded gravitation theory

C. K. Raju

Indian Institute of Education  
G. D. Parikh Centre, J. P. Naik Bhavan  
Mumbai University Kalina Campus  
Vidyanagari, Santacruz (E)  
Mumbai 400 098  
[ckr@ckraju.net](mailto:ckr@ckraju.net)

February 7, 2018

## Abstract

Lorentz covariance is essential to physics. Making Newtonian gravitation Lorentz covariant leads to retarded gravitation theory (RGT). RGT predicts Earth's rotational velocity would affect orbits of near-Earth objects, at the  $v/c$  level. Such effects, at just the  $v/c$  level, have already been observed for spacecraft ("NASA flyby anomaly"), and are too large to be explained by general relativity. We redo our earlier calculations for five spacecrafts using RGT, without simplifications. RGT explains the steep gain near perigee, but there are small quantitative differences from the reported data. The reported data, however, actually consist of theoretically un-modeled residues. Accordingly, a separate controlled experiment is necessary to test the validity of RGT. Our proposed experiment involves two satellites orbiting Earth in opposite directions in the equatorial plane in eccentric orbits. Earth's rotation should not affect the satellite velocity on (1) Newtonian gravitation and (2) the phenomenological formula of Anderson et al. However, (3) on RGT, one satellite gains and the other loses velocity, by typically a few cm/s/day, which is easily measurable by satellite laser ranging.

# 1 Introduction

Lorentz covariance is essential for physics, to be able to measure time.[1] Reformulating Newtonian gravitation to make it Lorentz covariant results in retarded gravitation theory (RGT).[2] The RGT gravitational force, being Lorentz covariant, cannot be purely position-dependent. Unlike the Newtonian gravitational force, it necessarily depends also on velocity.

This velocity dependence of the gravitational force is small, at the  $\frac{v}{c}$  level, for the two-body RGT force a simplified expression for which is the following.:

$$F \approx \frac{k}{r^2} \left( \frac{X}{r} + \frac{V}{c} \right). \quad (1)$$

Here  $F$  is the 4-force acting on the particle at time  $t$ ,  $k = GMm$ ,  $m$  is the (rest) mass of the “attracted” particle, while  $M$  is the (rest) mass of the “attracting” particle, and  $V$  its 4-velocity at *retarded time*  $t_r$ . This is the time at which the backward null cone from the position of the “attracted” particle at  $t$  intersects the world-line of the “attracting” particle.  $X$  is the relative 4-position vector of the “attracting” particle, also at retarded time, and  $r$  is the corresponding retarded distance. As usual,  $c$  denotes the speed of light.

Specifically, RGT predicts that the orbit of a small body ( $m$ ) moving in the vicinity of the Earth ( $M$ ), would be affected by the Earth’s rotational velocity ( $V$ ), at the  $\frac{V}{c}$  level. This is unlike Newtonian gravitation where the velocity does not affect the gravitational force between two bodies.

Such an “anomalous” effect of Earth’s rotation has already been observed.[3] The trajectories of five spacecraft (four from NASA and one from ESA) were affected as they flew past Earth. Popularly called the flyby anomaly, this effect on spacecraft velocities was tiny (a few mm/s compared to perigee velocities of a few km/s), but was outside observational error. The flyby anomaly has been related to Earth’s rotation, in another way, through a phenomenological formula.

$$\frac{\Delta V_\infty}{V_\infty} = K(\cos \delta_i - \cos \delta_o), \quad (2)$$

where  $\Delta V_\infty$  was the difference between the incoming and outgoing asymptotic velocity in a geocentric frame. (Conceptually,  $V_\infty$  is the hyperbolic excess velocity at infinity of an osculating Keplerian trajectory, so the difference ought to have been zero on the Newtonian

theory.) Further,  $\delta_i$  and  $\delta_o$  were the declinations of the incoming and outgoing asymptotic velocity vectors. The constant  $K = 3.099 \times 10^{-6}$  was expressed in terms of the Earth's angular rotational velocity  $\omega_E$  ( $7.292115 \times 10^{-5}$  rad/s), its mean radius  $R_E$  (6371 km) and the speed of light  $c$  by

$$K = \frac{2\omega_E R_E}{c}.$$

Anderson et al. did not attempt any physical explanation for why the Earth's rotation might affect the motion of spacecraft. However, because the formula 2 involves the declination of the incoming and outgoing asymptotic velocity vectors, the formula predicts a zero effect for equatorial orbits, unlike the case in RGT as considered in §4.

As Anderson et al. noted, general relativistic frame drag in a Kerr geometry is far too small to explain this observation.

Since the flyby anomaly is a minute effect, various doubts have been raised. The right way to settle these doubts is to repeat the experiment with greater sensitivity. This can easily be done by using satellites in elliptic orbits instead of spacecraft in open hyperbolic trajectories. Thus, by appropriately choosing the orbit of the satellite, we can already push up the predicted departures by a factor of 10 as explained in §4 (from a few mm/s to a few cm/s change of velocity per orbit). Secondly, with elliptic orbits, the RGT effect is *cumulative*. Accordingly, by carrying on observations over, say, 10 days, the precision of the experiment can be further increased tenfold, provided there are no extraneous perturbation. Combining both factors, we theoretically achieve a hundred-fold increase in sensitivity of the experiment. This would enable most doubts to be settled.

However, when measurements in deep space are made over a longer period of time, the satellite may drift, due to external perturbations, and the accuracy of about 1 m/s may again start seeming doubtful. Hence, we propose the use of two satellites, one in prograde and the other in retrograde orbit. Except for the sense in which they are described, the two orbits are as similar as practicable. A positive cumulative effect in one satellite and a negative cumulative effect in the other reduces doubts that any observed drift in the velocity of any one satellite may be due to chance perturbations.

We reiterate that our aim is to test RGT, not any and all theories of the flyby anomaly. One reason to restrict ourselves to RGT is that RGT assume nothing beyond Lorentz covariance, which is a theoretical necessity (to be able to measure time). The advantage of that is clear:

we learn something, whatever the outcome of the experiment. This is not the case, for example, with Anderson's phenomenological formula 2. If that were to be disproved by experiment that would have no fundamental consequence. However, if our two-satellite experiment reaches conclusions contrary to RGT, then we would have to fundamentally rethink Lorentz covariance and the measurement of time in physics.

Secondly, some comments are in order regarding the relation of RGT to General Relativity Theory (GRT). First, there are many situations where it is not practicable to use GRT, such as the many body problem for the galaxy. The current practice here is to fall back on Newtonian gravitation (NG) instead. But RGT, being Lorentz covariant, is theoretically superior to NG. Accordingly, we would be better off using RGT rather than NG in those contexts. That is RGT is certainly *more convenient* than GRT, for the many body problem, which may be readily solved with RGT. Changing over from NG to RGT may have important consequences for spiral galaxies where the velocity effect could add non-linearly on the rotation velocity of stars.[2] Secondly, certain observations such as the perihelion advance of Mercury, long regarded as tests of general relativity, may prove to be special relativistic effects, as preliminary calculations show. (Note that RGT is stable[4] unlike old theories of retarded gravitation used a century ago to explain the perihelion advance of Mercury.) Lastly, we may want to examine the possibility of constructing a generally covariant theory, similar to GRT, but with Lorentz covariant RGT instead of NG as the base theory.

Earlier, a simplified calculation was done for the flyby anomaly using the simplified expression 1 for the RGT force. The earlier calculation, however, involved a second simplification: it took  $r$  in 1 as just the instantaneous distance.

The results obtained with these simplifications were qualitatively correct: most of the gain or loss of energy takes place close to perigee, an observed feature which no other theory of the flyby anomaly has explained so far. The quantitative results too were very close, but there was no neat and exact fit to data. Thus, for Galileo's first Earth flyby (Galileo-1), the calculated gain on RGT was 5.96 mm/s compared to Anderson et al.'s reportedly observed figure of 3.9 mm/s. Likewise, for Cassini, the calculated loss was -3.2 mm/s compared to the figure of -2 mm/s reported by Anderson et al.

These calculated figures are so close that they strongly suggest that the RGT explanation is valid and that the flyby anomaly is indeed a  $\frac{v}{c}$  effect due to the rotation of the Earth, as predicted by RGT. However, to resolve doubts we need to perform further controlled experiments.

## 2 The revised calculation

To this end, we have now redone the calculation using the full RGT force:

$$F = -\frac{kc^3}{(X.V)^3}X + \frac{kc^3}{(X.V)^3} \frac{(X.U)}{(V.U)}V. \quad (3)$$

Here  $F$ ,  $k$ ,  $c$ ,  $X$ ,  $V$ , are as before, and  $U$  is the 4-velocity of the attracted particle. Since the dot products  $X.V$ ,  $X.U$ ,  $V.U$  are scalars or Lorentz invariant, the RGT expression for the 4-force is Lorentz covariant and can be used in any Galilean reference frame.

The full force law may be rewritten in a manner similar to (1) (but without any simplification) as

$$F = \frac{k}{r_r^2} * h_1 \left( \frac{X}{r_r} + \frac{V}{c} * h_2 \right) \quad (4)$$

where the notation  $r_r$  emphasizes the use of retarded distance, and

$$h_1 = [\gamma_v(1 + \frac{v}{c} \cos(w, v))]^{-3} \quad (5)$$

$$h_2 = \frac{1}{\gamma_v} \frac{1 + \frac{u}{c} \cos(w, u)}{1 - \frac{uv}{c^2} \cos(v, u)}. \quad (6)$$

Here  $\vec{w}$ ,  $\vec{u}$ ,  $\vec{v}$  are the 3-vectors corresponding respectively to the 4-vectors  $X$ ,  $U$ ,  $V$ . Explicitly,  $X = (ct, \vec{w})$ ,  $U = \gamma_u(c, \vec{u})$ , and  $V = \gamma_v(c, \vec{v})$ . Further,  $\gamma_v = \gamma(\vec{v}(t_r))$  is the Lorentz  $\gamma$  factor for  $\vec{v}(t_r)$ , and  $\vec{v}(t_r)$  and  $\vec{u}(t)$  are the 3-velocity vectors respectively of the attracting particle (at retarded time  $t_r$ ), and attracted particle (at current time  $t$ ). Finally,  $v = \|\vec{v}\|$ ,  $r_r = \|\vec{w}\|$ , and  $\cos(w, v) = \cos(\vec{w}, \vec{v})$  is the cosine of the angle between the 3-vectors  $\vec{w}$ , and  $\vec{v}$ .

For actual calculations, the terms  $\frac{v}{c} \cos(w, v)$  etc. are conveniently calculated using

$$\frac{v}{c} \cos(w, v) = \frac{\vec{w}}{r_r} \cdot \frac{\vec{v}}{c} = \frac{1}{r_r c} \vec{w} \cdot \vec{v}, \quad (7)$$

$$\frac{u}{c} \cos(w, u) = \frac{1}{r_r c} \vec{w} \cdot \vec{u}, \quad (8)$$

$$\frac{uv}{c^2} \cos(v, u) = \frac{1}{c^2} \vec{v} \cdot \vec{u} \quad (9)$$

We continue to ignore the effects of the spacecraft on the Earth, as in the simplified calculation.[2] In this 1-body case, the functional (delay) differential equations of motion in RGT reduce to ordinary differential equations. We solved these for the six flybys as before using the well known DOPRI code. The initial data were obtained from NASA Horizons interface in state vector format, for a geocentric frame with Earth mean equator and equinox of reference epoch, as displayed below (Table 1).

Table 1: Initial data from NASA Horizons used for trajectory calculations. For each spacecraft the first row contains the Julian day number and Gregorian date. The second and third rows contain the position and velocity vectors in units of km and km/s.

Galileo-1	2448234.000694444	= 1990-Dec-08 12:01:00.0000
X = 2.473779667197228E+04	Y = 2.813889802133052E+05	Z = 6.905794938512032E+04
VX = -4.941514457055581E-01	VY = -8.870912489618423E+00	VZ = -1.967648249648285E+00
Galileo-2	2448965.083333333	= 1992-Dec-08 14:00:00.0000
X = 3.533466991876042E+04	Y = 1.725196573562905E+04	Z = 2.099339477769745E+04
VX = -6.357173143445935E+00	VY = -5.098983436463288E+00	VZ = -5.502425976370658E+00
NEAR	2450836.500000000	= 1998-Jan-23 00:00:00.0000
X = 3.228651780539691E+04	Y = 1.824150970351419E+05	Z = 8.220032102022604E+04
VX = -1.018551223818951E+00	VY = -6.596090066205002E+00	VZ = -2.513531686187036E+00
Cassini	2451408.541666667	= 1999-Aug-18 01:00:00.0000
X = -1.338142479401416E+05	Y = 5.520901141486376E+04	Z = 2.971150587397455E+04
VX = 1.421307835849869E+01	VY = -6.833512053283764E+00	VZ = -3.615965629625016E+00
Rosetta	2453434.208333333	= 2005-Mar-04 17:00:00.0000
X = -9.443459287333746E+04	Y = 3.821864878059387E+04	Z = 1.506916188567711E+04
VX = 4.622615071909428E+00	VY = -1.111383971910594E+00	VZ = -2.124678157771555E-01
Messenger	2453584.958333333	= 2005-Aug-02 11:00:00.0000
X = -4.061872527992503E+04	Y = 1.331170620795427E+05	Z = -7.266170750518485E+04
VX = 1.566976521677327E+00	VY = -3.612768891021895E+00	VZ = 2.457281285018382E+00

The geocentric velocity gain or loss, calculated using RGT, for the various spacecraft are shown in Table 2, and the graphs of detailed trajectories are in Fig. 1.

Table 2: Velocity gain or loss for various spacecraft flybys.  $\vec{v}_r$  is the velocity calculated on RGT, and  $\vec{v}_n$  that on Newtonian gravitation. Column 2 shows the difference of scalar velocities. Comparing it with column 3 shows how much  $\vec{v}_r$  and  $\vec{v}_n$  differ in direction. The last two columns are the “observed” figures for the scalar difference of velocities (in mm/s) as reported by Anderson et al. and those calculated using their formula (2).

Spacecraft	$  \vec{v}_r   -   \vec{v}_n  $ (mm/s)	$  \vec{v}_r - \vec{v}_n  $ (mm/s)	“Observed”	Anderson et al.
Galileo-1	5.96	7.4	3.92	4.12
Galileo-2	7.97	7.98	-4.6	-4.67
Near	3.73	5.68	13.46	13.28
Cassini	-3.32	3.53	-2	-1.07
Rosetta	4.58	6.22	1.80	2.07
Messenger	6.85	9.17	0.02	0.06

To summarise, we get back the original results because the use of two simplifications (use of instantaneous distance and a simplified formula) compensated for each other, so our earlier simplified calculation gave the correct answer!

## 2.1 Discussion

All factors have now been taken into account, in the RGT calculation. This includes the oblateness of the Earth, which is a mere confounding effect, and does not add anything to the energy gain or loss at infinity. Even possible discretization errors have been checked by performing a more analytical calculation (Suvrat Raju, personal communication). So the above figures are valid predictions according to RGT. On the other hand, it seems unlikely that figures so close to the stated values are simply wrong.

## 3 The formula of Anderson et al.

The formula (2) is wrong as it stands since it has no distance dependence. The Rosetta spacecraft in its second flyby showed no such flyby effect. Anderson et al. later explained this was because it was very far away (height 5322 km) at perigee. However, their formula gives no indication of the distance at which it fails or why.

It is therefore paradoxical that the formula nevertheless provides a neat and seemingly exact fit with the data for the other six flybys, where the height of perigee varies all the way from 303 km (Galileo-2) to 2347 km (Messenger). **How could observations so neatly fit a wrong formula?**

### 3.1 Un-modeled residues

In fact the data fitted by Anderson et al. is not direct observational data, but consists of un-modeled residues. This reconstructed data involves various speculative estimates used in modeling, which can be tweaked. Undoubtedly the NASA orbit determination program is very sophisticated, but there are difficulties with the way Anderson et al. have used it.

#### 3.1.1 Case of Galileo-2

For example, consider the case of Galileo-2. According to RGT, there should have been a velocity gain, not the velocity loss calculated by Anderson et al.'s formula. (This, incidentally, is a clear difference between the two theories.)

Further, the perigee of Galileo-2 was very low at 303 km. At this height, atmospheric drag may lead to significant velocity loss. These atmospheric effects at high altitude depend upon unpredictable factors such as solar storms. Hence, retrospective modeling of atmospheric drag is not reliable. Further, we are not informed about the exact details of this modeling by Anderson et al., but are just presented with a speculative estimate.

#### 3.1.2 Case of Cassini

Again, a spacecraft is not a passive point mass but is a powered vehicle. In the case of Cassini, Anderson et al. note how effects of the flyby may have been masked by the effects of thruster firing near perigee. We have no information on the exact details of these thruster firings and have not taken them into account.

#### 3.1.3 Case of NEAR

Anderson et al.'s formula is only for the difference of *scalar* velocities. However, the flyby affects not only the speed, but also direction: in

our RGT calculations in some cases the norm of the vector velocity difference (column 3 of Table 2) is significantly larger than the difference of scalar velocities (column 2 of Table 2). In the case of NEAR this difference itself amounts to 1.95 mm/s, which is twice the extrapolation errors expected by Anderson et al.

Anderson et al. provide no way to estimate such change of direction. Further, even continuous observational data was not always available: as Anderson et al. note, in the case of NEAR there was a blackout of 3 hours 39 minutes in the data near perigee. In this time, the un-modeled change of direction at perigee could have easily been amplified by other effects, such as lunar and solar perturbations.

### 3.1.4 Choice of osculating orbital elements

We accept the observation by Anderson et al. that the post-perigee trajectory cannot be simply extrapolated from the pre-perigee Keplerian trajectory. But Anderson et al.’s calculation of the exact gain or loss of velocity at perigee depends upon the fitting of osculating orbital elements to pre-perigee and post-perigee data.

In contrast, in the above calculations with RGT, we consistently chose orbital elements several hours before perigee (“past infinity”) based on NASA Horizons data. Because details of thruster firing etc. are not available in the ephemeris data, that osculating trajectory at past infinity is not necessarily the best fit: for that we may need to fit osculating orbital elements close to perigee. Table 3 shows the difference between our data and Anderson et al.’s choices. Had we fitted osculating elements to the RGT trajectory, (a better approximation to the real trajectory) close to perigee, that change of initial data could significantly change the velocity gain or loss on RGT.

Table 3: Incoming declinations of velocity vectors of various spacecraft as used in RGT calculations compared with the data of Anderson et al.

Spacecraft	NASA Horizons data (Table 1)	Anderson et al.
Galileo-1	-12.487	-12.52
Galileo-2	-34.027	-34.26
Cassini	-12.914	-12.92
Rosetta	-2.559	-2.81
NEAR	-20.636	-20.76
Messenger	31.964	31.44

That is, the gain or loss in Anderson et al.’s calculations also depends on the exact fitting of osculating orbital elements before and after perigee. This is not a trivial matter, since the real trajectory is subject to relatively huge effects such as those of earth oblateness, or solar and lunar perturbations etc.

### 3.2 Discriminating between the two theories

Such doubts about the un-modeled residues cannot be settled by mere retrospective arguments. The best way forward is by performing a fresh controlled experiment to test both theories.

We suggest one such experiment below.

## 4 The two-satellite experiment

First, we note that RGT is a perfectly general replacement for Newtonian gravitation, and not limited to flybys. It predicts theoretical departures from Newtonian gravitation which can be tested all the way from the laboratory to the galaxy. In particular, experiments to test the gravitational effects of Earth’s rotation can be performed not only with spacecraft but also satellites. This is likely to be far more cost-effective. It also gives far greater precision, since the effect on satellites is larger and also cumulative.

Such a test between RGT and Newtonian gravitation can be easily expanded to include a test of Anderson et al.’s formula. This can be achieved as follows. Anderson et al.’s formula (2) differs from RGT on the effects of Earth’s rotation—this is already clear from the case of Galileo-2. But these differences are most prominent in the case of an equatorial orbit. Thus, the formula (2) involves a factor of  $(\cos \delta_i - \cos \delta_o)$ , where  $\delta_i$  and  $\delta_o$  are the declinations of the incoming and outgoing asymptotic velocity vectors. When the orbit is in the equatorial plane,  $\cos \delta_i - \cos \delta_o = 0$ , so there should be no flyby effect on their formula. In contrast, on RGT the velocity effect will persist in the equatorial plane. This suggests the following experimental design.

Suppose two satellites are established in orbits in the equatorial plane; one co-rotates with the Earth, and the other counter-rotates. On the Newtonian theory the sense of rotation should make no difference to the satellite orbit.

Suppose, further, that the orbits are highly eccentric, so that the approach to perigee can be regarded as effectively a flyby. On Anderson et al.'s formula, since the trajectory is in the equatorial plane, there should again be no flyby effect and no difference from the Newtonian theory.

However, on RGT, the velocity dependent force should accelerate the co-rotating satellite and decelerate the counter-rotating satellite even in the equatorial plane.

To calculate the amount of such gain or loss on RGT, for each orbit, we need the two state vectors, at one instant of time, say apogee. Since the orbit is in the equatorial plane, the  $z$  coordinate and velocity is zero. Let us suppose that the apogee is in the  $x$ -direction, so that, at apogee, the  $x$ -coordinate is just the apogee distance and the  $y$  coordinate is zero. Likewise the  $x$  velocity is zero at apogee, and the  $y$  velocity is, say,  $v_y$ .

To get some definite numbers, let us start off as usual with an initial Newtonian orbit. For this orbit, we take the perigee distance  $r_p = 8500$  km corresponding to a perigee at an altitude of around 2129 km where the atmospheric drag may be neglected. Further, let us take the eccentricity to be  $e = 0.5$ . If  $a$  is the semi-major axis,  $r_p = a(1 - e)$ , so that  $a = 2r_p = 17000$  km. The time period of the orbit  $T = 2\pi\sqrt{\frac{a^3}{\mu}}$  where  $\mu = GM_E \approx 3.98 \times 10^{14} \text{m}^3\text{s}^{-2}$ . Thus  $T = 2\pi\sqrt{\frac{17^3}{3.98}} \times 10^2 \text{s} = 553.352 \times 10^2 \text{s}$ , or around 15.37 hours. Accordingly, we ran the simulation for around 33 hours, starting from apogee.

From the energy equation  $\frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}$ , at apogee ( $r = a$ ) we have  $\frac{1}{2}v^2 = \frac{\mu}{2a}$  or  $v = \sqrt{\frac{\mu}{a}} = \sqrt{\frac{3.98 \times 10^{14}}{17 \times 10^6}} = 0.4838 \times 10^4 \text{m/s} = 4.838$  km/s. This velocity is in the  $y$  direction, either positive or negative, depending upon the sense of rotation of the satellite in its orbit. Thus, initial data are (in units of km, and km/s)

- case 1:
  - position: (17000, 0, 0),
  - velocity: (0, 4.838, 0), and
- case 2:
  - position: (17000, 0, 0),
  - velocity: (0, -4.838, 0).

On Newtonian gravitation, the orbit stays on the initial ellipse. On RGT, it deviates slightly, as follows. In case 1, using a best linear fit,

the satellite gains about 4.2 cm/s velocity in 33.33 hours, or about 3.02 cm/s/day. In case 2, it loses velocity at about the same rate. Such a difference is easily measurable with modern satellite laser ranging systems which have millimeter accuracy.

These are just illustrative figures: two counter-rotating satellites in exactly the same orbit would obviously collide! The design will need to be optimised for an actual experiment.

## 5 Conclusions

RGT is based on the theoretically compelling requirement of Lorentz covariance. Accordingly, the unmodeled residues reported by Anderson et al. cannot be accepted at face value, and controlled experiments are needed to test RGT. The two satellite experiment provides huge gains in precision by (a) choosing appropriate orbits to make the RGT velocity effect larger, and (b) because the RGT velocity effects in an elliptic orbit are cumulative across orbits. The predicted effects on satellite velocity, of the order of 1 m/s across 10 days, are easily measurable, and the problem of random perturbations is minimised by choosing a pair of satellites to get correlated gain and loss in velocity.

## Acknowledgment

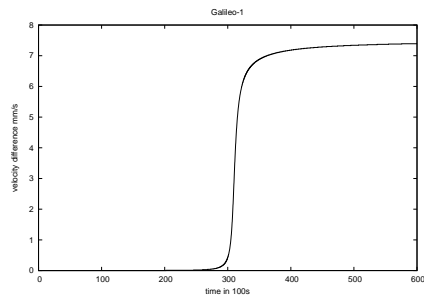
I am grateful to Dr Suvrat Raju for cross checking the validity of the flyby calculations with RGT by using another numerical technique. I am grateful to Archishman Raju for help in formulating the numerical model of the Earth.

## References

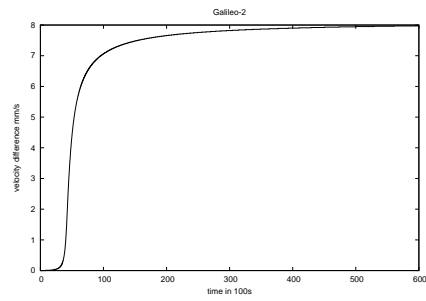
- [1] C. K. Raju. *Time: Towards a Consistent Theory*, volume 65 of *Fundamental Theories of Physics*. Kluwer Academic, Dordrecht, 1994.
- [2] C. K. Raju. Retarded gravitation theory. In Waldyr Rodrigues Jr, Richard Kerner, Gentil O. Pires, and Carlos Pinheiro, editors, *Sixth International School on Field Theory and Gravitation*, pages 260–276, New York, 2012. American Institute of Physics. 1102:2945v3.

- [3] John D. Anderson and et al. Anomalous orbital-energy changes observed during spacecraft flybys of earth. *Physical Review Letters*, 100:091102, 2008.
- [4] C. K. Raju. Functional differential equations-4. retarded gravitation. *Physics Education*, 31(2), April-June 2015. [http://www.physedu.in/uploads/publication/19/309/1-Functional-differential-equations-4-Retarded-gravitation-\(2\).pdf](http://www.physedu.in/uploads/publication/19/309/1-Functional-differential-equations-4-Retarded-gravitation-(2).pdf).

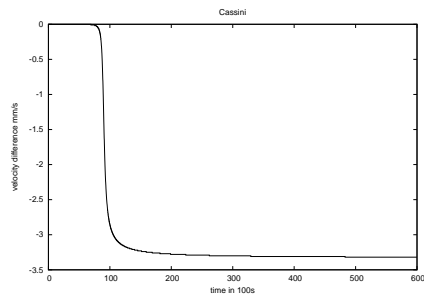
Figure 1: Graphs of velocity gain or loss for the six flybys. In (c) there was a velocity loss



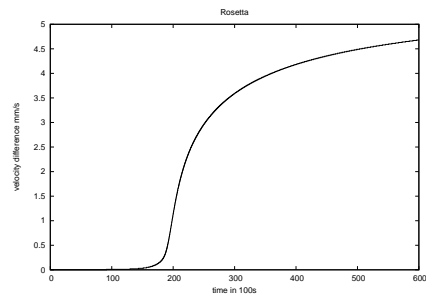
(a) Galileo-1



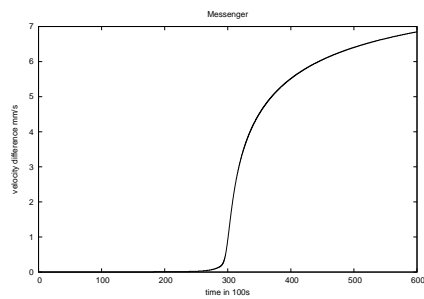
(b) Galileo-2



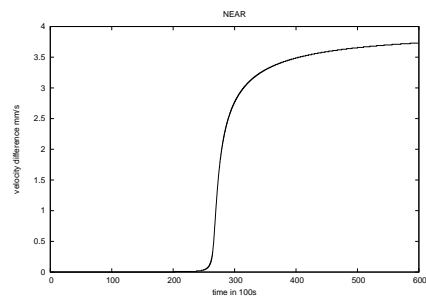
(c) Cassini



(d) Rosetta



(e) Messenger



(f) NEAR