Abstract for contributed talk

Rajju ganita (string geometry) vs "Euclidean" geometry

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Detailed abstract

The MYTH of "Euclid" and his "irrefragable" proofs is central to the Western mathematical tradition. This myth involves two **extreme lies** which are exposed by teaching rajju ganita or traditional Indian string geometry,
1. The first lie (e.g. in Indian class IX text[1]) is that all other mathematical traditions, such as ganita in India, lacked a notion of proof, and especially lacked the use of deductive reasoning.
2. The second lie is that the "Euclid" book actually has axiomatic proofs which use only deductive reasoning, but prohibit the empirical, on the church superstition[2] that deduction is infallible.

The easily verifiable fact contrary to lie 1 is that in Indian ganita (and philosophy) there is a notion of proof **explicitly** stated e.g. in the Nyaya sutra 2;[3] this accepts **both** the empirically manifest (प्रत्यक्ष) and deductive inference (अनुमान), as means of proof as does science.[4] Both means of proof are used in rajju ganita, e.g. in the proof of the "Pythagorean theorem". This exposes the gross propagandist lie ("others lacks deductive reasoning") in our school texts. Further, we recognize that the unique feature of axiomatic proof in current formal math is NOT the use of reason, as is trumpeted, but the **prohibition of the empirical**.

The other easily verifiable fact is that there are no axiomatic proofs in the "Euclid" book: its first and 4th propositions directly use empirical proofs, and its proof of the "Pythagorean" proposition relies on the 1st and 4th propositions. This was slyly publicly admitted in the West over a century ago when David Hilbert (1899)[5] rewrote the "Euclid" book, to provide the axiomatic proofs missing in it. Hilbert's rewrite mangled the original "Euclid" book and converted it into synthetic

1 NCERT, Mathematics, class IX, p. 79. For an extract, see http://ckraju.net/papers/presentations/images/NCERT-geometry-Ind-Bab-IX-p79.jpg.

2 C. K. Raju, 'Decolonising Mathematics', *AlterNation* 25, no. 2 (2018): 12–43b https://doi.org/10.29086/2519-5476/2018/v25n2a2; C. K. Raju, 'The Religious Roots of Mathematics', *Theory, Culture & Society* 23 (March 2006): 95–97.http://ckraju.net/papers/Religious-roots-of-math-TCS.pdf.

3 Satish Chandra Vidyabhushana, *The Nyaya Sutras of Gotama* (Allahabad: Pāninī Office, 1913). For examples, see http://ckraju.net/papers/presentations/images/Proof-table.html.

4 However, one pre-Buddhist Indian school of thought, the Lokayata or people's philosophers, rejected deductive inference, especially from metaphysical premises as in current formal math, as an unreliable and an inferior means of proof which does NOT result in valid knowledge. This proves that reasoning certainly existed in other traditions from long before any Greeks such as Aristotle whose syllogism copies the Nyaya syllogism. Buddhists and Jains rejected the 2-valued Nyaya logic. Raju, 'Decolonising Mathematics'; C. K. Raju, 'Logic', in *Encyclopedia of Non-Western Science, Technology and Medicine* (Springer, 2016 2008), 2564-70, http://ckraju.net/papers/Nonwestern-logic.pdf; C. K. Raju, 'Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the Yuktibhāṣā', *Philosophy East and West* 51, no. 3 (2001): 325–62, http://ckraju.net/papers/Hawaii.pdf.

geometry.[6] It also eliminated its political goal of equity by inventing the term "congruence" to replace the original term "equal".[7] Anyway, the fact remains that contrary to the myth, axiomatic proofs are **not** found in the original "Euclid" book,[8] and the attempt to rewrite the book as concerning axiomatic proofs violently distorts it.

The original "Euclid" book is also mangled by Birkhoff's metric axiomatisation,[9] since it trivializes the "Euclid" book, makes the order of propositions in it irrelevant, and eliminates the need to prove 46 propositions prior to proving the "Pythagorean proposition". Accepting Birkhoff suggests that the author of the "Euclid" book was a complete dolt to give needlessly prolix proofs. This distortion of the original was ignored by the School Mathematics Study Group,[10] set up after the "Sputnik crisis", which recommended teaching Birkhoff's metric axiomatic geometry.

Today, that axiomatic metric geometry is further confounded with empirical (metric) compass box geometry which is what school students take away. This is laughable: if the axiomatic and empirical approaches really were identical there would be no need for axiomatisation. Of course, Birkhoff's axioms assume Dedekind's axiomatic "real" numbers, which need a metaphysics of infinity not accessible to empirical methods, and distinct from normal or empirical real numbers such as $\sqrt{2}$ known to the sulba sutra-s. Else, Dedekind's axiomatisation and the related axiomatic set theory are redundant!

As the "Sputnik crisis" suggests, math is today taught for its applications to science, which is based on the experimental method, so why should the prohibition of the empirical be central to the philosophy of formal math? Does the prohibition of the empirical make proofs superior? Epistemically more secure? Does it make the theorems of formal math infallible (relative) truths? No! It is FALSE that prohibiting the empirical lead to infallibility or less fallibility.[11] But it has the **political** advantage that it puts Western axiom-makers in control of mathematical knowledge (e.g. asserting that Dedekind reals are essential for calculus).

Hence, formal mathematicians are unwilling or unable to publicly discuss this fundamental question of the non-political advantage of axiomatic math, and have been dodging engaging with it for the last 25 years.[12] Any such debate also threatens their livelihood. Further, the West has for centuries held all kinds of silly beliefs about what is "superior", such as racist beliefs. In math, the West failed to understand imported Indian (primary-school) arithmetic for 9 centuries, especially Western mathematicians persistently fumbled about negative numbers from Fibonacci[13] through Wallis[14] and

5 David Hilbert, *The Foundations of Geometry* (The Open Court Publishing Co., La Salle, 1950), http://ckraju.net/geometry/Hilbert-Foundations-of-Geometry.pdf.
6 E. A. Moise, *Elementary Geometry from an Advanced Standpoint* (Addison-Wesley, 1963).
7 E.g. H. M. Taylor, *Euclid's Elements of Geometry* (Cambridge: Cambridge University Press, 1893).
8 C. K. Raju, '"Euclid" Must Fall: The "Pythagorean" "Theorem" and the Rant of Racist and Civilizational Superiority—Part 2', *Arumaruka: Journal of Conversational Thinking* 1, no. 2 (2021): 57–105, https://doi.org/10.4314/ajct.v1i2.5.
9 George D Birkhoff, 'A Set of Postulates for Plane Geometry, Based on Scale and Protractor', *Annals of Mathematics* 33 (1932): 329–45.
10 School Mathematics Study Group, *Geometry* (Yale University Press, 1961).
11 Raju, 'Decolonising Mathematics'.
12 Raju, 'Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the Yuktibhāṣā'.
13 L.E. Sigler, *Fibonacci's Liber Abaci a Translation into Modern English of Leonardo Pisano's Book of Calculation* (New York NY: Springer, 2002).
14 John Wallis, *Arithmetica Infinitorum* (Oxford: Leon Lichfields, 1656).

Euler[15] to the 19th c. de Morgan,[16] and Peacock.[17]Newton failed to understand Indian calculus, and his confused[18] fluxions lie abandoned. So why should indefensible Western beliefs about what is "superior" matter for math teaching across the world?

The fact also is that the axiomatic method, or prohibiting the empirical, adds no value for applications of math to science and technology; instead it subtracts value.[19] It is a church method: prohibiting facts was politically convenient to the Crusading church invention of Christian rational theology, or reasoning MINUS facts, since the empirical is contrary to almost all church dogmas, and banning it allowed the church to control "truth" in theology. But formal mathematicians are loathe to discuss the possibility that this Crusading church method (of reasoning minus facts) was falsely read into the "Euclid" book when it first arrived in Europe in the 12th c. as a Crusading trophy.[20] This is again understandable because "Euclid" was central to the Western university system set up by the church during the Crusades,[21] and totally controlled[22] by the church until secular education began[23] in European primary schools after 1870. Admitting that the "Euclid" myth of axiomatic proofs is a fraud reflects poorly on that entire Western academic tradition.

In practical terms, the instruments of compass box geometry are unsuited to practical tasks, such as measuring the area of an agricultural plot with crooked boundaries, or to measure angles in space, such as the angle between the moon and the sun used for the scientific definition of a तिथि to teach the Indian calendar. Stock Indian calendrical calculations, traditionally done for the meridian of Ujjaini (copied by the meridian of Greenwich), must be calibrated for the local latitude and longitude, determining which require precise measurement of various angles such as the angle subtended at the eye by a mountain, or the angle of dip of the horizon. The Indian calendar is needed to determine most Indian festivals (including Buddhist, Jain, and Sikh festivals, not just Hindu festivals) as opposed to imposing the inferior,[24] unscientific and alienating Christian

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15 Leonhard Euler, 'De seriebus divergentibus' (Euler Archive - All Works, 1760), https://scholarlycommons.pacific.edu/euler-works/247.

16 C. K. Raju, 'In Black History Month: A Response to Nature's Editorials on Decolonising Mathematics', *Medium* (blog), 26 February 2023, https://medium.com/@c_k_raju/in-black-history-month-a-response-to-natures-editorials-on-decolonising-mathematics-cadc124fb2c8.

17 G. Peacock, *A treatise on algebra. Arithmetical algebra*, vol. 1 (Cambridge: J. & J.J. Deighton; G.F. & J. Rivington, 1842).

18 C. K. Raju, 'Marx and Mathematics. 4: The Epistemic Test', *Frontier Weekly*, 8 September 2020, https://www.frontierweekly.com/views/sep-20/8-9-20-Marx%20and%20mathematics-4.html.

19 C. K. Raju, 'California, Indian Calculus and the Technology Race. 1: The Indian Origin of Calculus and Its Transmission to Europe', *Boloji.Com*, 11 December 2021, https://www.boloji.com/articles/52924/california-indian-calculus; C. K. Raju, 'California, Indian Calculus and the Technology Race. 2: Don't Cancel the Calculus, Make It Easy!', *Boloji.Com*, 24 December 2021, https://www.boloji.com/articles/52950/california-indian-calculus-and.

20 C. K. Raju, *Euclid and Jesus: How and Why the Church Changed Mathematics and Christianity across Two Religious Wars* (Penang: Multiversity and Citizens International, 2012).

21 C. K. Raju, 'How to break the hegemony perpetuated by the university: decolonised courses in mathematics and the history and philosophy of science (Arabic)', in *Culturalisation of Humanities: Vision and Experiments. (Proceedings of the International Conference on Culturalization of the Humanities, held in Beirut on 20-21 November 2018.)* (Beirut: Al Maaref University, 2019), 77–114, http://ckraju.net/papers/Beirut-paper%20for%20iias%20journal.pdf.

22 trans Dana C. Munro, *Translations and Reprints from the Original Sources of European History, No. 3, The Medieval Student*, vol. II: No. 3 (Philadelphia: University of Pennsylvania Press, 1897). For a relevant extract that the Chancellor of Paris University, say, was appointed by and subordinate to the Bishop, see pope Gregory's edicts: http://ckraju.net/papers/presentations/images/Gregory-9-edict-for-Paris-1231.jpg.

23 T. Preston, *Elementary Education Act 1870* (London: William Amer, 1870), https://archive.org/details/elementaryeduca01britgoog.

24 C. K. Raju, 'A Tale of Two Calendars', in *Multicultural Knowledge and the University*, ed. Claude Alvares (Penang: Multiversity, 2014), 112–19, http://ckraju.net/papers/ckr-calendar.html; Also, *A Tale of Two Calendars - Dr C K Raju - India Inspires Talks* (New Delhi, 2015), https://www.youtube.com/watch?v=MvpuC7Dg4e0&feature=youtu.be.

(Gregorian) calendar. The immediate question here is of the mathematical pre-requisites needed to teach the Indian calendar which need to be included in the primary school curriculum.

In string geometry, as traditionally used in India (sulba-sutra) and Egypt ("rope stretchers", harpedonaptae), an angle is defined differently, as the relative length of a curved arc, not as involving a pair of straight lines.[25] However, the current geometry box has no instrument such as a flexible string, or flexible tape needed to measure the length of a curved line, and directly determine the angle in radians, say. The axiomatic definition of the length of a curve is very complex. A string can actually replace **all** the instruments in a compass box,[26] and gives a superior understanding of the two "Pythagorean" *calculations*.[27]

I also describe a traditional navigational instrument (kamal or rapalagai)[28] used to measure various "space" angles to much higher precision (a fraction of a degree) than provided by a protractor (converted into a hack sextant). The higher precision is needed for both navigation and astronomy. This is achieved by using using the two scale ("Vernier") principle applied to a pair of harmonic scales, a principle not understood by Vasco da Gama (and other Europeans) who copied the instrument from India but, as usual, failed to understand its sophisticated construction, down to present times.

This "rajju ganita" geometry course has been experimentally taught, to both students and teachers, in various Indian schools in several Indian states.[29] A draft text, and a draft teaching manual[30] are available. The attempt now is to incorporate the various mathematical pre-requisites in a structured way into the primary school curriculum, to enable also the teaching of the traditional Indian calendar to children. Teaching the Christian calendar, as done in colonial education, alienates people from their culture. That calendar has a crude and unscientific definition of months, delinked from the lunar cycle, due to persistent arithmetical incompetence of Europeans. It uses leap years to avoid fractions hence gets the tropical year right only on a thousand-year average. Worst of all, it lacks a way to determine the rainy season still so critical to the Indian economy.

25 *How Colonial Education Changed Our Math Teaching | C.K. Raju*, 2020, https://www.youtube.com/watch?v=Rm6d-bUmmGg; Raju, C. K., 'Śulba-Sūtra Geometry: Can We Teach It in School Today?' (Delhi, 23 November 2019), https://www.youtube.com/watch?v=rLI_UU6dfnE.

26 C. K. Raju, 'Towards Equity in Math Education 2. The Indian Rope Trick', *Bharatiya Samajik Chintan (New Series)* 7, no. 4 (2009): 265–69. http://ckraju.net/papers/MathEducation2RopeTrick.pdf.

27 C. K. Raju, 'Black Thoughts Matter: Decolonized Math, Academic Censorship, and the "Pythagorean" Proposition', *Journal of Black Studies* 48, no. 3 (2017): 256–78, https://doi.org/10.1177%2F0021934716688311.

28 C. K. Raju, *Cultural Foundations of Mathematics: The Nature of Mathematical Proof and the Transmission of Calculus from India to Europe in the 16th c, CE* (Pearson Longman, 2007); Also, a preliminary account in C. K. Raju, 'Kamal or Rapalagai', in *Indo Portuguese Encounters: Journeys in Science, Technology and Culture*, ed. Lotika Varadarajan, vol. II (Delhi, Lisbon: Indian National Science Academy and Centro de Historia de Alen-Mar, Universidade Nova de Lisboa, 2006), 483–504.

29 http://ckraju.net/geometry/Rajju-Ganit-poster.pdf.

30 http://ckraju.net/geometry/Rajju-ganit-draft-teacher-manual.pdf.