Teach religiously neutral mathematics

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The thought of a religious bias in mathematics may seem odd, especially to those who don't know mathematics well. *Why* is 2+2=4? People will typically line up 2 objects with 2 object to show 4 objects. That is no doubt practical and secular; but it is *not* mathematics, for empirical proofs are prohibited in the (formal) mathematics which is taught in schools and colleges today. In that mathematics, there is a demand for a proof that 2+2=4, and the proof is *required* to be metaphysical. Few people know what that metaphysics is or even what a mathematical proof is. How, then, do they know that this metaphysics is universal or certain (as mathematics is commonly believed to be) or even that it is religiously neutral?

Mathematics was associated with religious belief in Western thought from its beginnings: the very word "mathematics" derives from "mathesis" which means learning. According to Plato, "all learning is recollection" of the knowledge acquired by the soul in its previous lives. In the famous story of Socrates and the slave boy in Plato's *Meno*, Socrates first demonstrates that the slave boy has an innate knowledge of mathematics, and then claims he has thereby proved the existence of the soul and its past lives.[1] (The argument is that if the boy has knowledge of mathematics, but did not learn mathematics in this life, he must have learnt it in a previous life.) The belief that mathematics has an essential connection to the soul is driven home by Plato who asserts, in his *Republic*, that mathematics, like music, should be taught because it arouses the soul, and explicitly not for its practical value.[2] The idea of mathematics as a spiritual experience is still acknowledged indirectly, even by atheists like Russell, through claims of mathematics as an aesthetic experience, like music.

Exactly contrary to Plato, in most of the world, mathematics developed for its practical value, and it is for its practical value that students of sciences, engineering, economics etc. want to learn it today. Ironically, much of present-day school mathematics—such as arithmetic,[3] algebra,[4] trigonometry,[5] calculus[6] and statistics[7]—originated in India for its practical applications, as *ganita*. It was transmitted to Europe for its practical usefulness. However, Europeans tried to understand *ganita* in a religious perspective, and converted it to mathematics. During colonialism they returned that mathematics to us, together with a false history, both of which we have accepted without question.

For example, calculus started in India with Aryabhata of Patna (5th c.) who numerically solved a differential equation to calculate trigonometric values precise to 5 decimal places.[8] Such highly accurate trigonometric values were needed for both astronomical models and navigation (determining latitude, longitude, size of the earth[9]). Accurate astronomical models were, in turn, needed for the Indian *pancang* which determines the rainy season (the months of *sawan* and *bhadon*) which was (and still is[10]) needed for monsoon-dependent Indian agriculture, a key source of wealth in India. Navigation was needed for overseas trade which was then the other key source of wealth in India. That is, this *ganita* was of immediate practical value for the means of producing wealth. Over the next thousand years, the followers of Aryabhata in Kerala increased the precision of these trigonometric values to 9 decimal place, using infinite series.[11] Europeans then badly needed precise trigonometric values for navigation, because Europeans were then poor and dreamt of wealth through trade with India, which

needed accurate navigation. In the 16th c. Europeans could not even determine latitude at sea,[12] and had a hopelessly wrong value for the size of the earth (hopelessly wrong compared to the Indian or Arabic value from about a thousand years earlier). Hence, this Indian knowledge was translated and despatched to to Europe, in the 16th c., by Cochin-based Jesuits.[13] There calculus got connected with the names of Newton and Leibniz.[14]

Europeans immediately understood the practical value of calculus, but had difficulties in understanding the underlying notion of infinity. For example, the circumference of a circle, or the number today called π, is stated using infinite series: π = 3.14159.... Descartes rejected this, saying that the length of a curved line was beyond the capacity of the human mind.[15] In amusing contrast, in India, from the days of the *sulba sutra*, children were taught to measure curved lines with a flexible string. (Sulba means string.) Today, in blind imitation of the West, school children are all handed a compass box which has no instrument with which to measure a curved line.[16]

The religious basis of Descartes' objection may be understood as follows. Adding the infinite series term by term 3 + 1/10 + 4/100 + 1/1000 + … would take an infinite amount of time. If, however, we stop at some term (e.g., after two steps, at 3.14) that would not be eternal truth, hence not mathematics. In contrast, in Indian *ganita* there never was such a demand for eternal truth (and some Indian schools of thought would outright reject the very possibility of eternal truth as an absurdity). The value obtained for the circumference of a circle was explicitly accepted as *anitya*,(impermanent) in the *sulba-sutra*-s,[17] and as *asanna* (near value) by Aryabhata etc.[18]

The demand for eternal truth in mathematics is clearly *not* universal, but is linked to religious beliefs. Plato thought mathematics arouses the eternal soul just because it incorporates eternal truths. (This is on the further ground of sympathetic magic that like arouses like.) Those religious beliefs were transformed due to church politics as I have explained for the layperson in my book *Euclid and Jesus: how and why the church changed mathematics and Christianity across two religious wars*.[19]

Briefly, the Crusades were launched to convert Muslims by military force, the way Europe was earlier converted by military force. When military force failed (after the first Crusade, and beyond Spain) the church adopted reason as the only way to persuade Muslims (who accepted reason as in the prevalent *aql-i-kalam* or Islamic rational theology). But first reason had to be made a Christian weapon. To this end, Aquinas and his schoolmen modified Islamic rational theology, and reinterpreted Christianity as a doctrine of reason. On this Christian rational theology it came to be believed that reason binds God: that is God cannot create an illogical world, though he is free to create the facts of his choice. Accordingly, it came to be believed that logical proof (which binds God) relates to eternal truth, and is stronger than factual or empirical proof (for facts do not bind God). Hence, the demand for logical or deductive proof in mathematics, and the prohibition of empirical proofs.[20]

Exactly contrary to this, all early Indian schools of thought (such as Nyaya-Vaisesika, Samkhya-Yoga, Lokayata, Advaita Veanda, Buddhists, Jains) accept the empirically manifest (*pratyaksa*) as the first means of proof (*pramana*), and this is the only means of proof accepted by all systems. Likewise, Islam accepts *tajurba* (experience) and science accepts experiments. Empirical proofs were freely used in Indian *ganita* since the sulba sutra, through Aryabhata and down to the 16th c.[21] But today children learn in schools through mathematics, albeit indirectly, that empirical proofs are unreliable, and they should trust only the metaphysics of "rigorous" mathematical proof.

That is, school children are indirectly taught through mathematics not only to distrust the evidence of their senses, and to regard it as inferior to metaphysics, they are tacitly taught that all systems of Hindu,

Buddhist, Jain, and Islamic philosophy are hence unreliable, and that only Crusading Christian metaphysics is trustworthy since it rejects empirical evidence! That, in itself, is a dangerous teaching and is sufficient reason to change the way mathematics is taught today. Anyway, if these students are learning mathematics for its practical value (as they believe), why reject empirical proofs? The use of empirical proofs does not adversely affect any practical applications of mathematics, for all practical applications involve the empirical.

Now, every Western educated person learns to parrot that "deduction is more certain than induction". But *why* is that so? No doubt empirical proofs are fallible: the classic example in Indian thought is that one might mistake a snake for a rope or *vice versa*. But by prodding the rope-snake with a stick one can repeat the observation inductively and soon eliminate the doubt after a few observations. Only a "philosophical" (in the pejorative sense of unrealistic and impractical) doubt could persist even after the poor snake is long dead. But similar errors are very common in deductive proofs, as in common logical fallacies, or in the numerous wrong (but published) proofs of, say, the Riemann hypothesis.

Perhaps the best (but little known) example here is the *Elements* which, according to the myth, was written to demonstrate deductive proofs. However, the very first proposition of the *Elements* uses an empirical proof, not a deductive one! The 4th proposition too uses empirical proof in a way essential to the whole book. However, such was the hold of church myths over the Western mind that it took 7 *centuries* for Western intellects to notice these obvious facts! (And when they did notice it, they *still* clung to the myth and changed the facts instead: that is, Russell and Hilbert rewrote the book to conform to the myth changing the 4th proposition into a postulate! That is how geometry is taught in schools today.[22]) Anyway, the fact remains that errors in the most elementary deductive proofs have persisted for *centuries*. So there is a solid basis for doubt about possible errors in deductive proofs.[23]

Secondly, logical proofs can hardly be certain for logic itself is not certain. Thus, it is a mere Western superstition that logic binds even God hence it is universal. Kant amusingly said that logic had reached perfection since it had not changed since Aristotle.[24] But logic is certainly not universal as a matter of *fact*: the Buddhist logic of *catuskoti*, the Jain logic of *syadavada*, quantum logic, or the logic of natural language are all different from binary logic.[25] To use mathematics to teach binary logic as normatively universal is a clear religious bias against Buddhist and Jains at least.

Besides, with an infinity of different logics to choose from, how exactly does one decide which logic to use? If the use of binary logic is defended on empirical grounds (not metaphysical prejudice), then logical proofs can no longer be regarded as *more* certain than empirical proofs, for the nature of logic itself is decided empirically. Why, then, reject empirical proofs in mathematics? Besides it is not certain that binary logic is the only possible empirical choice: this is clear from the cases of quantum logic and the logic of natural language.

But if logic is not certain, how can deduction be? Mathematical theorems are today defined as the conclusions of deductive proofs. For many centuries Westerners superstitiously believed that the theorems of mathematics were certain and universal truths. After formalism it was conceded that the theorems are at best relative truths, relative to the postulates. However, the correct position is that the theorems of a formal axiomatic theory are truths relative to both the postulates and the logic. For example, with any of the above mentioned logics, all proofs by contradiction would fail. That is changing logic changes the theorems which can be deduced from the same set of postulates. That means "accepted" theorems of mathematics (such as the existence of a Lebesgue non-measurable set) would become invalid. To give an easier example, in binary logic, a contradictory proposition implies any other proposition, but in natural language from the proposition "This person is both good and bad"

one cannot conclude that “elephants are pink”. What, then, is the point of proving theorems?

A third source of religious bias is the postulates. Though, in principle, within formalism one is free to choose the (metaphysical) postulates of a mathematical theory, that is not true in practice. The postulates underlying present-day school mathematics were all decided by a few influential Western mathematicians, and are treated as inviolable. Certainly, students and teachers in our schools and colleges cannot change them. The calculus involves infinite series, and postulates regarding infinity (as in set theory) bring in a subtler religious bias, through a culture-specific concept of infinity/eternity.[26] As an easier example, college calculus teaches limits, hence postulates the continuum, contrary to the belief in atomicity among Islamic thinkers like al Ashari, suggesting that Muslim thinkers were naïve. Students are never told that calculus historically originated with a similar belief in atomicity, in India, or that it could be done differently today with a different set of postulates, even within formalism,[27] or as in calculations done on a computer which cannot handle infinity.

False history is a fourth source of religious bias. Today our NCERT school texts are full of pictures of Greeks all portrayed using a racist stereotype,[28] and NCERT is unable to produce primary evidence even for their existence leave alone the colour of their skin. This racist history originated in the Christian chauvinist history of the Crusades. Thus, during the earlier Christian Dark Age it was the practice of the church to burn all non-Christian books as heretical.[29] However, during the Crusades, this policy was reversed in an attempt to gather Arabic knowledge, and the books in a captured library at Toledo were mass translated, starting 1125 CE. Since this was a time of religious fanaticism, justification was required for the sudden change of policy from burning books to learning from the books of the religious enemy. The justification was provided by the stock church trick of distorting history, since Orosius. Secular knowledge among the Arabs was attributed wholesale to the theologically-correct *early* Greeks, and hence declared a valid Christian inheritance. Where needed, Greeks like Euclid were concocted for this purpose.[30] Later, racist historians tied this false history to the story of the Greek race.[31]

We have already seen that the book *Elements* does not support the belief that it was written to promote deductive proofs. Nor does it support the belief that “Euclid” was the author. Greek manuscripts of the *Elements* all attribute the book to Theon or say it is based on his lectures, so its author must come after Theon (4th c CE, or some 7 centuries after “Euclid's” purported date). Even Greek commentators speak anonymously of the “author of the Elements”. Today, the only known “evidence” for “Euclid” is a short passage from *another* book attributed to Proclus. Though Proclus himself lived in the 5th c. (some 8 centuries after “Euclid”), the manuscript of his book comes to us from another 800 years later, so we can hardly be sure that Proclus wrote every word in it. Indeed, that passage is clearly an interpolation, for it refers to a known interpolation in a late (13th c. or later) manuscript attributed to Archimedes, and hence must be an interpolation from even later.[32] To drive home the complete absence of evidence for “Euclid” I have offered a prize of MYR 10,000 (about Rs 1.8 lakh) for serious evidence about Euclid.[33] Nobody has claimed the prize but the myth of Euclid continues in our school texts to date.

Changing the author (or the date of the book, hence its social context) may change our understanding of the book. Thus, Proclus explicitly connects the *Elements* to Plato's religious understanding of mathematics, saying that mathematics leads to the “blessed life”. In Proclus' social circumstances, someone had to defend the Platonic notion of soul which was then under violent attack by the church. In this context of a religious war between Christians and pagans, the very first proposition of the *Elements* acquires a new significance: for it is the fish figure (incorporating the cross), used as a secret sign by Christians. So one may infer that the book is addressed to Christians, for it begins with their secret sign of the fish. The changed understanding of the *Elements* as a religious text, allows a simple

explanation for the empirical proofs at its beginning, and Proclus explicitly says that "proofs must vary with the kind of being". Indeed, on Proclus' understanding, proofs are *not* the key aim of mathematics, which is to drive attention inwards to arouse the soul (in the manner of yogic meditation, with exactly the same underlying religious beliefs).

The concoction of Euclid enabled the church to disconnect the *Elements* from Platonic beliefs and reinterpret it in support of its own Crusading doctrine of reason. As we have seen, this myth of "Euclid" and his deductive proofs blinded the Western intelligentsia to the reality that the very first proposition of the book uses an empirical proof, and is today a superb example of the fallibility of deductive proofs, even in the most elementary cases, which is dangerous since this fallibility may persist for centuries.

To summarise, because mathematics was connected to religious beliefs in the West, the practical mathematics imported by the West got coated with a variety of metaphysical biases. This was then declared as "superior" and re-exported during colonialism. We have so far swallowed it all without question. So what exactly will happen if we were to scrap all these religious biases in Western mathematics, and teach mathematics solely for its practical value?

It would not affect the practical applications at all: *for the Western metaphysics imposed on present-day mathematics is irrelevant for its practical applications.* To send a rocket to the moon, NASA or ISRO must calculate the trajectory of the rocket. That is still done by numerically solving differential equations: the same technique of calculation initiated by Aryabhata in the 5th c. CE.[34] Of course, the calculation is today done on a computer, and uses a faster algorithm etc. But the point is that the metaphysics of the continuum (as taught in our universities to mathematics students) is not relevant to the practical process of calculation. In fact, a computer cannot handle infinity. Hence, a computer calculation can *never* use the metaphysical continuum, and uses instead what are called floating point numbers. Thus, a *different* philosophy of mathematics, such as my realistic philosophy of zeroism,[35] better suits the actual practical process of numerical calculation.

Because blind imitation of the West in mathematics means constant engagement with metaphysics, formalist mathematics impedes the ability to do practical calculations. When I had to solve such practical computational problems (for ISRO among others), despite offering very high salaries, I could not find mathematicians able to do the job, for even the "applied mathematicians" among them had only learnt to prove theorems!

There is a big bonus for abandoning Western metaphysics. It makes math easy. On the principle that phylogeny is ontogeny, math teaching in the school classroom reproduces, in fast forward mode, the difficulties that the West had with imported mathematics (particularly with zero in arithmetic, and infinity in calculus). As we saw in the case of Descartes, those difficulties arose because the complexities of the Christian theology of eternity were interwoven into the mathematics of infinity since John Philoponus. Getting rid of that metaphysical head load makes math easy, as easy as measuring a curved line with a string. This makes the calculus very easy, as I have already demonstrated in teaching experiments[36] in three countries. Making math easy enables students to solve harder problems, which otherwise confuse both students and teachers.[37]

Eliminating the religious bias in present-day mathematics is not only practically advantageous, it is a constitutional necessity in India, for mathematics is compulsorily taught in schools, and hence ought not to have a religious bias. In this context, it is important to note the difference between "secular" and "religiously neutral". Thus, Islamic states may not accept secularism (meaning separation of religion and state) but may yet want religious neutrality, and may hence want to eliminate the religious bias in

present-day mathematics.

What we ought to do is amply clear. First, we ought to remove the false history of Euclid and other Greeks from our school texts. Second, we ought to eliminate the religious bias in mathematics and teach it purely for its practical applications, at least at the school level.[38] Mathematics is at the base of hard sciences, so decolonising mathematics education is essential to decolonising education. The question is: can we do it? Can we make such a break from the West? For, we still have to tackle the legacy of colonial mind-capture. Briefly there are two difficulties.

First, false history was the anchor of colonial "soft power": that is what enabled colonial education to be initiated.[39] Indoctrination helps to preserve that power. Children believe any story without evidence, so they believe the false history of Euclid etc. in their school texts. Later, they are enraged if the story changes, and hurl the charge of Hindu chauvinism against anyone who doubts that history (no evidence needed, of course, even of being Hindu). This is exactly the medieval church tactic of preserving its voluminous falsehoods just by denouncing all sceptics as heretics, atheists etc.

Second, "expert raj" is a means to preserve colonial institutions. Thus, today, everyone learns mathematics in school, but few learn enough to publicly offer an opinion about what mathematics should be taught. The students don't know why they learn the mathematics they do, they point to the teachers; the teachers, in turn, point to the government who has set the syllabus; as for the government, ministers and judges too don't usually know enough mathematics, so they point to the "experts". But, if all these people don't know mathematics, how then do they know who the real experts are? Obviously they don't. They just blindly trust those approved by the West. So our education system makes the vast majority of people doubt their own judgment and dependent on Western authority as the ultimate arbiter of truth.

That is, our whole education system still instills the two fundamental but unstated teachings of colonial education: (1) blindly imitate the West, and (2) blindly trust Western authority and those the West approves of.[40] There is a palpable fear among decision-makers if asked to do something non-imitative of the West. Strange that they so implicitly trust those who have ruthlessly exploited them for centuries! The barest applications of the mind show that there is a conflict of interests here: if "experts" are decided by the test of Western approval they cannot easily oppose the West and damage their own social value. Furthermore, few of these "experts" are honest enough even to openly acknowledge this conflict of interest. So, with the help of a handful of compromised "experts" a nation of a billion people is still held in thrall.

The solution to "expert raj", if we want it, is not very difficult. First, "experts" who hide their conflict of interests should be treated as criminals and prosecuted. We don't need any new law for this; section 420 is good enough! Second, given that there is a clear conflict of interests, it is essential to ensure transparency: the "experts" need to explain publicly their reasons for supporting a certain decision (or a certain kind of mathematics). Specifically, in this situation it is *not* adequate to convince a committee behind closed doors, for such committees are often easy to manipulate. Third, the justification for the decision (what mathematics is taught in schools) must be offered within the constitution. The "experts" cannot plead ignorance of any aspect, for if they are ignorant they should not continue as experts. Fourth, any decision must take into account the practical value of mathematics to the students and society. Indeed, the "experts" themselves should be chosen not by marks of Western approval, but for the practical benefits they have brought to society. If just this last bit is implemented most of these Western-approved "expert" mathematicians will overnight cease to be labelled "experts". That will be our first step towards real freedom, real swaraj.

1 Plato, *Meno*, In: *The Dialogues of Plato*, trans. B. Jowett, Encyclopedia Britannica, Chicago, 1994, pp. 179–180

2 Plato, *Republic*, VII.526. In: *The Dialogues of Plato*, trans. B. Jowett, Encyclopedia Britannica, Chicago, 1994, p. 394.

3 It is well known that arithmetic algorithms originated in India and were transmitted first to the Baghdad Bayt al Hikma (in the 9th c.), and then to Christian Europe via Cordoba in the 10th c. through al Khwarizmi's "Hisab al Hind", as "Arabic numerals" or algorismus (named after al Khwarizmi's Latinized name). What is not so well known is that the algorismus was poorly understood in Europe which was then accustomed to (additive) Roman numerals based on the abacus. An example of this lack of understanding is the special abacus which Gerbert constructed for "Arabic numerals" in 976, completely missing the point about the advantages of the place value system and its algorithms. By the 13th c., the Florentine merchants saw that algorithms gave a comparative advantage in commercial transactions and adopted it as a trade secret. Nevertheless, Europe took over 5 centuries to replace the primitive abacus with the algorismus. I have called this the first math war. See, C. K. Raju, "Math wars and the epistemic divide in mathematics", chp. 9 in *Cultural Foundations of Mathematics*, Pearson Longman, 2007. For an image of Pope Sylvester's 10th c. Arabic abacus see *Euclid and Jesus: how and why the church changed mathematics and Christianity across two religious wars*, Multiversity and Citizens International, 2012.

4 The origin of algebra in India since Brahmagupta (5th c.) and its transmission to Europe through al Khwarizmi's "Al jabr wa'al muqabala" is equally well known. See, e.g., *Algebra....from the Sanscrit of Brahmegupta and Bhascara* trans. H. T. Colebrooke, John Murray, London, 1817.

5 Many encyclopedias claim that trigonometry originated with Greeks and was transmitted to India. This is completely false for a variety of reasons. First, there is no proof that Claudius Ptolemy or the book he purportedly wrote ever existed in the Roman empire even as late as the 5th c. (For a quick summary of the arguments, see C. K. Raju *Is Science Western in Origin?*, Multiversity, 2009; for a fuller account, see *Cultural Foundations of Mathematics*, cited above.). Thus, the (Attic) Greek and Roman numerals were too primitive to express fractions, and the Roman calendar hence adopted the wrong length of the year (at 365 ¼ days) just because there was no way to express precise fractions with Roman numerals. This error led the date of Easter to slip within a century. The Christian calendar reformers of the 5th c. under Hilarius could not reform their calendar. They did not even adopt the (wrong but better) duration of the tropical year in the *Almagest,* so the text was unavailable to them despite their full state backing. Available manuscripts of the *Almagest* explicitly mention the difficulty with multiplication before its chord table, exactly like other Muslim astronomers of the 8th c. It chord is the sole function it uses. The words sine and cosine are well-known Toledan howlers which arose because the Latin translators of the 12th c. mistook the Arabic *jiba* (from the vernacular *jiva*, from the Sanskrit *jya*) for *jaib* meaning a pocket or a fold (sinus). Aryabhata's trigonometric values (accurate to 5 decimal places) were used in Europe until the 16th c. (e.g. by navigational theorists like Simon Stevin). Beginning, in the 17th c. they were replaced with the more accurate trigonometric values, also from India, but accurate to 9 decimal places.

6 For the origin of calculus in India and its transmission to Europe see C. K. Raju, *Cultural Foundations of Mathematics: the nature of mathematical proof and the transmission of calculus from India to Europe in the 16th c. CE*, Pearson Longman, 2007. Of course, the origin of *infinite series* in India has long been known to Western historian from even before the whig historian Macaulay's infamous minute. Charles M. Whish, paper presented in 1832: "On the Hindu quadrature of the circle and the infinite series of the proportion of the circumference to the diameter exhibited in the four Shastras, the Tantrasamgraham, Yukti-Bhasa, Carana Padhati and Sadratnamala", *Trans. R. Asiatic Soc. Gr. Britain and Ireland*, **3** (1835) 509–523. The account of an earlier discussion and the statement of Heyne is in J. Warren, *Kala Sankalita*, Madras, 1825.

7 For the origin of probability theory and statistics in India, see C. K. Raju, "Probability in Ancient India", chp. 37 in *Handbook of the Philosophy of Science, vol 7. Philosophy of Statistics*, ed, Dov M. Gabbay, Paul Thagard and John Woods. Elsevier, 2011, pp. 1175-1196 (http://www.ckraju.net/papers/Probability-in-Ancient-India.pdf.) The game of dice is mentioned in the Rgveda (aksa sukta), and was instrumental in the Mahabharata war.

8 Aryabhata's calculation is accurate to the first sexagesimal minute. For quotations from the Aryabhatiya and an explanation why Aryabhata's derivation of sine values involved a differential equation and *not* an algebraic equation, see "Infinite series and π", chp. 3 in *Cultural Foundations of Mathematics*, cited above.

9 For more details, see, "Time, latitude, longitude and the globe", chp. 4 in *Cultural foundations of Mathematics*, cited above.

10 See for example, the material posted on my website from *Cultural Foundations of Mathematics* chp. 4, Box 4.2, pp. 208-212. http://ckraju.net/papers/Calendar-from-Cultural-Foundations-of-Mathematics.pdf. There are also various recent newspaper reports related to the phenomenon of "delayed monsoons". See "A tale of two calendars", paper presented at the AIU meeting on traditional knowledge, in Claude Alvares ed., to appear, posted at http://ckraju.net/hps-aiu/ckr-calendar.html.

11 These later trigonometric values were precise to the third sexagesimal minute. See Table 3.1 "Madhava sine table", and Table 3.2 "Accuracy of Madhava's sine table", in *Cultural Foundations of Mathematics,* cited above, pp. 121-22.

12 From the journals of Columbus and Vasco da Gama it is clear that both failed to determine latitude correctly. For determining latitude at sea, Europeans had to wait until a reform of the calendar (in 1582) to be able to tell the day of the

equinox correctly (and infer latitude from solar declination). Straightening loxodromes (as in Mercator's chart) required accurate trigonometric values. Furthermore, Columbus' estimate of the earth was off by 40%. *Hence*, the longitude problem of European navigation persisted until the 18th c. See "Kamal or Rapalagai", chp. 5 in *Cultural Foundations of Mathematics*, cited above.

13 For a quick summary see "Cultural Foundations of Mathematics", *Ghadar Jari Hai*, **2**(1), 2007, pp. 26-29. http://ckraju.net/papers/GJH-book-review.pdf.

14 Nothing natural about this false history which was due to the religious fanaticism of the Inquisition. Afraid to acknowledge their non-Christian sources, Europeans like Mercator (arrested by the inquisition) and Copernicus (whose close friend Scultetus was arrested by the Inquisition) hid their non-Christian sources, like Tycho, and Kepler, and later Galileo, Cavalieri, Pascal and Fermat. For a quick account, see C. K. Raju, *Is science Western in origin?*, Multiversity and Citizens International, Penang, 2009. Newton makes a telling comment on Leibniz that he did not understand the infinite series named after him, and then asks how he could have invented something he did not understand. Newton himself did not claim credit for all calculus but only for the Madhava sine series and for putting the calculus on a rigorous footing. (Newton provided this account anonymously, in his summary of his priority dispute with Leibniz. Isaac Newton, "An account of the book entituled Commercium epistolicum collinii Et aliorum, de analysi promota", *Philosophical Transaction of the Royal Society of London*, No. 342. January and February 1714/15, pp. 173–224.) However, Newton's doctrine of fluxions (see *Cultural Foundations of Mathematics,* chp. 8. "Numbers in calculus, algorismus and computers" for a discussion) has long fallen by the wayside, showing that Newton himself was confused about the calculus. The effect of Newton's confusion about calculus led to errors in his physics, and how it should be corrected has been described by me elsewhere. For the latest, see, C. K. Raju, "Retarded gravitation theory", in: Waldyr Rodrigues Jr, Richard Kerner, Gentil O. Pires, and Carlos Pinheiro (ed.), *Sixth International School on Field Theory and Gravitation*, American Institute of Physics, New York, 2012, pp. 260-276. http://ckraju.net/papers/retarded_gravitation_theory-rio.pdf, and the references cited therein.

15 R. Descartes, *The Geometry*, trans. David Eugene and Marcia L. Latham, Encyclopaedia Britannica, Chicago, 1996, Book 2, p. 544.

16 C. K. Raju, "Towards Equity in Math Education 2. The Indian Rope Trick", *Bharatiya Samajik Chintan* (New Series) 7 (4) (2009) pp. 265–269.

17 Apastamba sulba sutra 3.2, Aryabhatiya, Ganita 10.

18 For Nilakantha's interesting commentary on this in his (Aryabhatiyabhasya, Trivandrum Sanskrit Series, 101, reprint 1977, p. 56) and its translation see, *Cultural Foundations of Mathematics*, pp. 125-26.

19 For more details, see http://ckraju.net/Euclid.

20 In formal mathematics this belief persists in a slightly modified form: using Wittgenstein-Tarski formal semantics it is claimed that the theorems of mathematics as *necessary* truth (true in all worlds) as opposed to empirical facts which are regarded as mere *contingent* truths (true in some worlds). The difference is only that we here speak of worlds as "logical worlds in the sense of Wittgenstein" instead of "worlds that God could create". For a quick account see C. K. Raju, "The Religious Roots of Mathematics", *Theory, Culture & Society* **23**(1–2) Jan-March 2006, Spl. Issue ed. Mike Featherstone, Couze Venn, Ryan Bishop, and John Phillips, pp. 95–97. http://ckraju.net/papers/Religious-roots-of-math-TCS.pdf.

21 For examples, see C. K. Raju, "Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the YuktiBhâsâ", *Philosophy East and West*, **51**(3) (2001) pp. 325–362. http://ckraju.net/papers/Hawaii.pdf.

22 For an account of empirical proofs in the Elements and how the 4th proposition (Side-angle-side theorem) was changed into a postulate by Hilbert, and why Hilbert's synthetic postulates do not fit the *Elements*, see, "Euclid and Hilbert", chp. 1 in *Cultural Foundations of Mathematics*, cited above. For an account for the layperson, see *Euclid and Jesus*, cited above.

23 This is especially true of complex deductive proofs such as the computer-aided proof of the 4-color theorem which cannot be easily checked, and where most people just go by the opinion of mathematical authority.

24 "That logic has advanced on this sure course, even from the earliest times, is apparent from the fact that, since Aristotle, it has been unable to advance a step and, thus, to all appearances has reached its completion." Immanuel Kant, *Critique of Pure Reason*, preface to the 2nd edn., 1787, trans. J. M. D. Mieklejohn, Encyclopedia Britannica, Chicago, 1996 (*Great Books of the Western World*, vol. 39), p. 5.

25 For a non-technical account of non-Western logics, see "Postcript on rationality", chp. 9, in C. K. Raju, *The Eleven Pictures of Time*, Sage, 2003. Another quick account is in my article on "Logic" for the *Encyclopedia of Non-Western Science, Technology, and Medicine*, Springer, 2008, available at http://ckraju.net/papers/Nonwestern-logic.pdf.

26 Infinity is fertile ground for theology, and the first creationist controversy concerned eternity not Darwin and took place in the 6th c. between John Philoponus (*De aeternitate mundi contra Proclum*) and Proclus (*Elements* [of Theology]): it actually concerned the eternity of the cosmos against the literally interpreted Biblical myth of its creation by a God in 7 days. Today, Stephen Hawking's claim that the cosmos was created (in a singularity) is based similarly on such manipulation of the notion of infinity. See, C. K. Raju, *The Eleven Pictures of Time*, Sage, 2003, or, for a technical

account, “Renormalization and shocks”, appendix to *Cultural Foundations of Mathematics*, cited above.

27 E.g. within formalism calculus can be done over a non-Archimedean field (which has “permanent” infinities and infinitesimals, unlike non-standard analysis, and which was actually used in the Indian calculus). Or one could work with a discrete number system, such as the floating point arithmetic of computers. See, C. K. Raju, “Retarded gravitation theory”, in: Waldyr Rodrigues Jr, Richard Kerner, Gentil O. Pires, and Carlos Pinheiro (ed.), *Sixth International School on Field Theory and Gravitation*, American Institute of Physics, New York, 2012, pp. 260-276. http://ckraju.net/papers/retarded_gravitation_theory-rio.pdf.

28 C. K. Raju, “Teaching racist history”, *Indian Journal of Secularism* **11**(4) (2008) pp. 25–28.

29 For some more details, see Clarence A. Forbes, “Books for the Burning”, *Transactions of the American Philological Society* **67**, 1936, pp. 114–25.

30 For some more details, see C. K. Raju, *Is Science Western in Origin?* Multiversity, also Daanish Books, 2009.

31 For a detailed account of the origins of racist history in Christian chauvinist history see, e.g, *Euclid and Jesus*, cited above. See also the quotation from Rouse Ball in “Goodbye Euclid!” cited below.

32 C. K. Raju, “Goodbye Euclid!”, *Bharatiya Samajik Chintan* 7 (4) (2009) pp. 255–264. (http://ckraju.net/papers/MathEducation1Euclid.pdf.) This was presented in front of the current NCERT Director at the Indian Social Science Congress in Mumbai, in 2007.

33 For a video of the repeat of this announcement in front of the Malaysian Deputy Education Minister, see “Goodbye Euclid!” http://ckraju.net/videos/gbe1.html, and http://ckraju.net/videos/gbe2.html, http://ckraju.net/videos/gbe3.html.

34 Aryabhata's method of numerical solution is today known as “Euler's method”. (Of course, Euler had studied Indian texts and written about time.) For a detailed description and discussion of Aryabhata's method of numerically solving difference equations to derive precise sine and cosine values, see C. K. Raju, “Infinite series and π”, chp. 3 in *Cultural Foundations of Mathematics*, Pearson Longman, 2007.

35 For a quick account of zeroism and its advantages over classical limits, see my paper on “Probability in Ancient India”, cited above. The notion of limits abjectly fails with the frequentist interpretation of probability as the limit of relative frequency. Thus, on the law of large numbers, according to Kolmogorov probabilities, relative frequency converges to probability only in a probabilistic sense (convergence in measure). That, of course, begs the question.

36 C. K. Raju, “Teaching mathematics with a different philosophy. Part 1: Formal mathematics as biased metaphysics.” *Science and Culture* **77** (7-8) (2011) pp. 274–279. http://www.scienceandculture-isna.org/July-aug-2011/03%20C%20K%20Raju.pdf, and “Part 2: Calculus without limits”, *Science and Culture* **77** (7-8) (2011) pp. 280–85. http://www.scienceandculture-isna.org/July-aug-2011/04%20C%20K%20Raju2.pdf.

37 Two hard mathematical problems which my own children solved, while in school (K-12), were (a) the variation in the time period of the simple pendulum (which involves Jacobian elliptic functions), and (b) the brachistochrone with resistance. For the pendulum, see, Suvrat Raju, http://ckraju.net/11picsoftime/pendulum.pdf. For the brachistochrone, see Archishman Raju, “A simple way to solve the brachistochrone problem with resistance”, *Physics Education* (India), **28** (3) (2012), http://www.physedu.in//uploads/publication/3/65/Archishman_Brachistochrone13July.pdf. For an anecdotal account of my difficulties with their school teachers, see C. K. Raju, “Time: what is it that it can be measured”, *Science and Education* **15** (2006) pp. 537-551. http://link.springer.com/article/10.1007%2Fs11191-005-5287-z#page-1 and http://ckraju.net/papers/ckr_pendu_1_paper.pdf.

38 A typical trick to avoid this conclusion is to jump to the aesthetic value of mathematics. Those who value the spiritual/aesthetic value of mathematics should actually switch back from church mathematics to Platonic mathematics, and they should seek funds from the department of fine arts, not the department of atomic energy. Anyway, they can learn whatever they wish, either church mathematics or Platonic mathematics, as an optional subject, the way people may learn Western music, but definitely that should not be compulsorily taught at any level.

39 That is, Indian intellectuals like Rammohun Roy, influenced by this false history, themselves sought Western education before Macaulay. See, e.g., C. K. Raju, *Ending Academic Imperialism: a Beginning*, Citizens International, Penang, 2011. http://multiworldindia.org/wp-content/uploads/2010/05/ckr-Tehran-talk-on-academic-imperialism.pdf.

40 That is hardly accidental, for the Western education system started off (during Crusading times) with a view to produce missionaries, and remained under church control for centuries. These two values are fundamental for that purpose. See, C. K. Raju, “Decolonising our universities: Time for a change”, GlobalHigherEd blog. http://globalhighered.wordpress.com/2011/09/11/decolonising-our-universities-time-for-change/