

Decolonising mathematics: towards an easier and better math

DRAFT¹

C.K.Raju

Indian Institute of Advanced Study

Rashtrapati Nivas, Shimla 17105

ckr@ckraju.net

Abstract

Mathematics is not universal: colonial education changes math teaching from traditional normal math (which uses both reasoning and empirical proofs) to formal math (which prohibits empirical proof). Formal math adds enormously to the difficulty of math, but nothing to its practical value. Decolonisation of math therefore advocates rejection of colonial/formal math and return to traditional normal math. This makes math easy while preserving its practical value.

The colonised fear that “it works”, and any change might cause harm. They fail to ask *which* math works: normal or formal math? They cannot explain exactly what practical value formal math adds to normal math which worked for millennia. For example, colonial education teaches that formal “real” numbers are essential for calculus. But the application of calculus to calculate even rocket trajectories uses the normal math of inexact numerical calculation. today done on computers which cannot handle “real” numbers, and use floating point numbers, which do not even obey the associative “law” for addition. Because decolonisation makes math easy, it enables students to solve harder problems, such as brachistochrone with resistance, not covered in university courses. This has been repeatedly demonstrated in teaching experiments on decolonised math conducted across various universities and countries, in the last decade.

Colonial math, due to its difficulty, makes most people ignorant of math. This adds political value to the coloniser by making the colonised mentally subservient to Western/White authority.

Formal reasoning, or reasoning without facts, was first invented by the church during the Crusades, to suit its convenience, in its theology of reason, which copied and modified the Islamic theology of reason. Further, to claim reason as a Christian inheritance, the church concocted the myth that early Greeks, such as “Euclid” and Pythagoras, had used formal reasoning in geometry. This is false. Amusingly, however, both colonised and Western minds, hegemonised by the church, went by the myths, and did not read “Euclid’s” Elements carefully, even in centuries. This resulted in the extreme foolishness of the Cambridge exam rules about “Euclid’s” geometry even in the 20th c. imitated by colonial math teaching. Ultimately, the truth was admitted, that the Elements has no valid deductive proofs, and Hilbert rewrote the book to make it fit the myth, and provided the first formal proof of the “Pythagorean theorem”! Hilbert’s synthetic geometry prohibits length measurement as an empirical process, but defines area without first defining length. These absurdities have been heightened in present-day colonial teaching of geometry, which both allows and disallows length measurement. The colonised need to stand up to such blatant absurdities, even at the school level, as taught to children today.

The difficulty of formal math is justified by claiming it has “superior” epistemic value. This claim of “superiority” is as baseless as the racist claim of superiority promoted for centuries by the church. We recap our earlier critical re-examination. Everyone accepts that empirical proofs are fallible, but it is mere superstition that deductive proofs are infallible. Complex deductive proofs are MORE fallible than empirical proofs, and the keenest human mind INVARIABLY errs when asked to make a long chain of deductions. Because a deductive proof may have errors, its validity can only be established inductively, by repeating it. Worse, most people simply accept the validity of a deductive proof on mathematical authority, more fallible than empirical proof. Hence, deductive proofs are actually more fallible than empirical and inductive proofs.

Further, even a valid deductive proof need not result in valid knowledge for the initial axioms or postulates may be invalid. But how do we check whether the axioms are valid? If we check empirically, deductive proofs do not have any additional epistemic value over empirical proofs. If, on the other hand, as in formal mathematics, the initial axioms are pure metaphysics, which cannot be checked empirically, then any nonsense proposition whatsoever can be proved as a formal mathematical theorem, as my rabbit theorem demonstrates.

Actually, the axioms of formal mathematics involve a metaphysics of infinity, allied to church dogmas of eternity, though this is too hard for most people to understand. However, this metaphysics has political value, for it allows church dogmas to creep into science, through formal mathematics: as in Stephen Hawking’s creationist/apocalyptic conclusions. Many people accept this conclusion on faith in Hawking’s authority and are misled to believe that science

¹ This article was dictated on to a computer. with a screen too far away to read easily, since the author was bed-ridden. It may contain typos.

has proved the truth of church dogmas. Hawking's co-author Ellis got the Templeton prize of a million US dollars, for putting together "science", and "religion". But when challenged to debate this conclusion, during my visit to the University of Cape Town, Ellis ran away from debate, and contested me only by the low-level method of planting slander against me in the press.

Much of this metaphysics of infinity was added on by West, to normal math, especially the calculus, which it imported from India. This metaphysics was added on to suit other religious superstitions long prevalent in Western ethnomathematics. Thus, for example, Plato related mathematics to the eternal soul. Proclus explains that mathematics arouses the eternal soul by sympathetic magic, because mathematics has eternal truths. Later, this developed into the Western superstition that God rules the world with eternal laws which are hence written in the language of eternal truth: mathematics. This led to the further Western superstition that mathematics is exact. However, there is nowhere in the world where even the "Pythagorean theorem" is exactly true. To offset lack of practical value, some appeal to the supposed aesthetic value of formal math. However, the fact is that millions of students today reject mathematics as ugly. (Plato prescribed both mathematics and music, and millions of students still like music even if they have never been taught music.) Talk about aesthetics in formal mathematics is, therefore, pure fantasy which benefits the formal mathematician.

Two key decolonised courses in math have been developed and have been tried out over the last decade: a course on decolonised calculus (at the university level), and a decolonised string geometry (at the school level). A textbook is ready for the geometry course, and extensive lecture notes are available for the calculus course. No apocalypse occurred, as a result of trying out these courses. But while the colonial masters, and their colonised allies, were enraged, decolonised math made the students happy, clarified their concepts, and made them more productive.

Mathematics is not universal: normal vs formal math

Mathematics is NOT universal, contrary to the widespread colonial superstition. Traditional normal mathematics, found in almost all cultures, differs from the formal mathematics brought by colonial education, and taught today in universities, and introduced from middle school onward.

What is the difference? Briefly, normal mathematics uses BOTH reasoning and empirical proofs, like science. An empirical proof is one which involves the senses: anything one can see, touch, hear etc. Formal mathematics however prohibits the use of facts, or anything empirical,² and demands that reasoning must begin from authoritatively laid down axioms.³

Most people are unaware that formal mathematics prohibits the empirical, but this is obvious from the definition of a formal mathematical proof. A formal mathematical proof is a sequence of statements (propositions) in which each statement is either an axiom or is derived from preceding statements by some rule of reasoning. Any statement in this sequence is a theorem. There is no scope to introduce in this sequence any statement of the sort, "I see this, therefore it is true". Another clarification: an axiom does NOT mean a self-evident truth. As the class IX Indian math text⁴ correctly explains, the term "axiom" is today used interchangeably with the word "postulate" to mean anything *assumed* to be true (irrespective of facts).

Formal math adds difficulty but no practical value

2 See, e.g., C. K. Raju, "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhāsā", *Philosophy East and West*, 51:3 (2001) pp. 325–362. <http://ckraju.net/papers/Hawaii.pdf>, And at least the abstract at <http://ckraju.net/papers/Hawaii-abstract.pdf>.

3 For the stock definition of a formal mathematical proof see, e.g., the text by E. Mendelson, *Introduction to mathematical logic*, van Nostrand, New York, 1964, p. 29.

4 NCERT, *Mathematics*, Class IX, <http://ncert.nic.in/textbook/textbook.htm?iemh1=5-15>, See, also, related material (Appendix 1) on "proof in mathematics".

So, what difference does it make to $1+1 = 2$? The difference is this. Children are taught $1+1 = 2$ in kindergarten by showing them (images of) two apples say. This is an empirical process (a process involving the senses) for one can see (or touch) the apples in question. This is normal mathematics.

However, the empirical is prohibited in formal mathematics. Consequently, in formal math, to prove $1+1 = 2$ axiomatically, Whitehead and Russell needed 378 pages in their *Principia*.⁵ This cannot be taught to children. Indeed, the vast majority of people in the world, including most professional mathematicians, would be unable to decipher even a single sentence on that page 378. So, clearly, formal mathematics makes it too difficult for most people to understand why $1+1=2$.

But this huge complexity adds nothing to the practical value of arithmetic. In a grocer's shop, for example, it is still necessary to relate numbers empirically to objects in the manner of normal mathematics. That formal math adds no practical value to normal arithmetic is also clear from the fact that people performed marvellous feats of engineering from Egyptian times, from long before the advent of formal math. Most people have managed lifelong to do their groceries without an inkling of formal mathematics or its proof of $1+1 = 2$.

Further, the actual level of difficulty is greater than most people imagine. In formal mathematics, 1 as a "natural number" differs from 1 as a "real" number. To prove $1+1 = 2$ in "real" numbers, from first principles, in the manner of Russell, and without assuming any results from axiomatic set theory, one might need 1000 pages. It has never been done. After the panel discussion on decolonising science in the University of Cape Town, in 2017, I challenged the formal mathematician on the panel to prove $1+1 = 2$ in "real" numbers. He could not do it. He had wrongly assumed that by 1 I meant the natural number 1, and complained that I had suddenly switched from natural numbers to "real" numbers! (The challenge is still open to anyone; try it on a professor of mathematics you imagine is knowledgeable.) So, formal mathematics makes simple math so enormously difficult, as to be beyond even senior professional mathematicians from Africa's supposedly top ranking university.

What is decolonised math?

This deep ignorance results in deep faith. People who know nothing of either formal math or decolonised math often assert that it is impossible to decolonise mathematics. Falsehoods are spread about it by the politically motivated.

Decolonised math is not some exotic monster; decolonised math simply reverts to normal mathematics, which existed and worked very well for thousands of years. Colonial education simply declared formal mathematics as "superior" without once comparing formal with normal mathematics, just as the West/Whites were declared superior for centuries based on silly superstitions. Because they have nothing better, the supporters of formal mathematics consistently avoid any reasoned comparison between formal and decolonised math, through public debate. This debate-avoidance itself corroborates that they very well know that formal mathematics is inferior.

To reiterate, all practical value comes from normal math, so decolonising math by reverting to normal math preserves all practical value of math. But it eliminates the needless and huge extra difficulties created by formal mathematics. Because it makes math easy, students of decolonised maths are able to solve harder problems not covered in usual university or school courses. This has been demonstrated in pedagogical experiments carried out in schools and universities across various countries, in the last decade. These experiments involved two courses in decolonised mathematics: university calculus, and school geometry. The teaching experiments on decolonised university

5 A. N. Whitehead and Bertrand Russell, *Principia Mathematica*, Cambridge University Press, 2nd ed. 1927 (reprint 1963).

calculus were carried out in India, Malaysia, and Iran. In Malaysia, it included four groups of students, including one group of postgraduate students in mathematics.⁶ In India, it included both undergraduate students of science and engineering, as well as postgraduate students in social sciences.⁷ In Iran, there was one mixed group.⁸ In Palestine, the attempt to teach decolonised math was regarded as so serious a threat that Israel blocked it by denying a visa.⁹ Lebanon has expressed interest,¹⁰ but no actual courses have started.

The current colonial courses on geometry are seriously deficient in practical value. For example, students do not learn the simple practical task of determining the area of an agricultural field the boundaries of which are not straight lines. Further, as our pre-test repeatedly demonstrated, most students today are unclear even about basic geometrical concepts such as point, straight-line, angle, etc.

A little-known reason for the inferiority of colonial math teaching is that it imitates Western cultural practices. Europeans were backward and inferior in mathematics: fractions were introduced in the Jesuit syllabus only in the 16th c. The European difficulty with fractions resulted in the defective Julian calendar, they could not easily articulate the right duration of the tropical year, and instead represented it by the simplified but incorrect fraction $365\frac{1}{4}$ days. Further, even the reformed Gregorian calendar of 1582 could not state the duration of the year as a precise fraction; it instead used a complicated system of leap years which left everyone confused whether the year 2000 was a leap year not.

Europeans were similarly inferior in geometry, and, hence, could not correctly determine the size of the earth until the 17th c. Consequently, they could not correctly determine longitude at sea until the 18th c. This led to a huge navigational problem (specific to Europeans) as acknowledged by various European governments, such as the British act of parliament of 1712, setting up a Board of Longitude, because of the huge loss of life and wealth that this ignorance of navigation resulted in.¹¹

6 C. K. Raju, “Teaching Mathematics with a Different Philosophy. 1: Formal mathematics as biased metaphysics”. *Science and Culture* 77 (2011) 275–80. . arxiv:1312.2099. “Teaching Mathematics with a Different Philosophy. 2: Calculus without limits”. *Science and Culture*, 77 (2011) 281–86. . arxiv:1312.2100. This is a report of teaching calculus at the Universiti Sains Malaysia to 4 groups 1 group of post-graduate students in math, 1 group of applied math students, 1 group of pure math students, and 1 group of non-math students.

7 C. K. Raju, “Calculus without Limits: Report of an Experiment” 2nd People’s Education Congress, HBCSE, TIFR, Mumbai, Oct 2009. In Proc. <http://ckraju.net/papers/calculus-without-limits-paper-2pce.pdf> This a report of teaching at the Central University of Tibetan Studies. See the related blog post on “The 5-day course on calculus without limits” at <http://ckraju.net/blog/?p=34>. See also the blog regarding the course on “Calculus for social scientists” at Ambedkar University, Delhi, <http://ckraju.net/blog/?p=83>. There was also a later course for science and engineering students at the SGT University, Delhi NCR, for which see the poster at <http://ckraju.net/sgt/poster-calculus-without-limits.pdf>, the video of the introductory lecture at <https://youtu.be/0sdimbGwUCA>, and a sample tutorial sheet at <http://ckraju.net/sgt/Tutorial-sgt.pdf>.

8 See the related blog post on ‘Calculus without limits in Tehran’, <http://ckraju.net/blog/?p=84>.

9 <http://ckraju.net/blog/?p=157>.

10 <http://ckraju.net/blog/?p=161>. C. K. Raju, “How to break the Western hegemony perpetuated by the university: decolonised courses in mathematics, and the history and philosophy of science”, <http://ckraju.net/papers/Beirut-paper%20for%20ias%20journal.pdf>. (Submitted, IAS journal.)

11 For more details, see C. K. Raju, *Cultural Foundations of Mathematics: the nature of mathematical proof and the transmission of calculus from India to Europe in the 16th c. CE*, Pearson Longman, 2007, chp. 4, “Latitude, longitude and the globe” .

However, the decolonised course on geometry¹² not only teaches students how to calculate the area of an agricultural field, with non-straight boundaries, it also teaches students how to determine latitude, longitude, and the size of the earth, using traditional techniques of string geometry, used in both Africa and India, since ancient times.

Similarly, before the decolonised course on calculus, a pre-test was administered. It repeatedly demonstrated that most university students of calculus are conceptually confused about EVERY basic notion of calculus such as derivative, integral, etc. Despite the extreme prolixity of current calculus texts (which typically run into over 1300 pages), for example Thomas¹³, Stewart¹⁴ etc., they do not learn simple practical tasks such as the correct angle at which to throw a javelin, so it will go furthest. (Because they learn only simplified math, they wrongly imagine it is 45 degrees.) They remain confused even about the first experiment in science, the simple pendulum, since the theory is deemed too hard to be taught in schools. Students go by the simplified formula, that the time period is independent of amplitude and not what they easily observe to the contrary. This is a very bad way to do science. In contrast, the decolonised course on calculus, teaches students how to solve these practical problems,¹⁵ how to fit theory to experiment, instead of fitting experiment to theory due to ignorance of mathematics.

Briefly, though the colonised mind is taught to be superstitiously terrified at the thought of doing something non-imitative, the fact is that decolonised courses in mathematics result in real knowledge of math and greater practical value.

However, by “practical value” I mean the practical value of mathematics for applications to science, engineering, and commerce etc.

The difficulty of formal math adds political value by teaching ignorance

But there is another sort of practical value, namely, *political* value. Students who complain about the difficulty of mathematics do not seem to understand that the difficulty of formal mathematics is a source of political power for the master. It results in widespread ignorance of mathematics among the colonised. The ignorant have no option but to ask someone else, someone they trust. And the colonised are taught to trust Western authority, despite its extreme dishonesty and exploitative nature. To teach people never to trust Western authority, I have posted an example of dishonesty at the highest level of formal mathematics.¹⁶)

As another common example, today most people, if they want to know something, will ask Google, and Google will take them to Wikipedia. Wikipedia officially accepts only secondary sources.

12 C. K. Raju, *Rajju Ganita: String geometry for class IX*. See, also, the cover (<http://ckraju.net/geometry/Rajju%20Ganita%20cover-front-back-r.pdf>), the table of contents (<http://ckraju.net/geometry/Rajju-Ganit-toc.pdf>), and a draft teacher's manual (<http://ckraju.net/geometry/Rajju-ganit-draft-teacher-manual.pdf>). For details of pedagogical experiments, and teacher-training workshops, see e.g., the poster at <http://ckraju.net/blog/?p=155>, and the media reports at <http://ckraju.net/blog/?p=156>.

13 G. B. Thomas, Maurice D. Weir, Joel Hass, Frank R. Giordano, *Thomas' Calculus*, Dorling Kindersley, 11th ed., 2008. (1228 + 34 + 80 + 14 + 6 + 6 + xvi = 1384 pages; size 11x 8.5 inches).

14 James Stewart, *Calculus: early Transcendentals*, Thomson books, 5th ed, 2007. (1168 + 134 + xxv pp. = 1327 pages; size 10 x 8.5 inches + CD).

15 See a typical tutorial sheet at <http://ckraju.net/sgt/Tutorial-sgt.pdf>.

16 For example, a math faculty of the University of Cape Town, one Murugan, dishonestly called the ability to solve harder problems as “Bantuization” with a view to mislead people, and ensure that the majority, especially of Blacks, remains perpetually ignorant of math. As another example of dishonesty of formal mathematicians at the highest level, see the following posts. “Plagiarism by ex-President of the Royal Society”. 1. The facts: <http://ckraju.net/blog/?p=183>, 2. The cover-up by the American Mathematical Society: <http://ckraju.net/blog/?p=184>, 3. Lessons for decolonisation of math: <http://ckraju.net/blog/?p=185>.

Unofficially, it declares that only Western sources are reliable, and that non-Western sources are unreliable (and deletes them). That is, ignorance begets obedience to (White/Western) authority, as the ultimate arbiter of truth. (The knowledgeable person could go by her own knowledge and revolt against White/Western authority.)

Ignorance plus such blind trust begets superstition: the colonised superstitiously venerate everything Western.

A common superstition: “it works” (but *which* math works?)

One aspect of this superstition commonly finds expression thus: “it works” they say, referring to the use of mathematics in science and engineering. But *which* mathematics works? Normal mathematics or formal mathematics? Without knowing the difference between the two, and without knowing how it works, the colonially educated, because of their veneration for the West, superstitiously attribute the success of science and engineering to the master, meaning formal (White/Western) mathematics.

They do not ask a common sense question: how does the prohibition of empirical facts contribute to building better bridges? They do not ask how bridges (or pyramids) were built before the advent of formal mathematics. Instead they do what colonial education taught them to do: defend the master by foolishly asking why are our bridges still standing? Obviously because all bridges are built using normal mathematics, but the colonised superstitiously give credit to formal mathematics. Let us dispel this superstition.

That formal math does not add anything to the practical value of normal mathematics is true not only of groceries but also true of rocket trajectories: they are calculated using normal mathematics, and NOT formal mathematics. However, this aspect is not clear to the colonised mind which often lacks the foggiest idea of how rocket trajectories are actually calculated, by NASA, say.

Briefly, rocket trajectories are calculated by numerically solving differential equations; today this is done on a computer. (Recall that it was through this method of numerically solving differential equations that the calculus first originated in India in the fifth century, as normal mathematics, a thousand years before Newton and Leibniz,¹⁷ and that is how decolonised courses in calculus have been taught in the last decade.) The new feature added by formal mathematics was “real” numbers. “Real” numbers are considered essential to the formal mathematics of differential equations, as taught in present-day university courses on calculus. But “real” numbers cannot be used in a numerical calculation because it may take an eternity of time even to write down a single real number. The human mind may evade this reality by diving into fantasies. But a computer can only numbers which are actually real, not merely deceptively called “real” numbers. Hence, a computer CANNOT use these unreal “real” numbers and instead uses what are called floating point numbers. Floating point numbers are quite different: for example, even the associative “law” for addition fails for floating point numbers.¹⁸

What is true for rocket trajectories is true for all practical applications of mathematics to science and engineering: they involve calculation, not formal proof. This should be evident even to the layperson from the fact that most of these are done today using computers. All calculations, whether done by hand or machine always done to finite precision. So, formal mathematics adds absolutely nothing to the practical value of mathematics for science and engineering, as the ignorant and superstitious colonised mind believes. We will see further reasons for this later on.

17 C. k. Raju, *Cultural Foundations of Mathematics: the nature of mathematical proof and the transmission of calculus from India to Europe in the 16th c. CE*, Pearson Longman, 2007.

18 See the Hawai’i paper cited above for a simple computer program demonstrating this failure. For an in-depth understanding of why the associative law fails, see my online lecture notes on floating point numbers: <http://ckraju.net/hps2-aiu/floats.pdf>.

Epistemic value? The social myths

Let us consider next the official claim that formal mathematics, even though it adds nothing to practical value, adds to the *epistemic* value of mathematics.

Before taking up this claim philosophically we first need to dispel a variety of myths used to spread this claim, which interfere with a proper understanding.

It is a common fallacy that the use of the empirical in normal math excludes the use of reasoning. Science, too, is based on empirical experiments, but does not exclude reasoning! Normal mathematics, likewise, uses both empirical proofs and proofs based on reasoning, just like science.

Another false corollary is that normal mathematics lacked proof. This too is false as shown by, say, the Indian proof of the so-called Pythagorean theorem.¹⁹ However, in the West, the church was interested in mathematics for its political value for persuasion, based on reasoning, not in science and engineering. Hence, it over-valued proof.

Because the church used reasoning in its rational theology, it glorified itself. Therefore, another common myth is that Western/formal mathematics is unique in using deductive reasoning. This is completely false. The fifth century Aryabhata, for example, deduced that the earth is round²⁰ from the *observation* that far off trees cannot be seen (and the horizon is circular).²¹ This involved a deduction: he did not travel to space to see the earth from space. Nor did he begin with any axioms but with empirical observations that can be easily made on earth.

Another related myth, systematically spread by colonial education (e.g. the class IX Indian math text) is that Greeks were unique in using deductive reasoning in math. This is a total fib: others did use deductive reasoning, as already pointed out. Further, as already explained in the earlier talk, “Euclid’s” Elements has axioms and proofs, but it does not have a single valid axiomatic proof²² which avoids the empirical. Its very first proposition uses an empirical proof. This is true also of the 4th proposition on which the proof of the “Pythagorean theorem” depends.

Western scholars hegemonised by the church did not read the book carefully (though it was a prescribed text) and just went by church myths about it. This resulted in the foolishness of the Cambridge math syllabus²³ until the 20th c. Thus, the Cambridge authorities failed to notice that the text on “Euclid” they specially commissioned uses empirical proofs in its first and fourth proposition (side angle side proposition). If empirical proofs are admitted, then there is no significance to the exact order or sequence of the propositions (as there is in formal proofs which rely on previously proved propositions). Hence, in Indian texts, the so-called Pythagorean proposition is the first proposition to be proved, and not the last (or second last) proposition as it is in “Euclid’s” Elements.

19 See the Hawai’i paper cited above. Also, “Mathematics and Culture”, in *History, Culture and Truth: Essays Presented to D. P. Chattopadhyaya*, ed. Daya Krishna and K. Satchidananda Murthy, Kalki Prakash, New Delhi, 1999, pp. 179–193.

20 आर्यभटीय, गोलपाद, 6-7 compares the earth to a kadamba flower and asserts it stands supportless in space.

21 लल्ल, शिष्यधीर्द्धिद, chp. 20, मिथ्याज्ञाननिराकरणम्.

22 B. Russell, “The Teaching of Euclid”, *The Mathematical Gazette* 2 (33) (1902), pp. 165-167.
<http://ckraju.net/geometry/Bertrand%20Russell%20on%20Euclid.htm>.

23 <http://ckraju.net/geometry/cambridge-note.html>.

But on the myth about the book, “Euclid’s” chief contribution was to arrange the propositions so that the proof of any proposition only relied on previously proved propositions. But, the Cambridge authorities did not read the actual text, or notice that it uses empirical proofs. They failed to apply their mind that this makes the order of the propositions irrelevant. Instead, going by the myth, they foolishly made an exam regulation that the order of the propositions in “Euclid’s” Elements must be followed! (This was in 1888, not the Christian “Dark Ages”, and was intended to “liberalise” the earlier Cambridge rule of rote learning! At this time, colonial education in math had already come to the colonies and imitated this foolishness.)

Hilbert’s synthetic geometry: and the resulting absurdities

It was finally admitted at the turn of the 20th c. that there are no valid axiomatic proofs in “Euclid’s” Elements. To “save the story”, it was argued that “Euclid” intended to give such such proofs, but erred in executing his intentions; how does one know the intentions of a mythical person? Anyway, the collapse of the “Euclid” story led to the birth of formal mathematics, with Hilbert’s attempt²⁴ to provide a valid formal proof of the “Pythagorean theorem” and of the other propositions in the Elements. Few are aware of it or the resulting absurdities in Hilbert and in present-day colonial education in geometry.

Hilbert’s geometry is called synthetic²⁵ for the following reason. Length measurement uses the empirical process of superposition. One is required to pick up a scale, and superpose it over a line segment, and *see* which markings on the scale coincide with the endpoints of the line segment. Since the empirical is outlawed in formal mathematics, so is superposition and length measurement, in Hilbert’s geometry. The word “synthetic” means non-metric, i.e., that length measurement is disallowed. As it is picturesquely put, the instruments of synthetic geometry are the unmarked straight-edge and the collapsible compasses (compasses which are a bit loose, so that they collapse when lifted from paper, so that distances cannot be picked and carried). Though length is not defined, nevertheless, area is defined, since that is required for the proof of the “Pythagorean theorem”. It requires considerable mental gymnastics to understand how to define area without first defining length, and we will not engage in these useless gymnastics here.

The key point this brings out is that (a) there was no axiomatic proof of the Pythagorean theorem prior to the 20th c., and Hilbert gave the first such proof, and (b) the uniquely distinctive feature of formal math is not the use of reason (which was used also in normal math) but the prohibition of the empirical, though the colonially educated are never clearly informed about it.

This results in further absurdities in present-day colonial education in geometry. Proposition 4 of the original Elements is known as the side-angle-side proposition, or simply SAS for short. In the original Elements SAS was proved empirically by superposition: placing one triangle on top of another triangle and *seeing* that the two are equal (NOT congruent, which is Hilbert’s terminology). (Indeed, that is how I learnt it in school in the 1960s.) This is an empirical proof, outlawed in formal math. After the Sputnik “crisis”, the US changed its math syllabus.²⁶ The rest of the colonised world, which has learnt to imitate uncritically, blindly imitated that change. Because the empirical is outlawed in formal math, SAS was changed from a theorem to a postulate. (As the class IX Indian school text correctly explains the word “postulate” is today synonymous with the word “axiom” and means anything that is assumed to be true.)

24 D. Hilbert, *Foundations of Geometry*, trans. E. J. Townsend, Open Court, La Salle, 1950.

25 E. A. Moise, *Elementary geometry from an advanced standpoint*, Addison-Wesley, 1963.

26 School Mathematics Study Group, *Geometry*. New Haven: Yale University Press, 1961.

However, this creates a peculiar problem: if the empirical process of superposition is disallowed then, to be consistent, the empirical process of length measurement by superposing a scale on a line segment should also be disallowed. But doing that would expose the whole game that formal mathematics prohibits the empirical. It would become apparent to even the most indoctrinated colonised mind that such mathematics cannot have any practical value for practical requirements such as measuring the length of a tennis court.

Therefore, our colonised educators swallow the contradiction in the manner of the church. Though SAS is a postulate, since superposition is disallowed, every child is furnished with a ritualistic compass box or geometry box, to enable measurement by superposition. So the empirical is both allowed and disallowed. This is done while swearing by the use of deduction: it is a well-known principle that from such a contradiction any nonsense proposition whatsoever can be deductively proved.

That is, the first book in formal mathematics, by Hilbert, attempted to rewrite “Euclid’s” Elements, to make it fit the church myths about it. This clearly resulted in loss of practical value (because length measurement was disallowed) irrespective of any gain in epistemic value. The resulting attempt in colonial education to reconcile the teaching of formal geometry with minimal practical value resulted in the most foolish imaginable absurdities.

Like rote learning, these absurdities actually suit colonial education. The function of colonial education is to teach mental obedience: teaching children to swallow this kind of stupidity, without protest, is a good way to teach obedience. The fact that this method worked is clear from the fact that among the millions of colonially educated, across two centuries, not a single person stood up to object to such absurdities in the teaching of elementary school mathematics.

Epistemic value of deduction? The philosophy

All these utter absurdities in our school texts are justified on the claim that formal deductive proofs (minus facts) have some special epistemic value. But this claim about the epistemic value of deduction is just another church superstition,²⁷ which has pervaded Western thought because of church hegemony.

Let us understand the genesis of this superstition. Everyone admits that errors in empirical proofs and observations happen. The classic example in Indian philosophy is that, in dim light, one may mistake a rope for a snake or vice versa.²⁸ Nevertheless, every traditional system of Indian philosophy, without exception, accepts empirical proof as the first means of proof. Likewise, the first thing one learns the science laboratory, is that there can be errors in observation, and how to handle them inductively. Nevertheless, science, too, is based on experiment and observations.

The naïve Western thinking seems to have been that eliminating the empirical would eliminate errors, And therefore that proofs based on deduction (reasoning minus the empirical) are infallible. The very word “infallible” should warn people about a related church superstition about papal infallibility. How do we know? Take for example the 378 page proof of $1+1 = 2$. As stated earlier, most people would not understand anything in the proof. How, then, do they know that this proof is error free? Not even one typo? Indeed, most people do not know it of their own knowledge; they just believe it because they have blind faith in mathematical authority, exactly like the superstitious

27 C. K. Raju, Decolonising mathematics, *AlterNation* 25(2) (2018), pp. 12-43b <https://doi.org/10.29086/2519-5476/2018/v25n2a2>.

28 E.g., Nyayavali 304 रज्जुसर्पन्यायः https://sanskritdocuments.org/doc_z_misc_major_works/nyaayaavali.html?lang=sa.

people who believe in papal infallibility. While the claim of papal infallibility may have lost credibility today, people still believe scientists and mathematicians. This claim is false. The number of erroneous proofs of the Riemann hypothesis, including those given by the most authoritative mathematicians, are legion. As another example, nobody knows for sure whether the purported formal proof of the four colour theorem is valid. (The four colour theorem says that at most four colours are needed to colour any map in such a way that no two adjacent countries have the same colour.)

For my PhD thesis, I had to do a long symbolic calculation which ran into over hundred pages, and took over a month. (Those were days before the personal computer.) I could not be sure that the calculation was right and free of errors. The best I could do was to repeat it until I got the same result twice. Obviously, this inductive method was no “infallible” proof of the validity of the conclusion. That is, a complex piece of deduction is almost sure to contain errors, and the only way to check is to repeat the deduction inductively. Therefore, deduction is fallible, and surely more fallible than induction, because inductive methods are needed to check the validity of deduction. But, to reiterate, the colonially educated go by myths, and fail to apply the merest common sense.

In fact, the mind is more fallible than the senses: therefore, it makes errors more easily. Take for example the game of chess, which is based on pure deduction. The best human players INVARIABLY make a mistake, and therefore lose to a machine. In contrast, one does not invariably mistake a rope for a snake or vice versa. That is, deduction is more fallible than empirical proof.

But that is not the end of the story. Let us suppose for the sake of argument that we are somehow sure that we have a valid deductive proof. But even a valid deductive proof is no guarantee of valid knowledge. This was argued from ancient times by the ancient Indian Lokayata or “people’s philosophers”. To demonstrate this, a man, at night, makes a wolf’s footprints on the ground. The next morning people see the wolf’s footprints and invalidly deduce that a wolf was around. That is, even if the deduction is valid, the conclusions or the theorem may be invalid because we began with the wrong assumption or postulates. That is, accepted by formal mathematicians that a mathematical theorem is at best relative truth, relative to the assumptions or axioms.

Therefore, to decide whether a formal mathematical theorem is valid knowledge we need to check whether the initial assumptions are valid knowledge. But how exactly do we check the validity of the axioms? If we do so empirically, then our belief in the axioms is fallible, since empirical proofs are fallible. And the deduced conclusions (or theorems) are just as fallible, because a chain is only as strong as its weakest link. Therefore, the 378 page proof of $1 + 1 = 2$ adds nothing to practical or epistemic value, but only a huge amount of prolixity and difficulty.

On the other hand, let us suppose that the axioms cannot be checked empirically, and, hence are pure metaphysics. (This is the case in formal mathematics. As we will see it is impossible to check the axioms of formal mathematics empirically, for they involve a metaphysics of infinity.) For the time being we go along with Russell who explained that we can begin with any assumptions which seem “amusing”.²⁹

Then, any nonsense proposition whatsoever can be deductively proved as a formal mathematical theorem. This is illustrated by my rabbit theorem. The nonsense proposition “all rabbits have two horns” is deductively proved as a theorem as follows.

1. All animals have two horns.
2. A rabbit is an animal.
3. Therefore, a rabbit has two horns.

29 Bertrand Russell, “Mathematics and the metaphysicians”, in: *Mysticism and logic and other essays*, Longman Green and Co., London, 1919, pp. 71-96.

The deduction is valid, but the premises are faulty. In particular, the first premise is faulty. But how exactly do we know it is faulty? We know this only empirically, for we can easily point to animals which don't have horns. But, we have already set aside the empirical method of deciding the validity of the initial assumptions. Our key criterion now is amusement, and I find it extremely amusing that all animals have two horns. But, as the reaction to my censored article showed, most colonially educated people fail to understand this. It is impossible for them to laugh at the persistent foolishness of Western authorities. Therefore, the correct way of putting it is this: absolutely any nonsense proposition whatsoever can be deductively proved as a formal mathematical theorem.

In fact, not only are the axioms decided metaphysically, logic too is decided metaphysically. Until the 1930's the West was unaware of the possibility of different logics which are not two valued. This was due to another church superstition. After the church adopted Christian rational theology, by copying from Islamic rational theology, it came to believe that God is bound by logic, and could not create an illogical world, though he could create the facts of his choice.³⁰ This is part of the reason to believe that proofs based on logic, which binds even God, are somehow "superior" to proofs based on facts. Obviously God would be confused by the fact of an infinity of different logics, or he would not know what logic he was bound by! (Actually, this church superstition was based on a misunderstanding of a casual concession made by al Ghazali whose opponent Ibn Rushd (Averroes) was greatly valued by the church.) However, logics which are not only not two valued but are not even truth functional have existed in other cultures, for example the Buddhist logic of *catuskoti*, or the Jain logic of *syadavada*.³¹

That is, a formal mathematical theorem is, at best, fallible relative truth, relative to both axioms and logic. Since two valued logic is still being used in formal mathematics, formal mathematical theorems are at best cultural truths. If, on the other hand, we were to decide logic empirically, this would again make deduction weaker than empirical proofs. Further, there is no guarantee that logic is empirically two-valued as the case of quantum logic³² shows.

Therefore, it is a mere foolish Western superstition that prohibiting the empirical somehow strengthens epistemic value. In fact, it damages epistemic value, while adding nothing to practical value (except political value).

In fact, prohibiting the empirical also from the axioms, as is done in formal mathematics, makes a formal mathematical theorem irrefutable, or pure metaphysics in the sense of Karl Popper. As pointed out earlier, this is true of Aquinas' angels. The church obviously preferred metaphysics because its validity can only be decided by authority. But irrefutability should not be confounded with infallibility.

Epistemic value of deduction? The history

We can also historically understand the genesis of this superstition underlying formal mathematics, and why the church adopted this peculiar method of reasoning minus facts. In the 11th c., prior to the Crusades, Muslims were the richest inhabitants of Europe. Regardless of "official" accounts, the real motive for the Crusades was to grab Muslim wealth by forcibly converting them to Christianity

30 This was a misreading of a casual concession made by al Ghazali and is the basis of the belief that logical truths are necessary truths, while facts are contingent truths. See detailed discussion in C. K. Raju, "The Religious Roots of Mathematics", *Theory, Culture & Society* 23 Jan-March 2006, Special Issue ed. Mike Featherstone, Couze Venn, Ryan Bishop, and John Phillips, pp. 95–97. <http://ckraju.net/papers/Religious-roots-of-math-TCS.pdf>.

31 See, *Encyclopedia of Non-Western Science, Technology and Medicine*, ed. Helaine Selin, Springer, 2008, 2014, 2016. article on Logic, pp. 2564–2570. (2016) <http://ckraju.net/papers/Nonwestern-logic.pdf>.

32 C. K. Raju, *Time: Towards a Consistent Theory*, Kluwer Academic, Dordrecht, 1994, Fundamental Theories of Physics, vol. 65, chp. 6B "Quantum-Mechanical Time".

the way “pagan” Europe was earlier converted to Christianity by force. However, even after the internal collapse of the Umayyad Khilafat, the combined military might of Christian Europe, under a religious banner, was inadequate for this purpose, for centuries. It could win only isolated battles, not the war. Therefore, the church had no option but to turn to persuasion.

However, the church lacked the tools to persuade Muslims who rejected the Bible as corrupted. Further, as pointed out by Christian spies like Adelard of Bath,³³ Muslims accepted only reason and not authority, certainly not the authority of the church. Therefore, the church created the Christian theology of reason by copying from the Islamic theology of reason (aql-i-kalam). To claim reason as its inheritance, hence theologically correct, it concocted a false Greek origin of reason. This is false. As already pointed out, others were using reason from long before even the historical Aristotle of Stagira commonly conflated with the mythical Aristotle of Toledo, invented in the 12th c.³⁴

However, while the church accepted reason, it could not at the same time accept facts, for facts are obviously contrary to any number of church dogmas, such as virgin birth. That is, the church invented this peculiar method of reasoning minus facts, to suit its political convenience. The first actual example of such reasoning minus facts is found in Aquinas. To prove Aquinas’ theorem, that any number of angels can fit on the head of a pin, he began, in the manner of formal mathematics, with the authoritative postulate that angels occupy no space. This method of axiomatic reasoning was, as we have seen, falsely attributed to “Euclid”, by “re-interpreting” a pagan religious text.

As already pointed out, though this was a prescribed text, for centuries Western scholars (including Cambridge University) did not read the text carefully, not even its first proposition, and simply went by the myth about it, as many Western scholars (such as Clagett, and Needham) and colonised minds still do. When it was finally admitted, in the 20th c., that this reinterpretation did not fit the text, Hilbert simply rewrote the book to fit its church reinterpretation resulting in various absurdities!

The church always glorified itself, and declared all its practices as superior; so deduction minus facts was glorified as “superior”. This message has been globalised by colonial education which came as church education. The church arguments in favour of such claims of superiority are as absurd as the arguments for the claim of white superiority. We have seen that the purported arguments for the superiority of formal mathematics (which rejects the empirical) crumble if examined critically.

However, the church method of prohibiting the empirical ensured that there is no empirical way to check the initial axioms. For example, how does one empirically check Aquinas’ axiom that angels don’t occupy any space? Obviously, angels are no part of empirical reality, the truth of the axiom is entirely a matter of authority. This sort of metaphysical reasoning, or faith-based reasoning, anchored in authority, totally suited the church. The church claimed “reason”, but subverted it to mean “authority”.

Because of centuries of church hegemony over the Western mind this sort of faith-based, metaphysical reasoning was accepted and incorporated in formal mathematics. Thus, the axioms of formal mathematics are equally metaphysics. For example the class VI Indian text on mathematics explains that geometric points must be invisible. The class IX Indian text states the postulate that exactly one invisible straight-line connects two invisible points. Obviously it is impossible to check this postulate empirically: if one works empirically with two visible dots then it is always possible

33 For detailed quotes from Adelard of Bath, and other background, see, C. K. Raju, *Euclid and Jesus*, Multiversity, 2012.

34 See, article on ‘Logic’. cited above.

to connect them with more than one straight-line. So, to the extent that postulate is verifiable it is false.

However, consider what happened after the panel discussion in Cape Town in 2017. I spoke of invisible point as an example of foolish metaphysics, and laughed at the superstitions of the West/Whites. A news reporter was deeply offended by my laughing at the West/Whites. He said he had confirmed with an unnamed mathematician that mathematicians routinely work with invisible points. He did not explain how, and most people too did not ask for an explanation and took it for granted that the authority on which he purportedly relied was correct. What is curious is that the whole lot of academics from the University of Cape Town accepted this foolish claim. This shows that formal math is just the faith-based math of those who lack common sense.

Math as eternal truth?

The Western tradition of mixing math with religious superstitions does not begin with the church. The first superstition goes all the way back to Plato, and his belief that mathematics arouses the soul to make it for recollect its eternal knowledge from past lives. Indeed, as Proclus explains, the word mathematics derives from mathesis meaning learning. On Plato's theory all learning is recollection of the eternal knowledge acquired by the soul in its previous lives. The further Western superstition in sympathetic magic, that "like arouses like", led to the belief that mathematics contains eternal truth, hence arouses the eternal soul.

This "soul arousal" relates to aesthetics, as in its colloquial meaning. And Plato thought that "soul arousal" makes people virtuous. Therefore, Plato prescribed both music and mathematics as compulsory subjects for the future citizens of his imagined utopia (the Republic) to make its citizens virtuous.

Acutely aware of its practical uselessness of formal math, formal mathematicians hence talk about aesthetics in mathematics. This is another myth contrary to observation. Every year, we observe millions of students dropping out of mathematics, because they find present-day formal mathematics exceedingly ugly. The same students still enjoy music, even if they have never been taught music. Much Western "knowledge" is about myths, contrary to facts. This is achieved by relying on a few "experts" with a vested interest in formal mathematics who will vouch that their subjective experience of this purported aesthetics is more important than the anti-aesthetic experience of millions. And there seems no shortage of people who are gullible enough believe them. How exactly do we know that they are not lying to keep their job?

Anyway, despite the church's extreme opposition to "pagans", this belief that mathematics has eternal truths somehow survived until after the Crusades. Aquinas said that God rules the world with eternal laws of nature,³⁵ and it came to be superstitiously believed that these purported eternal laws are written in the language of eternal truth: mathematics.

Math as exact?

The belief that mathematics contains eternal truths led to the further Western superstition that mathematics is exact: presumably because any inexactitude would be exposed some time or the other during eternity! And, today, we still find many scholarly journals devoted to "the exact sciences". However, where in the world is the Pythagorean theorem EXACTLY true? Certainly not

35 Thomas Aquinas, *Summa Theologica, First part of the Second Part*, 91,1, <http://www.newadvent.org/summa/2091.htm>.

on the surface of the earth which is curved. And, certainly not in space, which too is curved. The fantasy of exact truth applies only in a fantasy world, not anywhere in the real world.

In contrast, normal mathematics accepted inexactitude. The “Pythagorean theorem” can serve as a useful inexact or approximate knowledge, This is the case with all practical applications of math. In particular, calculations done on a computer are always inexact

Further, as already pointed out, the focus in normal mathematics is on calculation (needed for scientific applications) not proof needed by the church for persuasion, and an illusory claim of certainty. For example, to USE the “Pythagorean theorem” for navigation, to determine one’s position, we must do the “Pythagorean” calculation, not merely prove the “Pythagorean theorem”. The moment we recognize this, the illusory claim of exactitude is busted.

Thus, to calculate the diagonal of a rectangle (to use the earlier African and Indian form of the proposition), the unit square, say. we need to calculate $\sqrt{2}$. However, the algorithm to calculate $\sqrt{2}$ does not terminate (nor do the digits in its decimal expansion $\sqrt{2}=1.414\dots$ display any pattern or recur). Therefore, the Indian sulba sutra uses a special term for $\sqrt{2}$; savisesa, meaning “with a remainder”, or, in a word, inexact. This inexactitude presents absolutely no problem for practical applications of mathematics to science and engineering: as another example, to calculate rocket trajectories, one allows for a certain circular error probable, just as one allows for some small errors in weights in a grocery shop. What is needed for practical applications is precision, never exactitude.

It is this Western superstition about the exactitude of mathematics which led to the huge and useless metaphysics of “real” numbers, as the exact sum of an infinite series. Thus, $\sqrt{2}=1+\frac{4}{10}+\frac{1}{100}+\dots$, can be regarded as the sum of an infinite series. Carrying out this sum to finite number of terms is generally doable, but would result in some tiny errors. For example if one sums the series to the first n terms, this will result in an error of 1 part in 10^{n+1} . This is very tiny and of little practical consequence even if $n=20$. But an exact sum would take an eternity of time (to calculate or even write down).. However, Western philosophers went by the superstition that mathematics must be exact; they thought if it is not exact it cannot be mathematics. Thus, we have a similar infinite series, the infinite series for $\pi=3.1415\dots$. This arises as the ratio of a curved line (the circumference of a circle) to a straight-line (the diameter of the circle. Alluding to this ratio, Descartes³⁶ asserted that the ratios of curved and straight lines are not mathematics, since not exact. Likewise, Berkeley,³⁷ in his critique of Newton and Leibniz, asserted that “It is said, that the minutest Errors are not to be neglected in Mathematics”.

The metaphysics of infinity in formal math and the politics of eternity (creationism and apocalypse)

Obviously, if one does not allow the minutest errors, then one must sum an infinite series exactly. This is physically impossible for it would take an eternity of time, as Descartes pointed out. The Western way to reconcile the superstition about the exactitude of mathematics with the practical

36 René Descartes, *The Geometry*, trans. David Eugene and Marcia L. Latham, Encyclopaedia Britannica, Chicago, 1990) Book 2, p. 544.

37 George Berkeley, *The Analyst or a Discourse Addressed to an Infidel Mathematician*, ed. D. R. Wilkins, 1734, available online at <http://www.maths.tcd.ie/pub/HistMath/People/Berkeley/Analyst/Analyst.html>.

value of infinite series (e.g. for navigation) was to sum infinite series metaphysically. Absolutely anything is possible metaphysically (or in a fantasy world), as the church had long demonstrated.

The important thing to understand is that the infinite series and its practical value came earlier, and the metaphysics was added on later, exactly as happened in the case of $1+1=2$. To reiterate, this metaphysics of infinity added nothing to the practical value of the infinite series, for, in practice, it still cannot be summed exactly. But to the Western mind this metaphysics added to epistemic value and reconciled the superstition that mathematics is exact with the practical value of infinite series.

Since the validity of metaphysics can only be decided by authority, this metaphysics of infinity was aligned to Western prejudices arising from long-standing church dogmas about eternity.³⁸

The decolonised course, it is explained how non-Archimedean arithmetic can be more easily used to sum infinite series. This, however, accepts inexactitude.

One reason for the opposition to decolonisation is that the existing metaphysics of infinity, in formal math, has deep political value for the church. Consider the book by Stephen Hawking and G. F. R. Ellis.³⁹ Using general relativity, they claim to have mathematically proved the existence of a singularity. What is a singularity? The bottom line of the book characterizes it as “the actual point of creation, the singularity”. So, science has mathematically proved the truth of the Christian dogma of one-creation. For this conclusion, G. F. R. Ellis the million dollar Templeton award (given for putting together science and religion). This conclusion was stated in the blunt American way by Tipler, in his books, *The Physics of Immortality*,⁴⁰ and *Physics of Christianity*,⁴¹ which explains, “Stephen Hawking proved mathematically...the Singularity...The Cosmological Singularity is the Judeo-Christian God” (capitalist original).

Let us now apply what we have learnt above about the decolonisation of mathematics. The first thing is that most people are mathematically illiterate. Therefore, they cannot understand of their own knowledge what exactly is the mathematical theorem that Stephen Hawking proved. They uncritically accept it just based on the authority of Stephen Hawking. It does not strike them that the church worked hard to build up this authority, because of the political advantage derived from the claim that “science has proved the truth of Christianity”. Secondly, recall that deductive proofs may be erroneous. A major error in the singularity theory of Roger Penrose and Stephen Hawking, as pointed out during my two-day public debate with Roger Penrose, is the wrong assumption that the breakdown of university calculus means the beginning or end of the world. Contrary to what Hawking asserts, the laws of physics do NOT breakdown at a singularity; the equations of general relativity continue to make sense at a singularity, even within formal mathematics.⁴² Thirdly, a mathematical theorem is not valid knowledge: it all depends on the validity of the assumptions. Hawking and Ellis assume the chronology condition, closely related to Augustine’s theology, and to

38 C. K. Raju, “Eternity and Infinity: the Western misunderstanding of Indian mathematics and its consequences for science today.” *American Philosophical Association Newsletter on Asian and Asian American Philosophers and Philosophies* 14(2) (2015) pp. 27–33. <http://ckraju.net/papers/Eternity-and-infinity-Pages-from-APA.pdf>.

39 S. W. Hawking and G. F. R. Ellis, *The large scale structure of space-time*, Cambridge University Press, 1974, p. 364. (Closing sentence of the book.)

40 F. J. Tipler, *The physics of immortality. Modern cosmology, God, and the resurrection of the dead*, Macmillan, London, 1995,

41 F. J. Tipler, *The physics of Christianity*, Penguin Random House, 2018.

42 For a technical discussion of discontinuities in general relativity, see, C. K. Raju, “Junction Conditions in General Relativity.” *J. Phys. A: Math. Gen.* 15 (1982) pp. 1785–97. Also, C. K. Raju, “Distributional Matter Tensors in Relativity.” In: *Proceedings of the Fifth Marcel Grossman Meeting on General Relativity*, Perth, 1988, ed. D. Blair and M. J. Buckingham, R. Ruffini (series ed.), World Scientific, Singapore, 1989, pp. 421–23. arXiv:0804.1998.

the church curse on cyclic time.⁴³ They repeat Augustine's wrong arguments, in favour of this assumption, giving them scientific veneer.

Hence, I challenged this creationist conclusion as absurd, during the panel discussion in UCT, and offered to debate the bad math used by Hawking and Ellis in depth in the math department of UCT. Ellis was afraid of being exposed and ran away from the debate. In the last twenty years he could not refute a single point I have made in my book *Eleven Pictures of Time*. Such debate avoidance, like the censorship of an article without being able to refute a single point in it, is the method of crooks out to fool people by misusing authority.

The decolonised course in university calculus

To escape from being fooled in this way by authority, the colonised need to eliminate their ignorance of mathematics. This is only possible with decolonised mathematics.

The calculus is where the real difficulty of math begins. This is usually taught for two years school in class XI and XII. Then it is again taught to undergraduates, possibly followed by further courses in advanced calculus, analysis, and, finally, functional analysis. The typical undergraduate calculus texts are huge: running into over 1300 pages, A4 size, with small type, and double column, with a supplementary CD. At the end of this students learn very little: as our pre-test showed, they do not learn limits or formal real numbers, merely hear about them. Hence, they are unable to define the derivative or the integral. Indeed, though most students have absolutely no difficulty in answering the question: what is the derivative of e^x , this is sheer rote learning, for they are unable to define what is e^x ? They cannot even correctly define the function $\sin x$. (This is the level of total ignorance that the colonial education is striving hard to preserve.)

What the student takes away is that a function is a graph and the derivative is a tangent. But none of the fat calculus texts defines a tangent independently of the derivative. Therefore, when asked what is the tangent to a curve, the typical incorrect answer is that it is a line which "just touches" the curve at one point. When asked how many points the tangent to $x \sin x$, at $x = 0$, intersects the curve, they are unable to answer. And so on, and so on.

That is, students learn absolutely nothing of the formal theory of calculus, they are utterly confused about each and every concept, and the blame is laid at the door of the student or the teacher never foolishness of the subject being taught.

Besides, students are stuck to symbolically calculating derivatives and integrals, a completely useless mechanical skill which can be easily performed by computer.

Most students don't grow out of their conceptual confusion about calculus, even after they graduate. Thus in the group of postgraduate students at the University Sains Malaysia, the first question was to define a complete metric space. Most students left this question blank, though some tried to answer it by consulting Wikipedia. The thirty-five students (including one PhD) at initially registered, 18 dropped out after the pre-test. Needless to say, they had no understanding of the Lebesgue integral or the Schwartz derivative, needed to understand the limitations of the university calculus.

43 For an extended non-technical discussion of the Hawking-Augustine postulate and conclusions, and the church curse on 'cyclic' time, see, C. k. Raju, *The Eleven Pictures of Time*, Sage, 2003.

In contrast, the decolonised calculus course on calculus without limits has been run in just five days, including pre-and post-test. The “without limits” part of course refers to the use of non-Archimedean arithmetic in the way in which calculus developed in India a thousand years before Newton and Leibniz. The course emphasises numerical calculation of the solution of differential equations, not as numerical analysis, but to clarify basic concepts of the calculus, and the way it is applied in practice. Naturally, after the course, students are easily able to handle real-life problems involving non-elementary functions, such as elliptic functions needed for the motion of the simple pendulum. Further, to make sure that students can easily do all the symbolic calculation they learn in the usual three-year calculus course. This is not meant to teach any great skill but to emphasise the at uselessness of the skills that most students learn after those fat calculus texts.

As already stated colonial education ensures that most people are ignorant of the calculus. To reiterate the phrase “most people” is intended to include professional mathematicians. For example, one professional mathematician, a senior associate professor in one of the prestigious Indian institutions (IIT-BHU), walked out of my seminar, on hearing that a discontinuous function (such as the Heaviside function) could be differentiated. The poor fellow had been teaching undergraduate courses for far too long, and had come to believe that the theorem “a differentiable function is continuous” was true. He had, evidently not heard of the Schwartz theory of distributions, though students had definitely heard of the Dirac δ function. With the calculus without limits, one does not have to keep teaching limitations of certain metaphysics, and then teach more complicated metaphysics. Non-Archimedean arithmetic takes care of even the limitation of the Schwartz theory defining products of distributions

Non-Archimedean arithmetic with the new philosophy of zeroism⁴⁴ allows infinite series to be easily summed. Indeed, this was how to sum of the infinite geometric series was first obtained.

Because the decolonised course on calculus makes calculus so easy, it has also been taught to social scientists. One of the great intangible outcomes of the pedagogical experiments was their delight in mastering what they thought was forever out of their grasp.

Decolonised string geometry

Many people imagine that mathematics courses must be re-taught from the beginning. However this is impractical. It is better to initially work downwards from the University. There are several reasons for this. First of all University has autonomy, and can easily make changes or introduce new courses. Secondly, school level courses are often taught with an eye to what would be subsequently taught at the university level. For example, class XI school text teaches about “real” numbers (without actually teaching “real” numbers). This is evidently unnecessary if one intends to teach calculus without limits at the university level. Therefore, when trying to change the system, it is important to begin top-down.

Nevertheless, since the vast majority including mathematicians don’t understand the finer points of the calculus, it is necessary to demonstrate that mathematics can be decolonised even at the school level. The other aspect of the demonstration, is about the ritualistic nature of school geometry, and its ties to everything from the church quadrivium, to Hilbert’s synthetic geometry. As indicated

44 C. K. Raju, “Zeroism”, *Encyclopedia of Non-Western Science, Technology and Medicine*, Springer, ed. Helaine Selin, 2016. pp. 4604–4610. <http://ckraju.net/papers/Springer/zeroism-springer-f.pdf>.

earlier, this results in an incoherent and contradictory complete hotch-potch being taught to students, along with the ritualistic compass box which is utterly irrelevant to any real-life measurement. Further, it helps to demonstrate that traditional African and Indian geometry as far greater value than the geometry pushed by the church, and the Cambridge syllabus for centuries.

The new course teaches geometry in a completely empirical way. Because it uses a different instrument (the string or rope), it deviates fundamentally from Western geometry, by taking a curved line rather than a straight-line as the basis of geometry. For example, an angle is no longer defined as something (what thing?) to do with a pair of straight lines. But is defined in the traditional way as the relative length of an arc. Incidentally, this makes the radian measure of an angle easy to understand, because students are not made dependent on ready-made protractor. A string is obviously a more practical instrument to draw a semicircle in a football field say. It is obviously more eco-friendly and local.

Further, by reverting to traditional geometry it is possible to explain how to calculate latitude longitude and the size of the globe. This is not included in colonial geometry was obvious reason that Europeans could not make this calculations until very late. As stated earlier, a textbook is ready, and the course has been tried out with various groups of students and school teachers. Both students and teachers like the course very much. But, because school education state controlled, and this aspect of the state is controlled by Western and colonised “experts”, it is not clear how long this course will take to be introduced in schools. Unlike the calculus course, this course cannot be taught in parallel, first because students are young and immature, and secondly because the teaching of the course exposes the complete nonsense that is taught in school geometry today.

Because this course is pitched at the school level, it is easy enough for anyone to learn. The colonised will need to put in this effort to save themselves and their children from mental slavery, and challenge formal mathematicians and other allies of the coloniser. They should take heart from the fact that formal mathematicians are too frightened to debate publicly. Formal mathematics will collapse, the moment in open debate takes place. This collapse, exposing the myths and foolish superstitions of the West/Whites, will be the beginning of the end for Western/White intellectual dominance.

COLONISED PEOPLES OF THE WORLD, STAND UP TO DECOLONISE MATH; YOU HAVE NOTHING TO LOSE EXCEPT YOUR IGNORANCE!