Printing and Dyeing in India, Fig. 12 Block engraver at Pedana, 2013

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Probability in Non-Western Cultures

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The mathematical theory of probability begins with the theory of permutations and combinations, needed to calculate probabilities in games of chance, such as dice or cards. The earliest account of this theory is found in India. This theory is tied to the theory of the Vedic meter (and the theory of Indian music, in general).

The Vedic and post-Vedic composers used combinations of two syllables called *guru* (deep, long) and *laghu* (short). The earliest written account of this theory of meter is in Piñgala's *Chandahsūtra* (third c.CE), a book of aphorisms (*sūtra*-s) on the theory of meter (*chanda*). To calculate all possible combinations of these two syllables in a meter containing *n* syllables, Piñgala gives the following rule (which explicitly makes use of the symbol for zero) (*Chandahsūtra*, 1840): "(Place) two when halved;" "when unity is subtracted then (place) zero;" "multiply by two when zero;" "square when halved." In a worked example, Dutta and Singh (1962) show how this rule leads to the

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correct figure of 2^6 possibilities for the Gāyatrī meter with six syllables.

Piñgala's commentator, the tenth century CE Halayudha, clarifies that this involves the binomial expansion. Thus, in a 3-syllabic meter with two underlying syllables, *guru* and *laghu*, 3 *guru* sounds will occur once, 2 *gurus* and 1 *laghu* will occur twice, as will 1 *guru* and 2 *laghus*, while 3 *laghus* will occur once. Symbolically $(g + l)^3 = g^3 + 3g^2l + 3gl^2 + l^3$. To generalize this to the case of n underlying syllables, Halayudha explains the *meru-prastāra* (pyramidal expansion) scheme for calculation (Bag, 1979), which is identical to "Pascal's" triangle which first appeared in Europe over a thousand years after Piñgala and about a century before Pascal (on the title page of the *Arithmetic* of Apianus) and in China in the fourteenth century (Needham, 1981). An example, using the *Gāyatrī* meter is also found in Bhaskara's *Līlāvatī* (Bhaskara, *Līlāvatī*, trans. K. S. Patwardhan, S. A. Naimpally, and S. L. Singh, p. 102. The verse is numbered differently in different manuscripts. K. V. Sarma, in his critical edition of the sixteenth-century southern commentary *Kriyākramakarī* (VVRI, Hoshiarpur, 1975) on the *Līlāvatī*, gives this as verse number 133, while the other cited source has given it as verse number 121). Stock Western histories of mathematics (such as that by Smith, 1958) are unreliable and wrongly state that no attention was paid in India to the theory of permutations and combinations before Bhaskara II (twelfth century CE).

Although this theory of permutations and combinations is built into the Vedic meter, the earliest known written account actually comes from even before Piñgala and is found in the fourth-century Jain *Bhagwatī Sūtra*. Permutations were called *vikalpa-ganita* (the calculus of alternatives) and combinations *bhanga*. The text works out the number of combinations of n categories taken 2, 3, etc., at a time.

From the earliest Vedic tradition, there is a continuous series of manuscripts linking the first accounts of permutations and combinations with those of Bhaskara II (twelfth century) and later commentaries on his work up to the sixteenth century CE, such as the *Kriyākramkarī*. (It is these latter texts which arrived in Europe and first brought the mathematical theory of probability to Europe, as detailed later on, through Pascal, Fermat etc.) Thus, (Bag, 1979) the surgeon Suśruta (second century CE) in his compendium (*Suśruta-samhitā*) lists the total number of flavors derived from six flavors taken 1 at a time, 2 at a time, and so on. Likewise, Varāhamihira (sixth century) states the number of perfumes that can be made from 16 substances mixed in 1, 2, 3, and 4 proportions. Similar examples are found in the *Pāṭīgaṇita* (Slate Arithmetic) of Śridhar (tenth century), a widely used elementary school text, as its name suggests, Mahavira's (eighth century) *Gaṇita Sāra Saṁgraha* and Bhaskara II (*Līlāvatī*), etc. Bhaskara mentions that this formula has applications to the theory of meter, to architecture, medicine, and *khaṇḍameru* ("Pascal's triangle"). In these later texts, one finds explicitly stated formulae for permutations and combinations.

For example, to calculate $\binom{n}{r}$ values, Śridhar, in his text on slate arithmetic (*Pāṭigaṇita*, 1959), gives the following rule.

एकाद्युत्तरविधिना रस विन्यासे विलोमतो गुणयेत् ।
पूर्वेंण परं क्रमशो रूपादिचयैर्हरेर्विभ्जेत् ॥

This translates as follows (*Pāṭigaṇita*, 72, Eng. p. 58):

> Writing down the numbers beginning with 1 and increasing by 1 up to the (given) number of savours in the inverse order, divide them by the numbers beginning with 1 and increasing by 1 in the regular order, and then multiply successively by the preceding (quotient) the succeeding one. (This will give the number of combinations of the savours taken 1, 2, 3, … , all at a time respectively.) (*Pāṭigaṇita of Śridhar*, trans. K. S. Shukla. As he points out, similar articulations are found in the *Gaṇita Sāra Saṃgrah* of Mahavira, vi.218, *MahāSiddhānta* of Āryabhaṭa 2, xv, 45–46 etc.).

Thus, in the case of six savors, one writes down the numbers 1–6 in reverse order:

$$6, 5, 4, 3, 2, 1$$