Mathematics, infinity, and cosmos

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Abstract

Orientalist discourse since Max Mueller has viewed India as spiritual. The case of Indian mathematics shows this to be a complete distortion of reality. *Gaṇita*, or Indian mathematics, was entirely practical. There is no trace of spirituality in it over a 3000 year period, from the earliest Indian texts such as the *Vedānga Jyotiṣa* (a practical manual of timekeeping from −1350 CE), through the *śulba sūtra* (−500 CE), down the centuries through Āryabhaṭa (5th c. CE), Bhaskara, Lalla, Vaṭeśvara, Āryabhaṭa II, Bhaskara II, and right down to the 15th–16th c. CE texts such as the *Āryabhaṭīyabhāṣya*, the *Kriyākramakari*, and the *Yuktibhāṣa*.

In contrast, Western mathematics has been spiritual right from its inception. The early Greeks took mystery geometry, like their gods, from Egypt. The very word "mathematics" derives from *mathesis*, a notion which connects knowledge to beliefs about soul and cosmos. Socrates questions the slave boy in Plato's *Meno*, to demonstrate mathesis as a way to wake the soul and make it recollect its knowledge from previous lives. Mathesis means learning; hence the Platonic dictum that all learning is recollection. Mathematics was regarded as especially suited to this purpose, because the belief was that mathematics incorporates *eternal* truths which readily arouse the *immortal* soul. This view of mathematics as a mystery was promoted by the Pythagoreans, and Neoplatonists (down to Proclus).

Gaṇita and mathesis differed on the notion of proof. The current (formal) Western notion of mathematical proof, dating from Hilbert, is completely metaphysical. Greek geometry emphasized metaphysical proofs, so that the materialist Epicureans ridiculed it claiming that the theorems of geometry were known to any ass. In contrast, proof in Indian mathematics was just the same as the notion of proof which applied to anything else, such as physics, or law, or philosophical beliefs. The empirically manifest (*pratyakṣa*), was accepted as a means of mathematical proof, right since the days of the *śulba sūtra*-s, through Āryabhaṭa, and down to the *Yuktibhāṣa*. Consequently, Indian texts proved the "Pythagorean" theorem in one step, without needing 46 preceding theorems as in the *Elements*.

Mathesis and the underlying notion of soul involved beliefs about (quasi-cyclic) time which were very much part of early Christianity. The post-Nicene church changed the notion of the soul, and hence attacked the notion of soul underlying mathesis. The church cursed those beliefs (in a past life) in its anathemas against "pre-existence". Proclus, who thought the eternal truths of mathematics implied an eternal cosmos, was similarly declared a heretic, and all schools of philosophy were shut down in the Roman empire.

Mathematics was accepted back in the West after the Crusades, but only after making it theologically correct, by stripping it of its spiritual ideas, and reducing it to a mere tool to teach reasoning and metaphysical ways of persuasion useful to the church. Present-day formal mathematics is an apologia for Platonism from a post-Crusade theological perspective. The supposed value of theorem-proving is premised on the belief (of post-Crusade theology) that metaphysical deduction is certain and universal, binding even on God, and hence superior to empirical proof, or induction (since God is bound by logic, but free to create the world). This belief is carried over into the present-day formal claim that deductive proofs incorporate "necessary truth" or truth in all possible worlds (where "possible worlds", earlier understood as created or imagined by God, are now reinterpreted in the formal sense of Wittgenstein).

This naively assumed (like all Western philosophy) that logic is universal. However, Buddhist logic (*catuṣkoṭi*) and Jain logic (*syādavāda*) (and quantum logic) show that logic varies with culture (and physics), and is *not* binding on the non-West (or the cosmos). Consequently, deductive proofs are culturally limited, like Western music, and the theorems of present-day formal mathematics can never hope to be universal, unlike practical mathematics.

The two traditions of gaṇita and Western mathematics collided when Indian gaṇita was imported into Europe, starting from the "Arabic numerals" or "algorismus" in the 10th c, and going on to trigonometry and, then calculus, in the 16^{th} c. The most serious clash was over the calculus, and related to notions of the infinite. Descartes declared the length of a curved line to be beyond the human mind. Curiously Newton thought this difficulty with infinities and infinitesimals in the calculus could be rigorously resolved through "fluxions", or by supposing that *time flows* smoothly (whatever that might mean). *Hence*, he made time metaphysical (referring to "absolute, true, and mathematical time"). This mistake led eventually to the failure of Newtonian physics (when Poincaré corrected it by defining a proper clock, to physically measure equal intervals of time, thus formulating special relativity).

Nevertheless, this Western mathematics of infinity, related to the calculus, was globalised by colonialism, and modern mathematicians continue to believe that the infinities of the calculus can *only* be handled metaphysically by the postulates of set theory, and that for Newtonian physics to make sense, time (or the t in $\frac{d}{dt}$) *must* be like the "real line"! What if it is not, and time is structured, in reality? So, this is a current metaphysical imposition on physics.

In contrast, Indian "mathematicians" focused on the practical value of the calculus, especially by solving differential equations, as was first done

by Āryabhaṭa in the 5th c, using what I have called "the fundamental *algorithm* of calculus" (the fundamental *theorem* of calculus being a triviality in the context, and implied by the fact that Āryabhaṭa stated only sine *differences*). This was not "*mere* calculation" or "numerical *approximation*" as formalists keep jumping up to exclaim from their theological belief in mathematics as *exact* (since it was thought to incorporate *eternal* truths); there was a method to the inexactitude. Indian mathematics treated polynomials like numbers, and rational functions like fractions; hence it was able to handle infinities and infinitesimals rigorously by order counting, and without limits. (This used what formalists would today call the "non-Archimedean field of rational functions"; formally, any field larger than that of real numbers *must* be non-Archimedean; in such a field, limits still exist, but are non-unique, or unique only up to "infinitesimals".) This was how sums of various infinite series were first understood in India, starting from the infinite geometric series.

This systematic acceptance of (residual) non-uniqueness or inexactitude, from the (*sa anitya* = impermanent) of the *śulba sūtra*-s, to the (*āsanna* = near value) of Āryabhata and Nīlakaṇṭha involved a philosophy, most rigorously argued by Buddhist *śūnyavāda*, which accepts change as the fundamental reality (a unique entity does not endure for two instants). Hence, it denies the possibility of exact representation of *anything* across two instants of time. Hence, also, it rejects all ideal notions as erroneous simplifications, forever incapable of capturing reality. Zeroism is a de-textualised version of this philosophy, accepted purely for its practical value. Zeroism provides a superior alternative foundation for the calculus, though Western philosophers have failed to grasp this philosophy (and may never do so, for they have long banned serious non-Western philosophy from their curriculum).

Curiously, this down-to-earth and practical philosophy of mathematics and physics, coupled with the rejection of the Western metaphysical ideas of time coming down from Augustine and Newton, leads to a *better* physics, which is *non-mechanistic*, and admits spontaneity (as distinct from chance) as real. (Spontaneity allows a physical description of life, or a better physical model of biological organisms.) Even more curiously, while the underlying notion of time, in this new physics, is locally similar to Buddhist *paticca samuppāda*, globally it allows (but does not guarantee) the physical existence of a quasi-cyclic (time-reversing) cosmos of the kind assumed in the Upaniṣads or the mystery or the sūfī tradition.

(The first prominent person to notice the similarity of the Upaniṣads and sūfī tradition was, I believe, the Mughal prince, Dara Shukoh, who wrote about it in his *Sirr-ī-Akbar*—the *Great Secret*. In the process, he lost an empire, and had his head served on a platter to his father; but this led to the diffusion of the Upaniṣads in Europe, which, in turn, deeply influenced Orientalist thought about India.)