Tutorial-1
Calculus without Limits

1. (a) Define an angle
   (b) Convert 32°into radians.
   (c) Convert 0.78 radians to degrees.

2. (a) Solve the ODE $y' = y$ with $y(0) = 1$.
   (b) Hence, calculate the value of e.
   (c) Define the exponential function e^x.

3. (a) Convert the second order ODE $y'' = -y$ to two first order ODEs.
   (b) Solve the system of two simultaneous ODEs with the initial data $y(0) = 0$, $y'(0) = 1$.
   (c) Calculate π correct to 4 decimal places.

4. (a) Define the function $\cos(x)$.
   (b) Calculate $\cos(42°)$.

5. (a) The equation for damped harmonic motion is often written as

$$\ddot{y} = -k^2 y - b\dot{y}$$

   . Convert this to a system of 2 ODEs, and solve with the initial data $y(0) = 0$, and $k = 1$, and $b = 0.1$.
   (b) How does the solution change if we use the initial data $y(0) = 1$?
   (c) Re-calculate the solution for $b = 0.2$, $b = 0.3$. Can you guess the solution for a general b?

6. The equation of motion for a simple pendulum is

$$y'^2 = (1 - y^2)(1 - k^2 y^2). \tag{1}$$

   The substitutions

$$\begin{aligned} y &= \mathrm{sn}(x) = y_1 && (2)\\ 1 - y^2 &= \mathrm{cn}^2(x) = y_2^2 && (3)\\ 1 - k^2 y^2 &= \mathrm{dn}^2(x) = y_3^2. && (4) \end{aligned}$$

converts this to 3-equations in Jacobi's form

$$y_1' = y_2 y_3, \tag{5}$$
$$y_2' = -y_3 y_1, \tag{6}$$
$$y_3' = -k^2 y_1 y_2, \tag{7}$$

Solve the above equations with the initial data $y_1(0) = 0$, $y_2(0) = 1$, $y_3(0) = 1$, and parameter k=0.4.
(b) Compare the Jacobian elliptic function $\mathrm{sn}(x)$ with $\sin(x)$.
(c) The time period of the simple pendulum is the first zero of $\mathrm{sn}(x)$. Calculate it.

7. (a) Van der Pol's equation is

$$y'' + \epsilon(y^2 - 1)y' + y = 0, \tag{8}$$

Convert this equation to two first order ODEs.
(b) Solve the resulting ODEs for $y(0) = 2$, and $y'(0) = 0$, and parameter value $\epsilon = 1$

8. (a) Solve the system of equations for the Lorenz model

$$y_1' = -\sigma y_1 + \sigma y_2, \tag{9}$$
$$y_2' = -y_1 y_3 + r y_1 - y_2, \tag{10}$$
$$y_3' = y_1 y_2 - b y_3. \tag{11}$$

for the parameters $b = \frac{8}{3}$, $\sigma = 10$, $r = 28$, and initial data $y_1 = 8$, $y_2 = -8$, $y_3 = 27$, over the range [-2, 2].
(b) Draw the resulting phase plots.
(c) Switch to 3-d view, and animate.

9. A ball is thrown upwards at an angle θ from a height of 10 meters. Assuming a simple model of air resistance proportional to velocity, and assuming its coordinates at any instant are (y_1, y_2), the equations of motion are given by

$$y_1' = y_3, \tag{12}$$
$$y_2' = y_4, \tag{13}$$
$$y_3' = -\frac{b}{m} y_3, \tag{14}$$
$$y_4' = -g - \frac{b}{m} y4. \tag{15}$$

where b is the drag coefficient and m is the mass of the ball.
(a) The mass of a cricket ball is 155.9 gram and the mass of a tennis ball is 58.5 gram. Assume $b = 0.01$. Both balls are thrown with the same velocity 10 m/s, at an angle of 45°. Which ball will travel further? By how much?
(b) If the angle of throw is changed to 44° (for either ball) will it travel a larger or a smaller distance?