Calculus Transmission

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It has gone unnoticed that Āryabhaṭa's 24 sine values (see "► Calculus") involved a striking departure from the earlier geometric tradition and a paradigm shift to numerical techniques. The geometric tradition is useful only in simple situations where high symmetry is present. It cannot be used to calculate the sine of 1° but was earlier used to compute 6 sine values 15° apart.

In a second radical departure, Āryabhaṭa used difference equations, instead of algebraic equations. Indeed, in the tenth *gītikā* (this is called the tenth *gītikā* since the first two are invocations; Shukla & Sarma, 1976), of the *daśgītikā* section, he states only the sine *differences*:

मखि भखि फखि धखि णखि ञखि
ङखि हस्झ स्ककि किष्ग श्घकि किघ्व ।
घ्लकि किग्र हक्य धकि किच
स्ग श्झ ङ्व क्ल प्त फ छ कलार्धज्या ॥१२॥

The numbers involved here are expressed in Āryabhaṭa's novel numerical notation (see Shukla & Sarma, 1976 or Raju, 2007, Chap. 3). Thus, the verse may be translated:

> 225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, 7—[these are the] sine [lit. half-chord] differences [for the quadrant divided into as many equal parts, each part hence being 225′] [in] minutes.

The method of calculating these differences is explained in Gaṇita 12 as:

प्रथमाच्चापज्यार्धाद्यैरूनं खण्डितं द्वितियार्धम् ।
तत्प्रथमज्यार्धांशैस्तैस्तैरूनानि शेषाणि ॥१२॥

This may be translated:

> (12) The sine of the first arc divided by itself and diminished gives the second sine difference. That same first sine, when it divides successive sines gives the remaining [sine differences]. (Trans. by the author, based on the Hindi translation of Rai, 1976, pp. 42–43; cf. Shukla & Sarma, 1976, p. 51)

That is, if the quadrant of the circle is divided into, say, 24 equal parts, where $R_1, R_2, \ldots, R_{24}$ are the 24 corresponding sine values; δ_1 $(= R_1)$, $\delta_2, \ldots, \delta_{24}$ are the corresponding sine differences; and $\delta_i = R_i - R_{i-1}$, for $i \geq 2$, then Āryabhaṭa's rule consists of the following two parts:

$$\delta_2 - \delta_1 = -\frac{R_1}{R_1}, \tag{1}$$

$$\delta_{n+1} - \delta_n = -\frac{R_n}{R_1}. \tag{2}$$

This cannot be understood as an algebraic equation, since that would lead to the wrong answers. More precisely, it can be regarded as an algebraic equation for calculating the second difference but not for its proper purpose, which is to calculate sine differences (Raju, 2007).

Āryabhaṭa's technique of solving those difference equations was later modified by Nīlakaṇṭha to make it accurate to thirds. Thus, the above interpretation is also the one given by Nīlakaṇṭha in his *Āryabhaṭīyabhāṣya* (Sastri, 1930), except that Nīlakaṇṭha makes it more precise, by stating it in the form

$$\delta_n^{(2)} = -\left(\frac{R_n}{R_1}\right)\ (\delta_1 - \delta_2). \tag{3}$$

The difference here is that for Āryabhaṭa, working to the precision of minutes, $\delta_1 - \delta_2 = 225–224 = 1$, while this is no longer the case with Nīlakaṇṭha, working to the precision of thirds, who uses the values, $R_1 = [224;\ 50;\ 22]$ and $R_2 = [448;\ 42;\ 58]$, so that $\delta_2 = [223;\ 52;\ 36]$ and $\delta_1 - \delta_2 = [0;\ 57;\ 46]$.

Āryabhaṭa's method of solving differential equations by finite differences is today wrongly called "Euler's method" after Euler who studied the relevant Indian texts when he wrote an article on the Indian calendar in 1700. Further, the numerical solution of differential equations (rather than metaphysical existence and