Decolonised calculus

C. K. Raju

Indian Institute of Education
G. D. Parikh Centre, J. P. Naik Bhavan,
Mumbai University Kalina Campus
Santacruz (East), Mumbai 400 098
ckr@ckraju.net

Background

Calculus was invented in India by the 5th c. Aryabhata, who first used a method of finite differences ("Euler" method) to solve a differential equation and to derive 24 sine values accurate to the first sexagesimal minute (about 5 decimal places).[1,2,3] Numerical solution of non-linear differential equations is still at the core of all *practical* applications of the calculus today, though current calculus teaching at the undergraduate level teaches mostly tricks and formulae related to differentiation and integration only of *elementary* functions,[4] apart from *linear* differential equations. Some more details of the Indian origin of calculus, its transmission to Europe, and some current applications of changed calculus teaching are in my MIT talk.[5]

Aryabhata's technique developed over the next thousand years. First, Brahmagupta and Vateshwara used quadratic interpolation ("Stirling's formula"). Eventually, Aryabhata's followers in Kerala used 11th and 12th order interpolation/extrapolation, to increase Aryabhata's precision and obtain those 24 sine values accurate to the third sexagesimal minute (about 9 decimal places). Indeed, they developed infinite "Taylor" series expansion, including the sine series falsely claimed by Newton, and the series today wrongly known as Leibniz series. However, Newton and Leibniz did not understand how to sum *any* infinite series, while Indians did have a way to sum infinite series using

1 C. K. Raju, *Cultural Foundations of Mathematics: The Nature of Mathematical Proof and the Transmission of Calculus from India to Europe in the 16th c, CE* (Pearson Longman, 2007).

2 C. K. Raju, 'Calculus', in Encyclopedia of Non-Western Science, Technology and Medicine (Springer, 2016), 1010–15, http://ckraju.net/papers/Springer/ckr-Springer-encyclopedia-calculus-1-final.pdf; C. K. Raju, 'Calculus Transmission', in Encyclopedia of Non-Western Science, Technology, and Medicine (Springer, 2016), 1016–22, http://ckraju.net/papers/Springer/ckr-Springer-encyclopedia-calculus-2-final.pdf.

3 For a recent popular level account, concerning difficulties in calculus teaching in the context of technology, and the remedy, see C. K. Raju, 'California, Indian Calculus and the Technology Race. 1: The Indian Origin of Calculus and Its Transmission to Europe', *Boloji.Com*, 11 December 2021, https://www.boloji.com/articles/52924/california-indian-calculus; C. K. Raju, 'California, Indian Calculus and the Technology Race. 2: Don't Cancel the Calculus, Make It Easy!', *Boloji.Com*, 24 December 2021, https://www.boloji.com/articles/52950/california-indian-calculus-and.

4 See, e.g., http://ckraju.net/sgt/technical-presentations-faculty/ckr-sgt-tech-presentation-1.pdf.

5 Talk on "Calculus the real story", MIT, Cambridge, Mass. May 2015. Video: https://www.youtube.com/watch?v=IaodCGDjqzs, Presentation: http://ckraju.net/papers/presentations/MIT.pdf, Abstract: http://ckraju.net/papers/Calculus-story-abstract.html, blog: http://ckraju.net/blog/?p=106. See, also, blog of "Institute lecture and workshop" at IIT-BHU: http://ckraju.net/blog/?p=163, and video of lecture at Indian Institute of Science, Bengaluru, at http://ckraju.net/blog/?p=112.

Brahmagupta's non-Archimedean arithmetic of polynomials combined with zeroism.[6] In the 15th c., Nilakanth used this method to first explicitly sum the infinite geometric series.

On the principle that "phylogeny is ontogeny" students today learn calculus in the same sequence in which it historically developed in Europe. Consequently, while anyone is able to rattle off the formula that $\frac{d}{dx}e^x = e^x$, few can define e^x . While they may say try to define it using an infinite series, almost no one can explain how to sum that series (see this short video of students from IIT Ahmedabad) . Europeans accepted their lack of understanding of infinite series and, in the late 19th c., Dedekind developed formal "real" numbers and limits, which are supposedly rigorous. But real numbers initially used Cantorian set theory which was riddled with paradoxes and contradictions. Therefore, a proper formal theory of real numbers had to wait until axiomatic set theory developed in the 1930's. On the same principle of "phylogeny is ontogeny" few, even among professional mathematicians, understand this late development of axiomatic set theory, in relation to real numbers and calculus.

Hence, my repeated Cape Town challenge and reward of Rs 10 lakhs to prove 1+1=2 in real numbers from first principles.[7] But those who cannot do that teach calculus and real analysis in prestigious colleges and universities.

This difficulty of understanding the calculus was recently recognized by the California state department of education which effective cancelled the teaching of calculus in schools.

To go to the historical roots of this problem with (the Western lack of understanding of) calculus arose because the calculus was stolen from India. Those who steal knowledge may get credit for it, like students who cheat in an exam, but they don't fully understand what they steal. This is my epistemic test.[8]

Why did Europeans steal calculus? Because in the 16th c., Europeans, were mathematically (even arithmetically) backward. Fractions were introduced into the European syllabus only in 1575, though fractions were being taught thousands of years earlier in other places like Egypt (Ahmes papyrus) and India.[9] Indo-Arabic navigational techniques then involved significant mathematical sophistication. Hence, mathematically backward Europeans had a major navigational problem, starting from difficulty in understanding how to measure the size of the spherical earth, how to represent that curved surface on a plane two dimensional chart in a way compatible with European primitive techniques of plane navigation (problem of loxodromes), how to determine latitude and longitude at sea through astronomical observations, etc.

But Europeans, then, were also very poor and their sources and dreams of wealth were all overseas. They first sailed to India with the dream of getting rich. But that required a good technique of navigation to bring back the wealth earned overseas through trade or piracy. Because of bad European techniques of navigation many men and treasure ships were lost.

---

6 C. K. Raju, 'Zeroism', in *Encyclopedia of Non-Western Science, Technology and Medicine*, ed. Helaine Selin (Dordrecht: Springer, 2016), 4604–4610, http://ckraju.net/papers/Springer/zeroism-springer-f.pdf; Raju, 'Calculus'.

7 This reward was offered, for example, to the faculty in JNU, "Statistics for Social Science and Humanities: Should we Teach it Using Normal Math or Formal Math?" https://youtu.be/A9Og1k-Z5O4?t=662. The importance of the 1+1=2 test in real numbers was explained in a talk to Army War College, MHOW: "Ancient India's contribution to mathematics and its relevance to modern technology." See, full video or points.

8 C. K. Raju, 'Marx and Mathematics. 4: The Epistemic Test', *Frontier Weekly*, 8 September 2020, https://www.frontierweekly.com/views/sep-20/8-9-20-Marx%20and%20mathematics-4.html.

9 C. K. Raju, 'To Decolonise Math Stand up to Its False History and Bad Philosophy', in *Rhodes Must Fall: The Struggle to Decolonise the Racist Heart of Empire* (London: Zed Books, 2018), 265–70; C. K. Raju, 'To Decolonise Maths, Stand up to Its False History and Bad Philosophy', *The Wire*, 2016, https://thewire.in/history/to-decolonise-maths-stand-up-to-its-false-history.

Hence, from the 16th to the 18th c., many European governments offered large prizes for the solution of their navigational problem. Precise trigonometric values (and a reformed calendar) were the solution to all three European problems of determining loxodromes, latitude and longitude at sea. At this time (16th c.) the most precise trigonometric values in the world were those of the Aryabhata school in Kerala, located around Cochin.

Obtaining these precise trigonometrical values and astronomical knowledge, to solve their navigational problem, was the major **motivation** for the theft of the calculus from India, by European. But there was also ample **opportunity** for the theft of calculus, even by standards of criminal law.[10]

How did the West steal the calculus? Thus, the first Roman Catholic mission was opened in Cochin in 1501, after Vasco da Gama fled from Calicut and sought refuge in Cochin which was hostile to Calicut. Soon there was a "mission school" around this mission for the local Syrian Christians, naturally in the local language, Malayalam. Eventually Jesuits turned this school into a college around 1550. Indians never understood that Jesuit priests, these "holy men" of Christianity, doubled as military spies. They systematically gathered knowledge from local sources, with the help of the local Syrian Christians, with whom they formed a religious bond. Naturally, their greatest interest was in precise trigonometric values, the Indian calendar and related astronomical models, all of which knowledge was readily available in texts of the Aryabhata school in Kerala. The Jesuits translated this Indian knowledge and sent it back to Europe. That is how calculus was stolen.

There is ample **circumstantial and documentary evidence** of theft, since this stolen Indian knowledge started appearing in Europe from the end of the 16th c. For example, the astronomical model of Tycho Brahe, Astronomer Royal to the Holy Roman Empire, was a carbon copy of the astronomical model of Nilakantha, of the Aryabhata school in Kerala. The Gregorian calendar reform was based on inputs from India provided to Clavius, the Jesuit general who headed the calendar reform committee. In 1607 Clavius published trigonometric tables which were an explicitly interpolated version of Madhava's sine table. This was surprising because shortly earlier the navigational theorist Simon Stevin was quoting from al Khwarizmi who repeated Aryabhata's value of pi. (Recall that the development from Aryabhata's precision of sexagemisal minutes to Madhava's precision of sexagesimal thirds took many centuries in India.) Thus, there is ample evidence that the calculus was stolen.

However, colonial education constantly drilled into the colonized the need to imitate their "superior" masters, and obey them. Therefore, even though most teachers of calculus and real analysis do not understand even 1+1 = 2 in real numbers, they parrot "real numbers" and "rigor" imitation of whatever is done in the West. Further, colonial education made math so difficult that few people understand what it is about, but all think that the sole people to be trusted are the teachers of mathematics in our colleges and universities, who will insist that limits and real numbers are "rigorous", but cannot prove 1+1 = 2 in real numbers from first principles, even when large rewards are offered.

This imitation of West coupled with lack of understanding was a key objective of colonial education: it ensures a loyal band of followers who can never turn into true leaders, because leadership requires innovation which requires understanding. It is particularly bad for those who dream of participating in modern technology development. For example, there is a lot of recent interest in techniques of machine learning and artificial intelligence *innovations* in which require a

10 For a quick recent recap, see, C. K. Raju, 'Western Appropriations of Indian Ganita: Contemporary Consequences', in *TSSP Eminent Scholar Lecture Series*, 2022, https://www.youtube.com/watch?v=lmwZfMaNwfo.

good *understanding* of statistics which presupposes a good *understanding* of calculus. This is just an example, and numerous other examples can easily be found.

When Indians originally innovated the calculus, they did so with full understanding of what they were doing. Indian calculus involves **non-Archimedean arithmetic** (Brahmagupta's avyakt ganita of polynomials) in which there are no limits, unlike the case of "real" numbers. This makes calculus very easy: so easy that it can be taught in just five days, though it takes time (1-2 month) for students to get familiar and comfortable with all the ideas involved.

Though this is too advanced a topic to be explained here, the failure of college calculus for discontinuous functions, and the failure of the linear Schwartz theory of distributions (in the context of nonlinear partial differential equations in physics) requires non-Archimedean arithmetic for its proper solution.[11]

Another novel aspect of Indian calculus is **Zeroism** which may be briefly explained as follows.

Thus, on the Western philosophy of mathematics, mathematics is an **exact** science. However, consider the very elementary case of the "Pythagorean" theorem. It is supposed to be exactly true. But, where in the world does the Pythagorean theorem apply **exactly**? Certainly not for triangles drawn on the curved surface of the earth, because these triangles involved curved lines (and the sum of their interior angles is not 180°). Even the seventh century commentator Bhaskara 1 commented that the rule is gross, because applying the rule to the curved surface of the earth results in gross values (according to the disciples of Aryabhata). Today, we would say "Euclidean" geometry does not apply to the triangles drawn on the curved surface of the earth. But that is equally true for triangles drawn in space which is curved or non-Euclidean. So, the Pythagorean theorem does not apply exactly **anywhere** in the real world. The best we can say is that (1) Western or axiomatic or formal mathematics is exact only in a fantasy world which (2) sometimes applies *approximately* to the real world.

Likewise, "real" numbers involve a fantasy world. In practice, one never use "real" numbers in real life. For example, on a computer, one uses floating-point numbers, which, at best, approximate "real" numbers. However, the algebraic properties of floating-point numbers are fundamentally different: for example, the associative law for addition fails for floating-point numbers.[12]

However, there are other mathematical theorems, such as the Banach-Tarski theorem which says that a ball of gold can be subdivided into a finite number of parts which can be reassembled (without stretching or tearing) into two balls of gold identical to the first. In other words, the mathematics of set theory enables one to become infinitely rich. This is obvious nonsense, which does not apply even approximately to the real world. This is a major problem with formal mathematics: we construct an imaginary world which may or may not have anything to do with the real world.

The major advantage of formal or axiomatic mathematics is that it enables the West to control mathematical knowledge: because Western mathematicians laid on the exams (for example of set theory), and on the principle of secretive refereeing they pronounce in secret on whether or not a given proposition has been validly proven.

---

11 C. K. Raju, 'Distributional Matter Tensors in Relativity', in *Proceedings of the 5th Marcel Grossman Meeting*, ed. D. Blair and M. J. Buckingham (World Scientific, 1989), 421–23, arXiv:0804.1998; C. K. Raju, 'Products and Compositions with the Dirac Delta Function', *J. Phys. A: Math. Gen.* 15 (1982): 381–96.

12 For a C program demonstrating this, see C. K. Raju, 'Computers, Mathematics Education, and the Alternative Epistemology of the Calculus in the Yuktibhāṣā', *Philosophy East and West* 51, no. 3 (2001): 325–62, http://ckraju.net/papers/Hawaii.pdf. Or see the section on floats in my "Class notes in C".

In contrast, Zeroism openly accepts the fact that exactitude is impossible, because the real world changes every instant. Therefore, Zeroism accepts inexactitude as a fact of life, and strives to obtain better approximations. This was exactly the philosophy adopted in traditional Indian math, since the earliest times. Hence, the sulba sutra speaks of the ("real") number $\sqrt{2}$ as *savisesha* (= with something remaining). Likewise, Aryabhata describes his accurate value of π as *asanna* (near value). In his commentary on Aryabhata, the 15th-16th c. Nilakantha explained why the "real" value cannot be given.

This constant search for better approximations is exactly what is done in science. But this would abolishes the artificial distinction between and exact and metaphysical mathematics and inexact, physical science, a distinction which is of nil practical importance. Actually this distinction was created in the West because the church captured mathematics (during the Crusades) and twisted it to suit its political goals.[13] Colonial education teaches related silly myths, such as the myth of "Euclid", for whom there is no evidence,[14] and whose purported book does not have a single axiomatic proof.[15]

However, after colonial education Indians have become backward in math. Today they are indoctrinated into all those myths in childhood when they accept them without checking them, and eventually learn only to fanatically imitate the West without understanding, so that Indian turn into mental slaves who can never be leaders.

13 See, The church origins of (axiomatic) math.

14 See, "Decolonising history: Goodbye Euclid" special lecture at Universiti Sains Malaysia, chaired by the Malaysian k Deputy n Minister for Higher Education: http://ckraju.net/blog/?p=63. Or see https://youtu.be/sEK1FCrLHjU?t=3292.

15 C. K. Raju, '"Euclid" Must Fall: The "Pythagorean" "Theorem" and the Rant of Racist and Civilizational Superiority - Part 2', *Arumaruka: Journal of Conversational Thinking* 1, no. 2 (2021): 57–105, https://doi.org/10.4314/ajct.v1i2.5.