

He used second-order differences to propose a second-order interpolation formula, nowadays called “Stirling’s formula”. Brahmagupta’s formula for quadratic interpolation is stated as follows.⁴⁷

गतभोग्यखण्डकान्तरदलविकल वधाच्छतैर्नवभिराप्त्या ।
तद्युतिदलं युतोनं भोग्यादूनाधिकं भोग्यम् ॥ ४ ॥

This has been translated (using Bhaṭṭotpala’s 10th c. CE [Saka 888] commentary) as:

Multiply the *Vikalā* by half the difference of the *Gatakhaṇḍa* and the *Bhogyakhaṇḍa* and divide the product by 900. Add the results to half the sum of the *Gatakhaṇḍa* and the *Bhogyakhaṇḍa*, if their half sum is less than the *Bhogyakhaṇḍa*; subtract, if greater. [The result in each case is the *Sphuṭabhogyakhaṇḍa* or correct “tabular” difference.]

Here, the underlying table is that calculated for *khaṇḍajyā*-s or sine differences for intervals that are spaced h apart, where it is assumed that $h = 15^\circ$ or $900'$. The *gatakhaṇḍa* or “past difference” ($= \delta_n$) refers to the interval that has been crossed, and the *vikalā* ($= \theta$) is the amount in minutes by which it has been crossed at the point at which we want to interpolate. The *bhogyakhaṇḍa* ($= \delta_{n+1}$) is the one yet to come. Thus, the formula states:

$$\text{sphuṭabhogyakhaṇḍa} = \frac{\delta_n + \delta_{n+1}}{2} \pm \frac{\theta}{h} \frac{\delta_n - \delta_{n+1}}{2} \quad (3.34)$$

$$\text{Rsin}(nh + \theta) - \text{Rsin} nh = \frac{\theta}{h} \times \text{sphuṭabhogyakhaṇḍa}. \quad (3.35)$$

This amounts to

$$\text{Rsin}(nh + \theta) = \text{Rsin} nh + \frac{\theta}{h} \frac{\delta_n + \delta_{n+1}}{2} \pm \frac{\theta^2}{h^2} \frac{\delta_n - \delta_{n+1}}{2}. \quad (3.36)$$

This formula is nowadays called Stirling’s interpolation formula: just as linear interpolation leads to an Euler solver, so also quadratic interpolation easily extends to a second-order (Runge–Kutta) method of numerically solving an ordinary differential equation. (Indian tradition, of course, did not recognize differential equations, but it worked directly with difference equations from the time of Āryabhaṭa: this is still the way most differential equations are actually solved today, even though present-day mathematics pretends that differential equations are somehow superior to difference equations.)

Just as a Runge–Kutta method can take much larger steps than an Euler solver, while retaining the same level of accuracy, the higher accuracy of quadratic interpolation enabled Brahmagupta to work with values $900'$ apart.

But Vaṭeśvara (in 904 CE) works with arcs that are only $56'15''$ apart, and still uses quadratic interpolation, explicitly giving the second of the above formulae, among many others.⁴⁸