Ganita vs mathematics

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The new education policy (NEP) has laid considerable stress on the development of indigenous knowledge systems (IKS). However, many colonially educated people are convinced of total Western superiority in knowledge, hence believe that any engagement with India's intellectual IKS is necessarily obscurantist and chauvinistic.

The aim of the proposed meeting on 11 October 2023 is to initiate a critical and rational discourse in the current highly polarized atmosphere, in such a way that the outcome would benefit students. Our discussion on IKS would be limited to the issue of Indian ganita versus Western mathematics.

One aim is to not limit the discussion to just mathematicians, who have a vested interest in existing (Western/formal/axiomatic) mathematics, but to involve also all those who USE mathematics and statistics, such as physicists, computer scientists, economists, and including people in humanities (such as History and English, who use at least statistics).

In this case of ganita versus mathematics, the usual claim of the supremacy of White/Western knowledge fails because, for thousands of years, Western mathematics was obviously far behind Indian ganita.

The simplest case is that of arithmetic. The undeniable fact is that Greek and Roman names for numbers are simplified forms of Sanskrit names.[1] The other undeniable fact is that the early Greeks and Romans were backward in arithmetic: the largest number they knew was a myriad, which even today in English means an uncountably large number, as in "a myriad of stars". In contrast, the number parardha (trillion) is found in Yajurveda 17.2, and very large numbers such as tallakshana 10^{53} and beyond are found in Buddhist literature (Lalita Vistara,[2] chp. 12). Therefore, *the gradient of ignorance in arithmetic clearly points towards the West*, and knowledge flowed along this gradient.

What were the consequences of this Western backwardness in arithmetic? Large numbers are needed for precise fractions (which are ratios of two large numbers) needed for higher precision in science. Alas, early Greek and Roman arithmetic lacked all but the simplest fractions, therefore (in the science of astronomy) the reformed Roman calendar (Julian calendar) used the crude device of leap years as a substitute for fractions. This **error** (making every fourth year a leap year, so that 1 tropical year = $365\frac{1}{4}$ days) directly led to its gross inaccuracy, which was officially accepted in Europe only during the 1582 calendar reform, when 11 whole days were dropped from that highly inaccurate Julian calendar. Though Europe had repeatedly imported fractions from India (via Arabs and directly from Kochi) by that time (end 16th c.), fractions were still not in common use in **European pebble (or whole number) arithmetic of abacus**. Hence, to avoid fractions even the Gregorian reform of 1582 used a complex system of leap years instead of stating a precise fraction for the precise duration (it wanted to say year = 365.241 days), of the tropical year. This crude

1 C. K. Raju, *Refutation of the Aryan Race Conjecture: The Arithmetic Evidence and Conquest-of-Greeks Theory* (Delhi: Kant Academic Publishers, 2022).

2 Rajendralal Mitra, ed., *The Lalita Vistara (*ललित विस्तर*:* सुत्त*)* (Calcutta: Asiatic Society of Bengal, 1877); Vaidya P. L., ed., *Lalita-Vistara (*ललित-विस्तर*:)*, Buddhist Sanskrit Texts 1 (Darbhanga: Mithila Institute, 1958).

arithmetic technique of leap years (every fourth year is a leap year but every hundredth year is not a leap year, but every thousandth year is) resulted in bad science: the reformed Gregorian/Christian calendar (the current Western calendar) still does not get the equinox on the same day every year.

Europe had no rainy season, and the Gregorian calendar has no way to determine the monsoon[3] season so critical to the Indian economy, for it is a primitive solar calendar, not a sophisticated **lunisolar** calendar like the Indian calendar. However, after independence from the British, we adopted this inferior calendar on the superstitious colonial conviction that everything Western is superior and must be **uncritically** aped.

Returning to arithmetic proper, contrary to colonial indoctrination, the West itself understood that its Greek/Roman arithmetic system of abacus is **inferior**, and damaging to its **commercial** interests. Therefore, the West abandoned its primitive native arithmetic in favor of the sophisticated algorithms of Indian ganita. The word algorithm comes from the Latinized name of al Khwarizmi of Baghdad who wrote the book Hisab al Hind on the superior system of Indian place value arithmetic, used in India since the earliest Vedic times. This process of Europeans abandoning their inferior native system of arithmetic (abacus) in favor of superior Indian ganita (algorithms) took them almost a thousand years starting from Gerbert (10th c.), and Fibonacci (13th c.), and going on to Clavius (16th-17th c.) and de Morgan (19th c,)

Since our immediate perspective is that of math education, rather than history, the most important point here is this long period indicates the **difficulty the West had in understanding Indian ganita even at the most elementary level of arithmetic.** This difficulty in arithmetic faced by the West is important because colonial education is premised on the maxim "ape the West". Therefore, on the further principle that "phylogeny is ontogeny" these historical Western difficulties with Indian ganita are replicated in today's math classroom, in fast-forward mode.

To stick to the easily understood example of arithmetic: the abacus is a concrete system of pebble arithmetic, or a coin counter system, which reduces all arithmetic to counting, and **is hence so inefficient**. However, the transition from inefficient abacus to efficient algorithms involved difficulties in understanding. When Gerbert (later Pope Sylvester 2) imported "Arabic" numerals from Cordoba in the 10th c, he entirely failed to understand the efficiency of algorithms, as shown by the fact that he constructed an abacus for "Arabic" numerals! His lack of understanding of the difference between the two systems of arithmetic (abacus vs algorithms) is further clear from the very term "Arabic" numerals which suggests that for him the only difference was some magic in the *shape* of the numerals!

After three centuries, a Florentine merchant Fibonacci eventually understood that the efficiency of **Indian arithmetic (algorithms) offers a comparative advantage in commerce**. For example, the simple problem 78×89 takes 1422 operations on the Roman abacus, whereas it takes just 10 operations in ganita.[4] But Florentines had another difficulty with the place value system, especially zero, as clear from the Florentine law against zero of 1299 (which we still follow in writing cheques, that if Arabic numerals are used, the amount of a financial contract must be written also in words).

These Western difficulties with zero occurred due to the Western cultural commitment to inferior Greek and Roman **pebble arithmetic**. (We have already seen the European problem with fractions in the 16th c.) These Western difficulties with Indian ganita persisted until the 19th century. Thus,

3 C. K. Raju, 'A Tale of Two Calendars', in Multicultural Knowledge and the University, ed. Claude Alvares (Penang: Multiversity, 2014), 112–19, http://ckraju.net/papers/ckr-calendar.html; A Tale of Two Calendars - Dr C K Raju - India Inspires Talks (New Delhi, 2015), https://www.youtube.com/watch?v=MvpuC7Dg4e0&feature=youtu.be.

4 C. K. Raju, *Refutation of the Aryan Race Conjecture: The Arithmetic Evidence and Conquest-of-Greeks Theory*.

zero relates to negative numbers and as is well known, Brahmagupta used negative numbers in the 7th c. However, an august and very influential professor of mathematics at University College London, De Morgan, foolishly declared in the 19th century that negative numbers are impossible![5] He also got confused about ordering among negative numbers declaring belief in witches 10000 times more possible than $-9<0$!

On the maxim "phylogeny is ontogeny" these silly European difficulties with Indian ganita are reflected in the difficulties in current arithmetic teaching to school students: transition to algorithms, fractions, zero and negative numbers.

While the above examples are limited to arithmetic, for easy comprehension by non-mathematicians, the very same problems of Western difficulty in understanding imported ganita relate also to **algebra, trigonometry, calculus, probability and statistics**, as the meeting would disuss. That Europe got all these aspects of ganita from India is clear from the fact that the very term for √2 is SURD from Latin surdus = DEAF from Arabic asumu. Why? In Indian शुल्ब सूत्र √2 = DIAGONAL (कर्ण) of unit square. But word कर्ण also means ear (=कान), hence bad कर्ण was mistranslated as bad ear = deaf by Arabs and this mistranslation was preserved when Europeans translated it from Arabic to Latin). Likewise, the foolish term sine (from sinus=जेब a misreading of Arabic जीबा from Sanskrit जीवा). The related conceptual misunderstanding is clear from the very word **trigonometry**: if the sine function is defined in relation to a right-angled triangle (as NCERT does), a simple thing like sin 92° is not immediately defined. But the term जीवा from ज्या makes clear that we are talking of circular functions which are defined for any value (positive or negative) of the related arc (चाप=angle).

However, the key principle of colonial education is that we must ape the West and trust it because the major teaching of colonial/church education was the political teaching that the West must always be deemed superior, and colonised must be subservient to its authority. Admitting that Europeans copied trigonometry from India, and failed to understand it, goes against this central dogma of colonial education for it shows the West in the harsh light of the truth. Therefore, any related facts must be set aside and **never allowed to be discussed**![6]

The related European misunderstanding of algebra (Brahmagupta's अव्यक्त गणित) and the related notion of ordering of polynomials had subtler consequences ,because polynomial arithmetic is non-Archimedean, contrary to the arithmetic of real numbers today used to teach calculus.

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5 Augustus de de Morgan, *On the Study and Difficulties of Mathematics*, New Ed. (Chicago: Open Court Pub, 1898); Augustus de Morgan, *Elements of Algebra: Preliminary to the Differential Calculus*, 2nd ed. (London: Taylor and Walton, 1837).

6 E.g. see this article, in the *Conversation*, South Africa, which went viral, was reproduced world-wide, and was then censored worldwide. Science 2.0 retained it, in India, the Wire, one of the two portals which reproduced it, putit back. C. K. Raju, 'To Decolonise Maths, Stand up to Its False History and Bad Philosophy', *The Wire*, 2016, https://thewire.in/history/to-decolonise-maths-stand-up-to-its-false-history; C. K. Raju, 'Was Euclid A Black Woman? Sorting Through The False History And Bad Philosophy Of Mathematics | Science 2.0', 24 October 2016, https://www.science20.com/the_conversation/was_euclid_a_black_woman_sorting_through_the_false_history_and_bad_philosophy_of_mathematics-180581; C. K. Raju, *Mathematics, Decolonisation and Censorship*, 2017, https://kafila.online/2017/06/25/mathematics-and-censorship-c-k-raju/; C. K. Raju, 'Black Thoughts Matter: Decolonized Math, Academic Censorship, and the "Pythagorean" Proposition', *Journal of Black Studies* 48, no. 3 (2017): 256–78, https://doi.org/10.1177%2F0021934716688311; C. K. Raju, 'To Decolonise Math Stand up to Its False History and Bad Philosophy', in *Rhodes Must Fall: The Struggle to Decolonise the Racist Heart of Empire* (London: Zed Books, 2018), 265–70.

The Indian origin of calculus is by now well documented.[7] (a) So many of the purported Western discoveries such as Euler's method (Aryabhata method)[8], Fermat's challenge problem (a solved exercise in Bhaskar 2),[9] Pascal's triangle, Stirling's formula (from Brahmagupta, Vateshwar), Newton's sine series,[10] Leibniz series[11] etc. are all found in India long before the people after whom they are named. (b) These purported "discoveries" by Westerners all post-date the extensive contacts between Europe and India in the 16th century, during which the Jesuits in Kochi were systematically collecting Indian books, translating them and sending them back to Europe. (c) There is ample evidence of theft of calculus on the basis of criminal law: opportunity, motivation (European navigational problem), circumstantial evidence (e.g. Fermat's challenge problem) and documentary evidence.[12] (d) Most importantly, in the present context of mathematics-teaching, the demand of colonial Western bhakts to call these as "independent rediscoveries" is not valid because of my epistemic test: those who steal knowledge, like students who cheat in a test and then claim "independent rediscovery" failed to FULLY grasp the knowledge they steal.

The Western failure to fully grasp the stolen Indian calculus is clear from the fact that 3 centuries after Newton's purported "discovery" of calculus, Dedekind invented real numbers since calculus was self admittedly NOT understood by Europeans (including Marx) in the 19th century: they failed to understand how to sum infinite series. Cantor's set theory used by Dedekind was full of paradoxes, such as Russell's paradox, therefore the final formulation of real numbers had to await axiomatic set theory which developed only in the 1930s. This aspect of real numbers (and the related axiomatic set theory) makes calculus so very complicated that it is almost impossible for most students (even of real analysis) to understand it fully. As a result, California recently canceled calculus teaching in schools. We ape the Western understanding of calculus; should we ape the cancellation of its teaching? The difficulty with axiomatic real numbers is also clear from my Cape Town challenge to prove 1+1=2 in real numbers from first principles (without assuming any theorems of set theory) for which I offered a prize of Rs 10 lakhs in JNU. (The prize went unclaimed.) Or should we use ganita to make the teaching of calculus very easy as has been demonstrated? This is a key point which needs an extensive discussion. This point was recently discussed at IIT: Kanpur, which is likely to start a course on calculus as ganita.

We note incidentally that today most practical applications of the calculus are done on computers. (I can vouch for this since I was in charge of implementing applications of national importance on the first Param supercomputer built by C-DAC.) For example, this is true for the calculation of rocket trajectories for Chandrayaan say. The issue here is that computers CANNOT use real numbers, since they require infinite memory. Hence, computers use floating point numbers instead typically with 32 or 64 bits. Floating point numbers do not obey the associative "law" for addition so basic to the algebraic structure of real numbers. The failure of the associative law happens because addition of floating point numbers involves bit shifting of the mantissa, to equalize exponents, when the exponents are widely different. Hence, if ε is a small number (1E-8 for 32-bit floats, or 1E-16 for 64-bit floats, etc.) then $\varepsilon+1 = 1$, so that $(\varepsilon+1) - 1 = 0 \neq \varepsilon = \varepsilon + (1 - 1)$. So, rejection of real numbers does not affect **any** practical application of the calculus (or statistics) done on computers.

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7 C. K. Raju, *Cultural Foundations of Mathematics: The Nature of Mathematical Proof and the Transmission of the Calculus from India to Europe in the 16th c. CE* (Pearson Longman, 2007).

8 Raju.

9 C. K. Raju, 'Calculus Transmission', in *Encyclopedia of Non-Western Science, Technology, and Medicine* (Springer, 2016), 1016–22, http://ckraju.net/papers/Springer/ckr-Springer-encyclopedia-calculus-2-final.pdf.

10 C. K. Raju, 'Calculus', in *Encyclopedia of Non-Western Science, Technology and Medicine* (Springer, 2016), 1010–15, http://ckraju.net/papers/Springer/ckr-Springer-encyclopedia-calculus-1-final.pdf.

11 Raju, *Cultural Foundations of Mathematics: The Nature of Mathematical Proof and the Transmission of the Calculus from India to Europe in the 16th c. CE*.

12 Matteo Ricci, 'Letter to Petri Maffei', 1581, ff. 129r-130v, Goa; Matteo Ricci, 'Letter to Petri Maffei', in *Documenta Indica*, vol. XII, 1581, 472–77.

The first recorded sum of infinite geometric series was obtained in Indian ganita by Nilakanth (15th-16th c.)[13] using Brahmagupta's non-Archimedean arithmetic[14], in a way which was not grasped by Europeans. Teaching calculus as Indian ganita (with non-Archimedean arithmetic) makes it enormously easy, as has been demonstrated in pedagogical experiments with 8 groups in 5 universities in 3 countries.[15]

The case of probability and statistics is similar.[16] Probability originated with the game of dice the first mention of which is found in the aksh sukta of the Rgveda. The theory of permutations and combinations, and the binomial expansion (meru prastar) was all developed in ancient India. The important point is that the game of dice in the Mahabharata involves the notion of fair play (or unbiased dice), hence the notion of probability. The Mahabharata also involves an illustration of statistics or sampling theory to count the number of fruits on the branch of a tree. As is well known, the so-called Pascal's triangle for the coefficients of the binomial expansion copies the meru prastar or khanda meru of Indian ganita.

In any case, the key point here is that probability could **not** be understood in the calculus sense of limits. Thus, on the law of large numbers the best one can say is that relative frequency converges to probability only in a probabilistic sense. Therefore, the frequentist understanding of probability fails (since it begs the question to define probability as the "limit" of relative frequency, since this notion of "limit in probability" presupposes the notion of probability). The subjectivist interpretation ("probability is the degree of subject belief") also fails because quantum probabilities[17] are objective, like a diffraction pattern. Finally, the measure-theoretic understanding of probability fails, because if probabilities are defined on a quantum logic[18], that logic (like Buddhist or Jaina quasi-truth-functional logic[19]) does not form even a Boolean algebra, let alone a σ-algebra.

All these considerations deserve extensive discussion, but any attempt to do so tends to get censored. The reason for censorship or taboo is **to avoid any discussion on the superstitious dogma of Western supremacy at the core of colonial education** which could collapse on examination. As pointed out in my Tübingen-Pretoria keynote/article[20] the claim of Western supremacy is a politically convenient **mutation** during colonialism, from the earlier superstitious claim of White supremacy during slavery, which was itself a politically convenient **mutation of the still earlier claim of Christian supremacy** used to justify both genocide and slavery on the American continents. The immediate point is that the same false history (with a change of labels) was used to support all three related claims of supremacy: Christian, White, Western supremacy.

The most important such claim of Christian/White/Western supremacy in mathematics is the claim in our class IX NCERT school text (chp. 5)that the "Greeks" ("Euclid") did a superior form of (axiomatic) geometry which all mathematics must imitate. Of course, NCERT has no primary evidence for the existence of "Euclid" or any evidence of any early Greek text which names Euclid

13 C. K. Raju, 'Calculus', in *Encyclopedia of Non-Western Science, Technology and Medicine*, ed. Helaine Selin (Dordrecht: Springer, 2016), http://ckraju.net/papers/Springer/ckr-Springer-encyclopedia-calculus-1-final.pdf.
14 C. K. Raju, 'Decolonising Mathematics', *AlterNation* 25, no. 2 (2018): 12–43b.
15 Raju.
16 C. K. Raju, 'Probability in Ancient India', in *Handbook of Philosophy of Statistics*, ed. Paul Thagard Dov M. Gabbay and John Woods, vol. 7, Handbook of Philosophy of Science (Elsevier, 2011), 1175–96, http://ckraju.net/papers/Probability-in-Ancient-India.pdf; C. K. Raju, 'Probability', in *Encyclopedia of Non-Western Science, Technology and Medicine*, ed. Helaine Selin (Dordrecht: Springer, 2016), http://ckraju.net/papers/Springer/Probability-springer.pdf.
17 C. K. Raju, 'Quantum-Mechanical Time', in *Time: Towards a Consistent Theory* (Kluwer Academic, 1994).
18 Raju.
19 C. K. Raju, *The Eleven Pictures of Time: The Physics, Philosophy and Politics of Time Beliefs* (Sage, 2003).
20 C. K. Raju, '"Euclid" Must Fall: The "Pythagorean" "Theorem" and the Rant of Racist and Civilizational Superiority - Part 2', *Arumaruka: Journal of Conversational Thinking* 1, no. 2 (2021): 57–105, https://doi.org/10.4314/ajct.v1i2.5.

as the author of the text attributed to him. For all such history we are required to have faith in Western authority, not seek evidence, overlooking the fact that the West was under hegemonic church rule for over a thousand years. The existence of “Euclid” is important to judge the intentions of the author of the book, which are so different from the beliefs about geometry prevalent among early Greeks. And also because the “Euclid” book, contrary to the story told about it, does not have a single axiomatic proof in it,[21] although this falsehood was believed by Europeans (e.g. Cambridge University) from 1125 CE when the Euclid book first came to Europe during the Crusades, until 1899 when Hilbert wrote on the foundations of geometry[22] to supply the axiomatic proofs missing in the “Euclid” book.

What makes this absence of axiomatic proofs in the “Euclid” book very much more problematic is that axiomatic proofs **are** found and were used in church rational theology.[23] Axiomatic proofs prohibit facts, and since facts are contrary to most church dogmas, this is a very convenient method of proof for the church. Anyway, if mathematics is taught for the sake of science which accepts empirical proofs, it is not clear how prohibiting the empirical in mathematics makes for a “superior” notion of proof. However, as pointed out above, asserting its own superiority to the point of genocide and slavery has been a very persistent aspect of church dogma. During colonialism this was used to make the colonised mentally subservient.

It would be great to have a rational discussion on the issue of why reason plus empirical, as used in ganita and science, is inferior to reason MINUS empirical as used for proof in axiomatic mathematics. As we already saw, axiomatics does not add any practical value.

Colonial power was based on the assertion of Western supremacy, which is a key aspect of colonial education, to keep the colonized mentally subservient. An open debate may undermine this Western mental dominance which continues after independence, hence it has been taboo. But an open debate is needed since independent India has a right to strive for freedom from mental domination which persists even after political independence from the British.

21 Raju.
22 David Hilbert, The Foundations of Geometry (The Open Court Publishing Co., La Salle, 1950), http://ckraju.net/geometry/Hilbert-Foundations-of-Geometry.pdf.
23 Thomas Aquinas, *Sumnma Theologica*, n.d., http://www.newadvent.org/summa/1052.htm#article3.