Calculus: gaṇita or math?

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- Will first state the key points, then offer proof.

Key points-1

Calculus: the real story

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- they failed to understand calculus (17th–19th c.),
- hence converted it from *gaṇita* to math.

Key points-2

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- and returned to India through colonial education.

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Science as superstition

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  - the claim that math is "superior" to *gaṇita*.
- Just accepted it all: science (in practice) = blind faith in the West.

Key points-4

The colonised mind

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  - the false Western history of calculus
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- Colonised academics say "we don't understand",
- they say "good science = certificates of Western approval".

Key points-5

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  - suits all existing practical application of calculus, and
  - suits advanced applications to science.

A general story

Not calculus alone

- Not only calculus but most present-day school math

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- Let us start with the easier story of arithmetic.

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- Roman and early Greek way of representing numerals is inferior: requires 12 symbols instead of 4.
- Challenge: can you write 10^{53} in Roman numerals?

Indian place value system

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- Numbers such as 10^{29} in Jaina literature.

इमा मेऽअग्नऽइष्टका धेनवः सन्त्वेका च दश च दश च शतञ्च शतञ्च सहस्रञ्च सहस्रञ्चायुतञ्चायुतञ्च नियुतञ्च नियुतञ्च प्रयुतञ्चार्बुदञ्च न्यर्बुदञ्च समुद्रश्च मध्यञ्चान्तश्च परार्धश्चैता मेऽअग्नऽइष्टका धेनवः सन्त्वमुत्रामुष्मिँल्लोके २

Shukla Yajurveda 17.2

एका च दश च दश च शतं च शतं च सहस्त्रं च सहस्त्रं च अयुतं च अयुतं च नियुतं च नियुतं च प्रयुतं च अर्बुदं च न्यर्बुदं च समुद्रः च मध्यं च अन्त्यः च परार्धः च

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- followed by Śridhar (*Pātigaṇita*), Mahavira (*Gaṇitasārasangrah*), Bhaskara (*Lilāvatī*).

Transmission to Europe

via Arabs

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- Indian arithmetic methods today called "algorithms", after al Khwarizmi's Latin name Algorismus.

- Q. Why did Europeans import Indian arithmetic?

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- A. Because their own arithmetic was inferior even in the 16th c.

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- Romans did multiplication by repeated addition.
- Simpler question: How do you add XXIX and LXXVIII?

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- Whew!

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- by Crusading, racist and colonial historians based on very late texts (in another language from another place).

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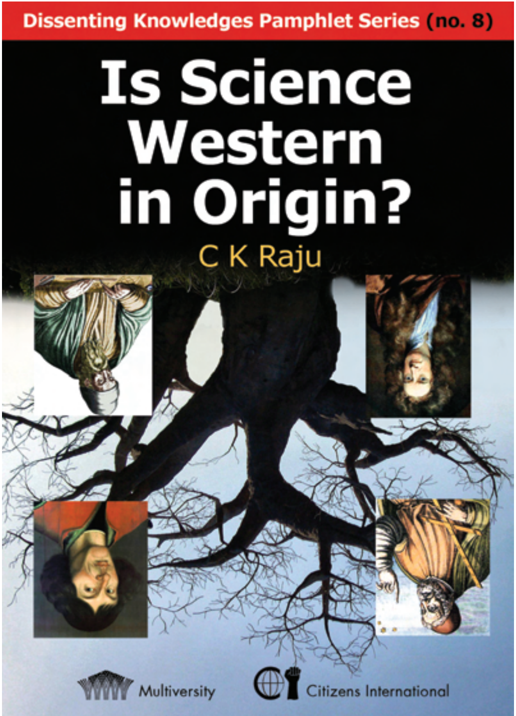
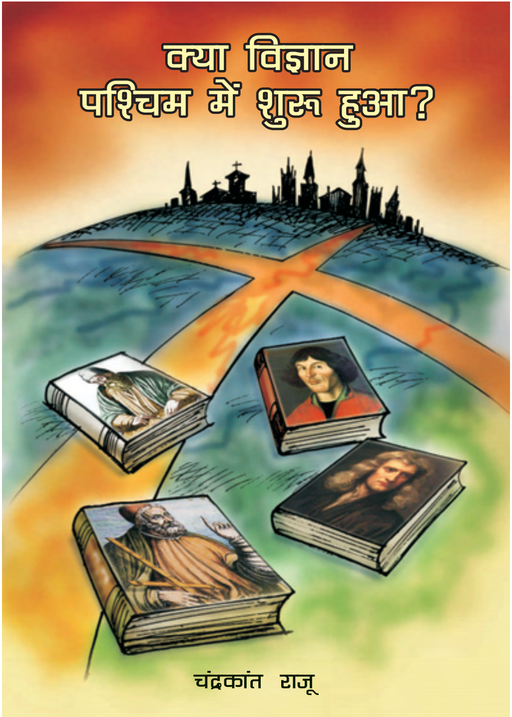
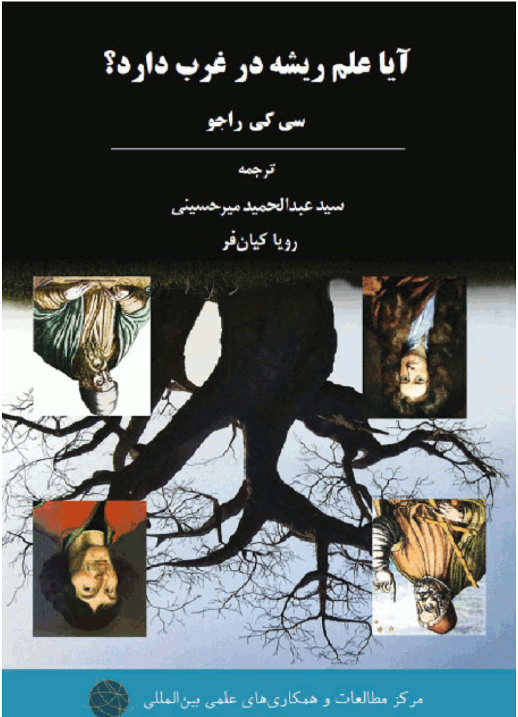
- in my booklet *Is Science Western in Origin?*


Dissenting Knowledges Pamphlet Series (no. 8)
Is Science Western in Origin?
C K Raju
Multiversity
Citizens International


क्या विज्ञान
पश्चिम में शुरू हुआ?
चंद्रकांत राजू


آیا علم ریشه در غرب دارد؟
سی کی راجو
ترجمه
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from India to Europe

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- Because their own Graeco-Roman arithmetic was inferior
- therefore efficient Indian arithmetic was imported by West
- first via Arabs in Cordoba (10th c.)
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- Contribution of India was efficient arithmetic, not just zero.

Failure to understand imported knowledge

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- The West misunderstood the imported Indian methods of arithmetic, for centuries.
- This story suppressed by Western historians,
- but the story emerges from the very terms "Arabic numerals", and "zero".

The pope's mistake

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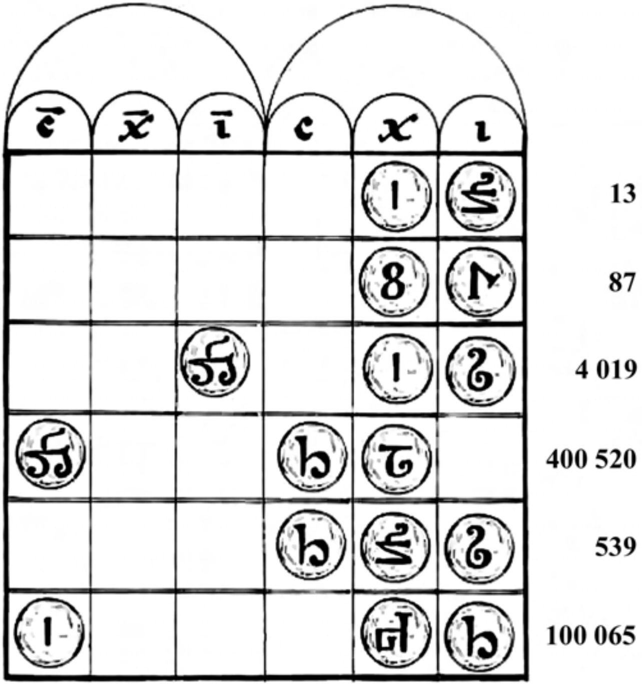
- Hence, the term "Arabic numerals".



C̄ X̄ Ī C X I
13
87
4 019
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- Roman numerals are additive, like counters: XII means $X + I + I = 12$.
- Place value numerals are not additive: $120 \neq 1 + 2 + 0 = 3$.

The mysterious zero

contd

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- A contract for Rs 120 could easily be changed to Rs 1200.
- Hence, the Florentine law that contracts (cheques) must be written out in words.

Algorismus vs abacus

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- Thus confusion about Indian arithmetic persisted for six centuries in Europe.

Arithmetic:summary

Origin, transmission, misunderstanding

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- Europeans misunderstood that arithmetic in various ways, and that confusion persisted for over six centuries.

Transmission via Toledo

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- The word sine derives from the Arabic *jaib* (as in जेब meaning pocket).

A pocketful of sines

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- *jiba* was from the Sanskrit *jya* or *jiva*, meaning chord.

Conceptual mistake

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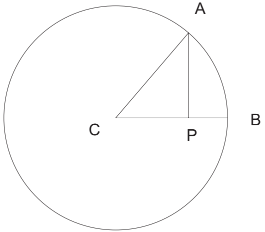

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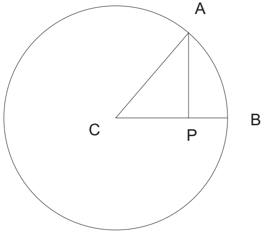

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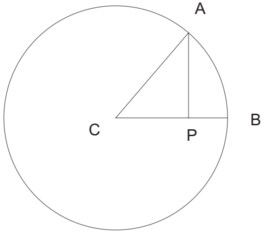

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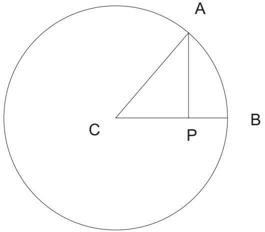

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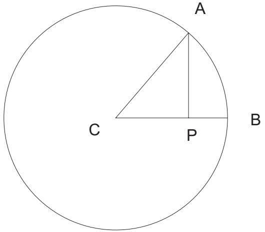
- Circle involves a curved line.



Conceptual mistake

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- If you use a protractor to measure angles, does the size of the protractor matter?

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- How to define an angle of 1° axiomatically?, etc

Measuring curved lines

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- Hence, also, *śulba sūtra*-s easily describe a measure of π.

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- This story old in my book *Cultural Foundations of Mathematics* (Pearson Longman, 2007)


PEARSON LONGMAN
History of Science, Philosophy and Culture in Indian Civilization
General Editor D. P. Chattopadhyaya
Volume X Part 4
Cultural Foundations of Mathematics
The Nature of Mathematical Proof and the Transmission of the Calculus from India to Europe in the 16th c. CE
C. K. RAJU
PHISPC
Centre for Studies in Civilizations

Āryabhaṭa's value of π

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  अयुतद्वय विश्कम्भस्यासन्नो व्रित्तपरिणाहः ॥१०॥

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- 100 plus 4 multiplied by 8, and added to 62,000: this is the near [*asanna*] measure of the circumference of a circle whose diameter is 20,000.

Āryabhaṭa's value of π

contd.

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- This value of π repeated, by al Khwarizmi,
- and a thousand years later, in 16th c. Europe, by Simon Stevin.

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- These may be obtained geometrically using symmetry considerations.
- Āryabhaṭa made a striking departure by calculating sine values numerically, 3.75° apart.
- But our trigonometry texts, obtained from the West, are still 1500 years behind the times.

Āryabhaṭa's method

Difference/differential equations

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- (only metaphysically different from differential equations).
- He solved them using only linear interpolation or the rule of 3.
- (Today wrongly called "Euler's method" of solving ordinary differential equations, after Euler who, like other Westerners, never acknowledged his Indian (non-Christian) sources.)

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- Differences 3.75° apart.
- Differences can be directly used to interpolate (by rule of 3)
- and calculate any sine value such as $\sin 1^\circ$.

Āryabhaṭa's table of sine differences

- मखि भखि फखि धखि णखि ञखि
  ङखि हस्झ स्ककि किष्ग श्घकि किघ्व ।
  घ्लकि किग्र हक्य धकि किच
  स्ग शझ ङ्व क्ल प्त फ छ कलार्धज्या ॥१२॥

Translation

- 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, 7—[these are the] Rsine [differences] [for the quadrant divided into as many equal parts, each part hence being 225′] [in] minutes.

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- (Circumference of the circle in minutes is $360 \times 60 = 21,600$.)

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- Translation: (12) The Rsine of the first arc divided by itself and negated gives the second Rsine difference. That same first Rsine, when it divides successive Rsines gives the remaining [Rsine differences].

Mathematical translation

- R_i = sine values, $\delta_i = R_i - R_{i-1}$ sine differences. Then Āryabhaṭa's rule consists of two parts

$$\delta_2 - \delta_1 = -\frac{R_1}{R_1}, \quad (1)$$

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- Note 1: Second differences have been brought in.
- Note 2: Brahmagupta also uses 2nd differences for quadratic interpolation.

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- E.g. for $n = 23$, $\delta_{23} = 22$, $\delta_{24} = 7$, while $R_1 = 225$, so that we should have $R_{23} = (\delta_{23} - \delta_{24}) \times R_1 = 15 \times 225 = 3735 \neq 3431$ the 23rd sine value.

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- Difference in each case, since no value is a multiple of 225.

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- by extending Āryabhaṭa's recursive process to infinite series.

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- In contrast, calculus immaculately conceived in Europe!

Madhava's value of π

- Value of π also stated in older *bhūta saṁkhyā* system in Nīlakaṇṭha's *ĀryabhaṭīyaBhaṣya*
- सङ्गमग्रामजो माधवः पुनरत्यासन्नां परिधिसंख्यामुक्तवान् –
  विबुधनेत्रगजाहिहुताशनत्रिगुणवेदभवारणबाहवः ।
  नवनिखर्वमिते वृतिविस्तरे परिधिमानमिदं जगदुर्बुधाः॥
- Mādhava of Saṅgamagrāma spoke the approximate [*āsanna*] number of the circumference of a circle: *vibudha* [33] *netra* [2] *gaja* [8] *ahi* [8] *hutāśana* [3] *tri* [3] *guṇa* [3] *veda* [4] *bhavāraṇa* [27] *bāhavaḥ* [28], i.e., [2,827,433,388,233] is the measure of a circle of diameter *nava* [9] *nikharva* [100,000,000,000].

Value of π contd.

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Value of π contd.

- Corresponds to value of $\pi = 3.141,592,653,5922\ldots$,
- accurate to 11 decimal places
- with the 12th and 13th places (92 respectively) differing slightly from their accurate value (89).

Madhava's sine table

श्रेष्ठं नाम वरिष्ठानां हिमाद्रिर्वेदभावनः ।
तपनो भानुसूक्तज्ञो मध्यमं विद्धि दोहनम् ॥
धिगाज्यो नाशनं कष्टं छन्नभोगाशयाम्बिका ।
म्रिगाहारो नरेशोऽयं वीरो रणजयोत्सुकः ॥
...
छायालयो गजो नीलो निर्मलो नास्ति सत्कुले ।
रात्रौ दर्पणमभ्राङ्गं नागस्तुङ्गनखो बली ॥
धीरो युवा कथालोलः पूज्यो नारीजनैर्भगः ।
कन्यागारे नागवल्ली देवो विश्वस्थली भृगुः ॥
तत्परादिकलान्तास्तु महाज्या माधवोदिताः ।
स्वस्वपूर्वविशुद्धे तु शिष्टास्तत्खण्डमौर्विकाः ॥ २.९.५ ॥

Madhava's sine table

Table : Mādhava's sine values

No.	Kaṭapayādi	kalā (′)	vikalā(″)	tatparā(‴)
1	श्रेष्ठं नाम वरिष्ठानां	224	50	22
2	हिमाद्रिर्वेदभावनः	448	42	58
3	तपनो भानुसूक्तज्ञो	670	40	16
4	मध्यमं विद्धि दोहनम्	889	45	15
...	...	...	...	...
21	धीरो युवा कथालोलः	3371	41	29
22	पूज्यो नारीजनैर्भगः	3408	20	11
23	कन्यागारे नागवल्ली	3430	23	11
24	देवो विश्वस्थली भृगुः	3437	44	48

Accuracy of Madhava's sine values

Table : Accuracy of Mādhava's sine table.

No.	Mādhava's sine value	Difference
1	0.0654031452	0.0000000160
2	0.1305262297	0.0000000375
3	0.1950903240	0.0000000020
4	0.2588190035	−0.0000000416
...	...	...
...	...	...
21	0.9807852980	0.0000000176
22	0.9914448967	0.0000000353
23	0.9978589819	0.0000000587
24	1.0000000000	0.0000000000

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- needed for the two key means of wealth in India
- overseas trade (needs good navigation)
- agriculture (needs a good calendar to tell the rainy season).
- (Note: Gregorian calendar is a bad, unscientific, religious calendar which continues to ruin our economic interests to this day: see video and presentation: "A tale of two calendars".)

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- Also needed to to determine the size of the earth, and
- use that to solve longitude triangles from knowledge of latitude difference and departures (Maha Bhaskariya, II.3–4), (of Bhaskara I, 7th c.)

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- Recorded that the pilot was telling the distance by his teeth!

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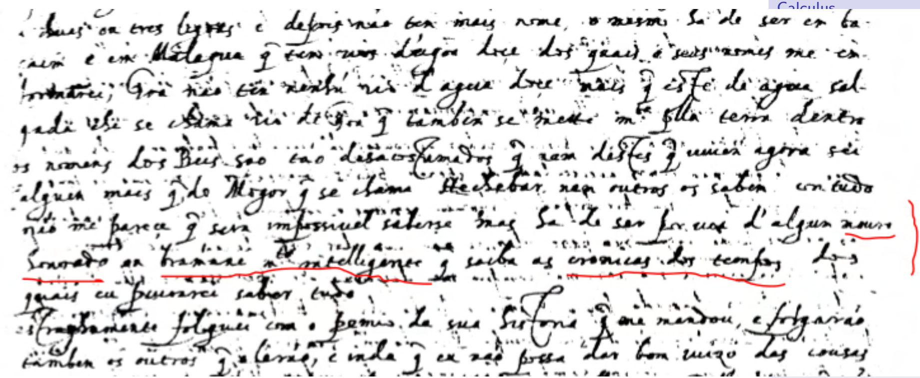
- on the Toledo model of mass translations, but using local Syrian Christians as intermediaries.
- This import solved the latitude and loxodrome problems which required precise sine values,
- also used Indian length of tropical year for Gregorian calendar reform of 1582 (not based on fresh observation, not immediately accepted by Protestants)

Ricci's letter of 1581

- In 1581 Ricci was in Cochin and wrote that he was looking for "an honorable Moor or an intelligent Brahmin to tell him about Indian methods of timekeeping".



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- Hence, Europe could not solve the longitude problem this way (used inaccurate "Dead reckoning").

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- Royal Society, French Royal Academy were set up for this purpose.

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- Infinite geometric series had appeared in India by the 14th-15th c.

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- $a + \frac{a}{d} + \frac{a}{d^2} + \cdots = \frac{ad}{d-1}$.
  (Assuming $d > 1$, so common ratio less than 1.)

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- Translation: To the diameter multiplied by 4 alternately add and subtract in order the diameter multiplied by 4 and divided separately by the odd numbers 3, 5, etc.

"Leibniz" series

contd.

- Mathematical translation: if d is the diameter of the circle, then

$$\text{circumference} = 4d - \frac{4d}{3} + \frac{4d}{5} - \frac{4d}{7} + \cdots . \quad (3)$$

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- What baffled all Western thinkers (Descartes, Galileo, Newton, Berkeley . . .) was this: how to do this infinite sum "perfectly".

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- See my article: "Towards equity in math education. 2: The Indian rope trick" (google, CKR, rope trick)

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- He thought for this we must physically sum the series term by term, which would take an infinite amount of time.

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- This empirical knowledge (*gaṇita*) of π called non-eternal (सानित्य, Apastamba 3.12) and imperfect (सविशेषः, Katyayana, 2.12)

Contrast between *gaṇita* and math

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- and that it must be perfect.

The meaning of mathematics

A mere religious belief

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- according to Plato "all learning is recollection" of eternal ideas in the soul.
- The belief was that mathematics contained eternal truths, hence aroused the eternal soul by sympathetic magic.

- For more on the religious beliefs underlying Western mathematics

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- see my book *Euclid and Jesus: How and why the church changed mathematics and Christianity across two religious wars*.


Euclid and Jesus
C. K. Raju
Multiversity
Citizens International

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- Berkeley, arguing against Newton, on calculus, stated that "the minutest Errors are not to be neglected in mathematics".

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- *Gaṇita* admits that its knowledge is non-eternal and imperfect.

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and other religious delusions in mathematics

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- *Gaṇita* accepts empirical proofs, mathematics prohibits them.

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- If logic is not universal how can reason be?

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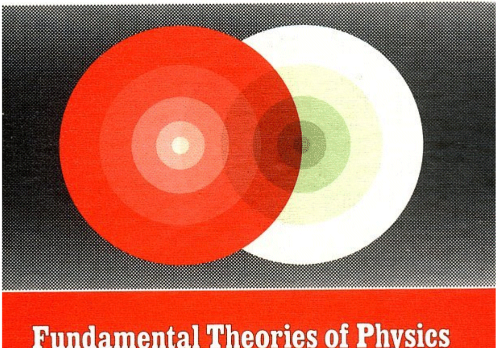
- if time ("absolute", "true", and "mathematical", or the time known to his God) flowed on!
- Newtonian physics failed because he made the notion of time metaphysical.

Time: Towards a Consistent Theory

by

C. K. Raju

Kluwer Academic Publishers



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- (How can time itself flow? e.g. Śriharṣa and his Western copycat (McTaggart's paradox)).
- However, belief persists that calculus requires metaphysical "real" numbers $\mathbb{R}$for existence of limits.
- That belief is taught in school and undergraduate calculus courses today (without actually teaching about $\mathbb{R}$ except to math majors!).

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- which have a different arithmetic (no associative "law", etc.)

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- Similarly issues in renormalization problem of quantum field theory.

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- Needs my philosophy of zeroism not formalism.

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- in *Yuktidīipikā* to accelerate convergence of slowly convergent series, like "Leibniz" series.

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- (See the series of expository articles in *Physics Education*, India)

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- Solution is to go back to the way that math developed in India, and reject the way it was misunderstood in Europe.

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- can be taught in a mere five days as I have demonstrated with 8 groups in 5 universities in 3 countries.


Central University of Tibetan Studies
Workshop on "Calculus without limits"
22nd - 28th September, 2009

نشست علمی "ریاضیات از منظری دیگر"، پروفسور سی.کی. راجو
مرکز مطالعات و همکاری‌های علمی بین‌المللی، تهران، ۱۳۹۱

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  - and is still taught today.

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- Those claims just involve glorification of a particular religious metaphysics.

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- since India is a secular country.

References

- *Gaṇita* vs math. Abstract posted on blog. `http://ckraju.net/blog/?p=111`

References

- *Gaṇita* vs math. Abstract posted on blog.
  http://ckraju.net/blog/?p=111
- A reading list (and list of videos) is posted at
  http://ckraju.net/papers/Reading-list-Bengaluru.html