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Time: What is it That it can be Measured?*

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Abstract: Experiments with the simple pendulum are easy, but its motion is nevertheless confounded with simple harmonic motion. However, refined theoretical models of the pendulum can, today, be easily taught using software like CALCODE. Similarly, the cycloidal pendulum is isochronous only in simplified theory.

But what *are* theoretically equal intervals of time? Newton accepted Barrow's even tenor hypothesis, but conceded that 'equal motions' did not exist—the refutability of Newtonian physics is independent of time measurement.

However, time measurement was the key difficulty in reconciling Newtonian physics with electrodynamics. On Poincaré's criterion of convenience, equal intervals of time ought be so defined as to make the enunciation of physics simple. Hence he *postulated* constancy of the speed of light. (The Michelson-Morley experiment was not critical.) The theory of relativity followed. But does there exist a proper clock?

Introduction

The simple pendulum is practically the first 'real' physics experiment that school students perform—or ought to perform—in standard 7 or 8. It is an

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introduction to the experimental method in science—to the idea that science proceeds from the empirically manifest, and from inferences from it, and that these are more reliable means of knowledge than mere authority.

However, in my experience both as a parent and as one who has taught 'refresher courses' to school teachers, science teachers in even the most elite Indian schools tend to be persistently confused about this first step. An anecdote quickly illustrates the state of affairs.

My elder son came to me with his 7th standard school text which stated that the time period of the simple pendulum is independent of amplitude, even when the angular amplitude was 90°. The text encouraged children to perform various experiments to test this 'fact' for themselves. It went on to state that grandfather clocks used a simple pendulum.

I was horrified for various reasons. The author of the text, who was also a teacher in the school, stated that she had taught this for ten years before penning the text. It was apparent that, in these ten years, the author of the text had never actually performed the first experiment that she encouraged the students to perform. It was also apparent that she never had enough curiosity to take apart a grandfather clock and see the kind of pendulum that it actually used. The science she taught did not, for her, relate to any of her actual experiences.

This being one of the most elite schools in Delhi, with a singular principal, on my protest the school eventually agreed to correct the text—though nothing obviously could be done for the students who had learnt from it for ten years. My son went on to actually perform an experiment on this, and to compare with the theoretical variation of the time period of the pendulum with amplitude.[1].

The next part of the anecdote concerns my younger son, eleven years later, then also in the 7th standard. During a routine interaction with his science teacher, I suggested ways in which science teaching could be made more interesting, by relating it to the actual experiences of the child, and pointed to my earlier experience, and how that led my elder son to develop an interest in science. The science teacher listened patiently. I thought I had convinced her. The next day, she told my son in front of the class that she had listened to me only out of politeness, and that actually what I was saying was completely wrong, since she had verified that the time period of the simple pendulum was independent of its amplitude!

Let me admit straightaway that this anecdotal evidence is not statistically representative—the situation in most schools in India is undoubtedly far worse! In fact, even the samples of teachers to whom I taught refresher courses were not statistically representative.

However, it seems to me fair to generalise as follows. The simple pendulum, widely regarded as an excellent tool for science teaching, helps to bring out also the infirmities of science teaching. At least in India, there is widespread confusion regarding it: confusion which demonstrates confusion about the very nature of science in the minds of school science teachers. To check the truth about even so simple a thing like the pendulum, the teacher relies not on an experiment performed by hand, but on the authority of a text. The problem with this process is of course well known—under these circumstances, the teacher cannot have a real idea of how authoritative or accurate a text is. Thus, even in as simple a matter as the simple pendulum, wrong notions are persistently taught. Exactly how widespread is this confusion in the minds of science teachers obviously requires further statistical studies, but I would not be surprised if such studies came up with a figure exceeding 95%.

The theoretical confusion about the pendulum obviously originates in the simplified theory of the pendulum, for small oscillations, for which, using the approximation $\sin\theta = \theta$, one obtains its equation of motion as the equation of simple harmonic motion

$$\ddot{\theta} = p^2\theta. \tag{1}$$

Here $p^2 = \frac{g}{l}$, g is the acceleration due to gravity and l is the length of the pendulum, so that its time period $T = \frac{2\pi}{p}$ is independent of the amplitude, and is usually written

$$T = 2\pi\sqrt{\frac{l}{g}}. \tag{2}$$

Variation in the time period of the simple pendulum

Why is this simplified theory so widespread? Because the more precise theory is regarded as something too difficult to teach in schools. But is that really the case?

For larger oscillations, one can no longer use the approximation $\sin\theta = \theta$, and the equation of motion of the simple pendulum is given by (Synge and

Griffiths 1959, p. 334)

$$\dot{y}^2 = p^2(1-y^2)(1-k^2y^2), \tag{3}$$

where,

$$\begin{aligned} k &= \sin\frac{\alpha}{2} \\ y &= \frac{1}{k}\sin\frac{\theta}{2}, \end{aligned} \tag{4}$$

and α is the angular amplitude of the pendulum.

Strangely, the difficult part of solving this equation is to specify the sign of the square root in (3). (Thus, only one sign cannot be specified, since the sign of the square root must change as y crosses a maximum.) To do this, it is helpful to first rescale the equation, using $x = pt$, so that it becomes,

$$\left(\frac{dy}{dx}\right)^2 = (1-y^2)(1-k^2y^2). \tag{5}$$

Jacobi's method of specifying the choice of square root was to introduce three new functions. The function $\mathrm{sn}(x)$ is specified as a continuously differentiable solution of (5) which further satisfies,

$$\mathrm{sn}(0) = 0, \quad \mathrm{sn}'(0) > 0. \tag{6}$$

The continuously differentiable functions $\mathrm{cn}(x)$, $\mathrm{dn}(x)$ are defined by the conditions

$$\mathrm{cn}^2(x) = (1-\mathrm{sn}^2(x)), \quad \mathrm{cn}(0) = 1 \tag{7}$$

$$\mathrm{dn}^2(x) = (1-k^2\mathrm{sn}^2(x)), \quad \mathrm{dn}(0) = 1. \tag{8}$$

These are the Jacobian elliptic functions. Since sn is a solution of (5), we can now specify the square root by regarding the three Jacobian elliptic functions as solutions of the three simultaneous differential equations:

$$\frac{d}{dx}\mathrm{sn}(x) = \mathrm{cn}(x)\mathrm{dn}(x) \tag{9}$$

$$\frac{d}{dx}\mathrm{cn}(x) = -\mathrm{sn}(x)\mathrm{dn}(x) \tag{10}$$

$$\frac{d}{dx}\mathrm{dn}(x) = -k^2\mathrm{sn}(x)\mathrm{cn}(x). \tag{11}$$

We already know that the solution of the motion of the simple pendulum is given by the $y = \mathrm{sn}(x) + c$, so that time period of the simple pendulum relates to the time period of the sn function. This is known to be given by the symbolic expression

$$4K = \int_0^1 \frac{dy}{(1-y^2)(1-k^2y^2)}. \tag{12}$$

involving a non-elementary elliptic integral.

There was a time, not so long ago, when people used to write advanced tomes devoted mostly to tabulating these widely used but hard-to-evaluate integrals (Byrd 1971). This probably explains why many school science teachers, who may not even be specialists in physics, are completely unaware of the exact theory of the simple pendulum, and incorrectly believe, like Galileo, that the simple pendulum is isochronous.

However, today, the job of numerically evaluating elliptic integrals can be done by a simple computer program like my CALCODE,[2] which accepts symbolic input and provides a graphic output, and permits further calculation with the numerical output. Since the derivation of the equations is not a particularly hard matter, neither is their solution. So it *is* possible today to teach the more precise theory of the pendulum in schools, and along with it a better idea of what science is about, and the difficulties that attend to confirming or refuting theory by experiment.

Note that a numerical computation of this sort may be expected to be significantly superior to the usual simplified formula for the amplitude dependence of the time period of the pendulum which is obtained using a power series expansion to give

$$T = 2\pi\sqrt{\left(\frac{l}{g}\right)}\left\{1 + \left(\frac{1}{2}\right)\sin^2\left(\frac{\alpha}{2}\right) + \left(\frac{1\cdot 3}{2\cdot 4}\right)\sin^4\left(\frac{\alpha}{2}\right) + \ldots\right\} \tag{13}$$

$$\approx 2\pi\sqrt{\left(\frac{l}{g}\right)}\left(1 + \frac{\alpha^2}{16}\right). \tag{14}$$

The difference between the formulae (2) and (14) is tabulated in the unnumbered table on p. 20 of the project already referred to. The difference in the second decimal place is actually quite large, for it means a difference of one second in 100 oscillations, which would take hardly 30 minutes. So, if one is thinking of building a clock with this theory, the difference would amount to something like a minute in the course of a day. If one uses this clock for navigation, to determine longitude at sea, that could be decidedly fatal.

Further, as is clear from that student project already referred to, one needs either greater experimental or theoretical sophistication: experimental sophistication to make the apparatus conform to the assumptions, or theoretical sophistication to make the calculations conform to the state of experimental affairs. In my opinion, it is harder to refine the experiments, but easy today to sharpen the theory. For this, one would want to take into account things like friction at the point of suspension, and air resistance etc., and though this would make the symbolic solution well nigh impossible—and certainly far out of the reach of students—the calculations are easy to perform via software like CALCODE.

This, then, is my first conclusion. That software available today can make the task of science teaching and learning much easier and more interesting, by freeing both teachers and students from the need to make various unrealistic simplifying theoretical assumptions, especially as regards Newtonian mechanics. It is possible today to allow the the student to explore theoretically and experimentally a variety of factors—the sort of exploration that could not have been contemplated twenty years ago—which exploration can provide a far more realistic idea of the nitty-gritty of science, instead of projecting science as all grand (but unrealistic) theory.

The cycloidal pendulum

Of course, software like CALCODE can also be used to study the cycloidal pendulum. The cycloidal pendulum designed by Huygens is so constructed that the bob is obliged to move along a cycloid. A cycloid is the curve traced out by a point on the circumference of a circle, when the centre of the circle moves in a straight line with constant velocity. For example, a pebble stuck in the tyre of a cycle traces out a cycloid, when the cycle moves with constant velocity. The parametric equations of a cycloid are given by

$$x = r(\theta - \sin\theta) \tag{15}$$

$$y = r(1 - \cos\theta). \tag{16}$$

The cycloid is a very interesting curve from a number of viewpoints, and its mathematical properties have been explored in great detail, and one can readily find many excellent expositions of it, for example, in the nice book by Simmons (1972). Briefly, the cycloid is the brachistochrone (*brachistos* = shortest, *chronos* = time; path of quickest descent), and also the tautochrone (*tauto* = same; hence path of equal descent, on which a falling object will reach the bottom at the same amount of time, no matter from where it starts). (Note that a 'taut' string also has the tautochronous property, in the sense that whether it is plucked a little or a lot, it makes the same sound, i.e., vibrates with the same time period.) The last property is related to the isochronous nature of the cycloidal pendulum, namely that its time period is independent of the amplitude, and is given by

$$T = 4\pi\sqrt{\frac{r}{g}}, \tag{17}$$

where r is the radius of the generating circle of the cycloid, and g is the acceleration due to gravity as before.

The typical description of the cycloidal pendulum relies on Huygens' mechanism of two cycloidal arches or 'cheeks' used to constrain the motion of the pendulum. This has costly implications in terms of the friction along the arcs which which generates effects larger than the circular error corrected by cycloidal motion (Gardner 1984). Also, practically speaking, it is hard to construct the arches so as to give a mathematical cusp, so it also requires the pendulum to oscillate through a large amplitude.[3]

The point I am making is, I hope, perfectly clear. Exactly like my children's school teachers, most teachers and books around the world seem satisfied with presenting a neat and satisfying, but over-simplified theoretical account of the isochrony of the cycloidal pendulum, as opposed to a real-life account of it. The attitude is the same, the level of technicality is different. Perhaps I should clarify at this stage that I have no objection to simplification, as such—it seems to me perfectly reasonable to present to a child an abridged version of a big novel. I also do not deny the possibility that in some cases, like Toynbee's *A Study of History*, an abridgement might decidedly improve upon an excessively prolix original, though this is often not the case. In any case, no one—not even a child—would confuse the abridgement with the original. Therefore, at a more fundamental level, my complaint is this: a simplified account is being taught as an 'ideal' account.

Let me explain what I mean. Historically speaking, Huygens' claim of the isochrony of a real cycloidal pendulum was disestablished only a few years after Galileo's claim of the isochrony of the simple pendulum was refuted. As Huygens recounts,

> at the request of the Directors of the Indies Company I undertook for finding longitudes to construct clocks of . . . sure and constant motion.[4](Mahoney 1980)

There were various difficulties in testing the clocks which were finally put to proper test on the ship *Alkmaar* in 1687. According to the clocks, the *Alkmaar* had sailed right through Ireland and Scotland rather than around them!

Huygens had heard of the reported phenomenon of the variation in the 'length' of a pendulum with latitude, and though this had been discounted by people like Jean Picard, Huygens suggested a theoretical explanation for it in terms of the centrifugal force due to the rotation of the earth. He reasoned that the centrifugal force diminished the weight of bodies depending on latitude. He calculated that a pendulum clock regulated to mean time at the poles would fall behind by about 2.5 minutes at the equator. If longitude were determined using such a clock, then if the clock were carried along the same meridian from poles to equator, it would indicate an apparent shift of longitude to the east. Nevertheless, the result of a second trial of Huygens' clocks in 1690–92 also disappointed everyone, and Huygens admitted that 'I have found the business much more difficult than I thought at the outset.'

Nevertheless, the current attitude is that Galileo was somehow 'wronger' than Huygens—because Galileo was 'mathematically' wrong while Huygens was only 'physically' wrong! While it is apparent that neither kind of pendulum resulted in an accurate clock, the belief is that the simple pendulum is not isochronous even from a theoretical viewpoint, while the cycloidal pendulum is. Therefore, the simplified theory of the cycloidal pendulum continues to be taught, while most accounts will mention that the linear theory of the simple pendulum is a simplification.

But the fact is that the simplified theory of the cycloidal pendulum is equally unrealistic. In fact, so far as clock makers were concerned, many of them took the position that the variation of the time period of the simple pendulum with amplitude was of no consequence.

> Although this [Huygens' cycloidal pendulum] is a perfect theoretical solution to the problem of circular error, it has never been put into practice successfully. It will be appreciated that the existence of circular error as such is of no importance so far as timekeeping is concerned; it is only the variations in error which matter. Hence, circular error would be satisfactorily accounted for by maintaining a constant amplitude. (Bishop 1955)

Thus, clockmakers focussed on what would happen to the rate of the clock if small 'kicks' were administered to the simple pendulum to keep its amplitude constant. These 'kicks' were conventionally known as impulse, and were supplied through the escapement. Thus, the corresponding error came to be known as 'escapement error'. Airy suggested in 1827 that the escapement error could be made zero to the first order if impulses were supplied symmetrically around the zero position of the pendulum.

Equal intervals of time in Newtonian mechanics

It should be fairly clear by now that, from a practical point of view, the cycloidal pendulum could never be made to mark equal intervals of time. For both Galileo and Huygens, the real problem of isochrony was the *practical* problem of determining longitude at sea, using clocks, and from this practical perspective neither the simple pendulum nor the cycloidal pendulum was good enough.[5]

But, exactly what is meant by the statement that the cycloidal pendulum theoretically marks equal intervals of time? What does it mean to say that intervals of time are theoretically equal? A diligent student who performs a variety of experiments with the pendulum is bound to come around and eventually ask the big question: how does one know that the pendulum is or is not marking equal intervals of time? After all, most laboratory experiments in school simply involve measuring time using a stop watch, so how does one know that the stop watch is correct, and it is the pendulum that is wrong? That is, what exactly is meant by equal intervals of time?

This is a question that ought to be addressed, at least at the undergraduate level. Note that Newtonian mechanics is the theory which is used without comment today to assert that Galileo was theoretically wrong and Huygens was theoretically right. However, note also that this theory came up long after Galileo's death, and that Galileo could conceivably have answered this question in a way incompatible with Newtonian mechanics.

THE REFUTABILITY OF NEWTON'S 'LAWS' OF MOTION

Now, it is today recognized that Newton's 'laws' of motion are not, by themselves, physics, since there is no clear cut way to refute them. However, it is usually taught that Newton's first 'law of motion' defines an inertial reference frame, and the second law of motion defines the notion of 'force'. How good are these even as definitions? Specifically, how good are these definitions in the absence of a definition of the notion of equal intervals of time (Raju 1994, 1991a, 2003)? Without a definition of the notion of equal intervals of time, there is no way to check whether or not a body is in a state of uniform motion, hence there is no way to determine whether or how much force is acting on the body. For example, if a teenager's pulse is used to define equal intervals of time, the passing by of an attractive person of the opposite sex is likely to lead to large forces!

According to Popper (1982), the refutability of Newtonian physics comes about by combining the 'laws of motion' with the Newtonian 'law of gravitation' (Raju 1991a), or the theory of falling bodies with the theory of planetary motion. Thus, for example, on Galilean physics, the path of a stone thrown in the air is a parabola, while it is usually an ellipse according to Newtonian physics (although the small portion of the ellipse that is visible provides a close approximation to the parabola). However, it is noticeable that the procedure suggested by Popper involves examining world lines (of planets) or trajectories of particles (projectiles), *without reference to time.* That planets move in elliptic orbits around the sun was, of course, believed to be true prior to Newton (and prior to Kepler), so what the Newtonian theory must be credited with is the unification of the theory of falling bodies with the theory of planetary motion.

BARROW'S EVEN TENOR HYPOTHESIS

We note also that for Newton himself the definition of equal intervals of time did not pose a problem. Newton's teacher, Barrow, had devoted much thought to time, and this was the topic with which he commenced his lectures on geometry. His opening volley (?)Barrow) was an ironic reference to Augustine's 'very trite Saying' ('What, then, is time?'): 'If no one asks me I know; but if any Person should require me to tell him, I cannot.' He thought this escape route was not available to 'Mathematicians' since they 'frequently make use of Time, they ought to have a distinct Idea of the meaning of that Word, otherwise they are Quacks'!

He then introduced the even-tenor hypothesis, 'whether things move... or stand still; whether we sleep or wake, Time flows perpetually with an equal Tenor.' (Barrow 1976, p. 205) Barrow's argument was that a quantity has a reality independent of the means used to measure it.[6] His other argument was that the imperceptible need not be non-existent: '*When we wake we cannot perceive or tell how much Time has passed during our Sleep;* which is certainly true: But it cannot be justly inferr'd from thence: *We do not perceive the Thing, therefore there is no such Thing,* that is a false Illusion, a deceitful Dream, that woud cause us to join together two remote Instants of Time.'(Barrow 1976, p. 205)[Italics original] [7]

Since time flows in 'an equal Channel, not by Starts', it could be measured only by a special class of motions, called 'equal motions', such as those of the sun or moon, adapted for that purpose by 'the divine Will of the Creator'. Was there any reason, apart from 'divine Testimony', to call this an 'equal motion'? Barrow appeals to the principle of sufficient reason: these motions could be compared using clocks, 'as, for Instance, an Hour-Glass... because the Water or Sand containd in it remain entirely the same as to Quantity, Figure and Force of descending, and the Vessel that contains them, as likewise the little Hole they run thro' don't undergo any Kind of Mutation, at least in a short Space of Time, and the State of Air much the same; there is no Manner of Reason for us not to allow the Times of every running out of the Water or Sand to be equal.' In short, Barrow's formula for equal intervals of time is that *the same causes take the same time to produce the same effects*.

NEWTON ON BARROW'S EVEN TENOR HYPOTHESIS

Newton simply accepted Barrow's even tenor hypothesis. Newton stated that he did 'not define time, space, place, and motion, as being well known to all' (Newton 1962), but sought to remove 'certain prejudices' amongst 'common people', by restating the even-tenor hypothesis: 'Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration. . . '

Newton readily conceded that any actual clock one might think of would be erroneous. He granted that solar motion could not be used to measure equal intervals of time, 'For the natural days are truly unequal, though they are commonly considered as equal, and used for a measure of time.' What, then, are 'equal motions'? This requires an inertial frame, and the universality of gravitation makes Newton doubt the existence of any such 'equal motions': 'It may be, that there is no such thing as an equable motion. . . . All motion may be accelerated and retarded. . . .' So far as he is concerned, this make no difference, for the flow of time has a reality independent of the means used to measure it, for things endure all the same, 'whether the motions are swift or slow, or none at all'.

EQUAL INTERVALS OF TIME AND THE ORIGIN OF RELATIVITY

In any case, all this still does not tell us what is the theoretical definition of equal intervals of time in Newtonian mechanics, and how one should go about determining whether two intervals of time are equal or unequal. It is perhaps not so well known that it was the absence of a definition of equal intervals of time that created a crisis in reconciling Newtonian physics with electrodynamics, at the turn of the previous century.

It was the resolution of this theoretical difficulty that led to the theory of (special) relativity. Contrary to what is incorrectly stated in numerous physics texts, the Michelson-Morley experiment was only of indirect relevance—it was performed *not* to test the existence of the aether, or the constancy of the speed of light, but only to discriminate between the aether theories of Stokes and Fresnel. It came out in support of Stokes' theory (Raju 1991b). This experimental conclusion was theoretically unacceptable, because the kind of motion visualised by Stokes was mathematically impossible.[8] Hence, Lorentz stated,

> The difficulties which this [Stokes'] theory encounters in explaining aberration seem too great for me to share this [Michelson's] opinion. (Lorentz et al. 1952, p. 4)

and went on to postulate length contraction as an alternative way to explain the experimental conclusion.

Poincaré recognized that this was an *ad hoc* hypothesis, and that the central issue was the reconciliation between Newtonian mechanics and electromagnetic theory. Poincaré also recognized that the central feature in this reconciliation was a definition of the notion of equal intervals of time, or, equivalently, a notion of simultaneity. This was eventually achieved by Poincaré's criterion of convenience (Poincaré 1958). Poincaré acknowledged that any definition of equal intervals of time would involve an element of arbitrariness.

> *We have no direct intuition of the equality of two intervals of time.* Those persons who believe they possess this intuition are dupes of an illusion. When I say, from noon to one the same time passes as from two to three, what meaning has this affirmation? The least reflection shows that by itself it has none at all. It will only have that which I choose to give it, by a definition which will certainly possess a certain degree of arbitrariness. (Poincaré 1958, p. 27) [Emphasis original]

To resolve this arbitrariness, he proposed the *criterion of convenience* (which has been needlessly ridiculed by those who do not understand it). According to this criterion, equal intervals of time ought be so defined as to make the enunciation of physics as simple as possible.

> The simultaneity of two events, or the order of their succession, or the equality of two durations, are to be so defined that the enunciation of the natural laws may be as simple as possible. (Poincaré 1958, p. 36)

This directly led him to *postulate* the constancy of the speed of light. (A photon bouncing between parallel mirrors hence marks equal intervals of time.) The relativity of simultaneity and the consequent need for a clock to measure length, follow as easy consequences of this postulate (Raju 1992).

However, from a practical point of view, the construction of a clock using a bouncing photon requires parallel geodesics (assuming the geodesic hypothesis). It is not clear in which part of the cosmos one can hope to find parallel geodesics, and whether the parallelism endures for any length of time. To put matters in another way: which physical clock is a proper clock? The answer is not known.

Does there exist a proper clock? This remains one of the five important unsolved problems related to time (Raju 1994, p. 208).

Conclusions

1. The availability of numerical computing software like CALCODE today allows the teaching and exploration of more realistic theoretical models of pendulum motion closer to experimental conditions.

2. Galileo's claim of the isochrony of the simple pendulum is no 'wronger' than Huygens' claim of the the isochrony of the cycloidal pendulum: from a practical point of view, both failed to solve the longitude problem of European navigation.

3. From a theoretical point of view, the very notion of equal intervals of time has no real definition in Newtonian mechanics.

4. The need to unify Newtonian mechanics with electromagnetic theory led, by Poincaré's criterion of convenience, to the postulated constancy of the speed of light, and the consequent definition of equal intervals of time using a photon bouncing between parallel mirrors.

5. Such a proper clock might not exist in reality, so it may not be possible to devise any practical method of accurately measuring time.

Notes

[1] The project report file may be downloaded from `http://11PicsOfTime.com/pendulum.pdf`. Questions regarding the project may be sent directly to `suvrat@physics.harvard.edu`

[2] CALCODE is a 'calculator' for ordinary differential equations (ODE), which accepts symbolically defined ODE's as input and provides a graphical output of their solution. It also performs many common tasks of Newtonian physics which all relate to the numerical solution of ODE's. [See also Hairer et al. (1991).] A free evaluation version of this software can be obtained by sending me an email. The Windows version will be developed, if there is enough interest.

[3] I should perhaps add that, in my childhood experiments with opening up watches, I came across what seems to me a somewhat neater mechanism (which, of course, I did not then understand, and am not sure I now recall correctly). The mechanism consisted of suspending the pendulum between twin knife edges, each of which was backed by a spring. As the pendulum moves, its motion pushes the knife edges in either direction, which retreat along a cycloidal path (as I now reconstruct it). I imagine that this mechanism reduces the friction due to the wrapping of the string around the arches, but I have not seen any proper description or analysis of this in the literature, and investigating this would make an interesting student project.

[4]This is an interesting aspect of the Indian contribution to the study of time!

[5]This problem was specific to European techniques of navigation, using charts. In fact, other ways then existed of determining longitude at sea, and these ways had long been known to other traditions, and these are recounted, for example in the *Laghu Bhaskarīya* I.29 (Bhaskara I 629a), or *Mahābhāskarīya* II.8, II.3–4 (Bhaskara I 629b), but though Europeans had access to these texts by the 16th c. CE, they could not use these methods because the one most easily usable at sea involved the size of the earth, and Europeans then lacked a precise estimate of the size of the earth, which was long available in Indo-Arabic sources. This estimate was lacking because of an 'error' made by Columbus. Hence, the Portuguese banned the use of the globe aboard ships, from 1500, and Picard's estimates of 1672 were not initially credible to navigators.

[6]'Magnitudes themselves are absolute Quantums Independent on all Kinds of Measure tho' indeed we cannot tell what their Quantity is, unless we measure them; so Time is likewise a Quantum in itself, tho' in Order to find the Quantity of it, we are obliged to call in Motion to our Assistance.' Barrow (1976, p. 204)

[7] A similar argument was advanced earlier by Giordono Bruno.

[8]See Raju (1991b). Briefly, Stokes thought that the aether instead of passing through the earth, as in Fresnel's theory, was dragged along by the earth in its motion. To ensure that plane wave fronts remained plane, he required the motion of the aether to be irrotational, and he also required it to be at rest relative to the earth. Such a flow was mathematically impossible, because the irrotational flow of an incompressible fluid is a potential flow. And the potential equation (Laplace equation) admits a unique solution if the normal derivative is specified at the boundary (Neuman problem). The aether is a key construct in the ancient Indian Nyāya-Vaiśeṣika system, and related to their doctrine of action by contact, as also in Descartes. Curiously, exactly these two hypothesis about the possible motion of the aether relative to the earth were also used by Varāhamihīra, in the 6th c. CE *Pancsiddhāntikā*, to contest Āryabhaṭa's claim that the earth rotated. If the aether streams through the earth (as in Fresnel's theory), Varāhamihīra argues, clothes on a clothesline would be blown off. If the aether in the vicinity of the earth is dragged (as in Stokes' theory) then, Varāhamihīra reasons, at least the falcon which flies high in the air would not be able to return to its nest. This notion of the aether is also found in post-10th c. CE Arabic works attributed to Aristotle, and a paraphrase of these arguments against the rotation of the earth is also found in the 11th c. CE Arabic works attributed to 'Claudius Ptolemy'.

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