Results

- Experiment performed with 1 group in Central University of Tibetan Studies, Benaras

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- 4 groups in Universiti Sains Malaysia, Penang

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- 4 groups in Universiti Sains Malaysia, Penang
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- Running as a regular course in SGT University, Delhi (72 students).

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- (even PG students, half of whom hence dropped out).

SGT University: calculus without limits

Pre-test

Answer all questions.
Blank answer fetches 0. Wrong answer gets negative marks.
Classes refer to NCERT texts. You may have learnt from a different text in school.

1. **Points.** You were taught about points in class VI.
   (a) Define a point.
   (b) What is the size of a point?
   (c) Can something with no size be seen? If something is invisible, how do you know where it is?
   (d) What is the difference between a fraction and a rational number?

2. **Numbers.** You were taught "real" numbers in class IX and class X.
   (a) Define a real number.
   (b) Write down the EXACT value of $\sqrt{2}$. If x is that exact value, show by direct calculation that $x^2=2$. (Note: this should not be 1.9999999999, but exactly 2.)
   (c) Can a complex number be written as the ratio of two integers? If i is the complex number such that $i^2=-1$ then is i irrational? Is it rational?
   (d) Are there numbers which are neither rational nor irrational? If your answer is yes, go back and re-check your definition at (a). If your answer is no, explain how -1 can have a real square root.

3. **Sets.** You were taught about sets in class X.
   (a) Define a set.
   (b) If you defined a set as a "collection of objects", define "collection" and define "object". Is "object" the same as in object-oriented programming? If not, what is the difference?
   (c) Let $R = \{ x \mid x \notin x \}$. Is it true that $R \in R$? Is it true that $R \notin R$?
   (d) Can a set have an infinite number of elements? How can you be sure?

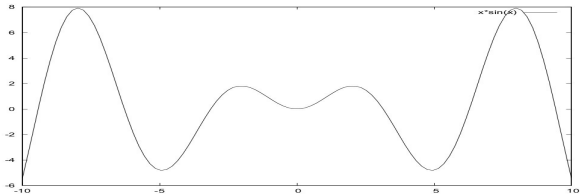
4. **Trigonometry.** You were taught about trigonometric functions in class IX.
   (a) Define sin(x).
   (b) Use that definition to calculate sin (0.3°).
   (c) Is sin (92°) defined? According to my calculator, sin (92°) = 0.9993. Is this right? Explain.
   (d) Define a radian. Exactly how many degrees is 1 radian?

5. **Limits.** You were taught about limits in class XI and XII.
   (a) Define limit.
   (b) According to my calculator $\sqrt{2}$ = 1.414213562373095048801688724209 7. Does the sequence 1, 1.4, 1.41, 1.414... have a limit?
   (c) What is the infinite sum of all natural numbers, 1+2+3+4+... ? Can it be a negative number?
   (d) What is the infinite sum 1-1+1-1+1-1+...?

6. **Derivative.** You were taught about derivatives in class XI and XII.
   (a) Define the derivative of a function.
   (b) Let N denote the set of natural numbers, and let f : N → N be given by $f(x)=x^2$.

What is the derivative of f ?

(c) Define the tangent to a curve at a point. Consider the function x sin(x) whose graph is



displayed. Write the equation of the tangent to the curve at x=0. At how many points does this line intersect the curve? Can you list these points?

(d) What is the derivative of atanh(x) (hyperbolic arc tangent) with respect to x?

7. **Integral.** You were taught about integrals in class XII.
(a) Define the integral of a function.
(b) Shown below is a piece of land with irregular boundaries. How will you calculate its area?

(c) Calculate $\int_{-1}^{-2} \frac{dx}{x}$.

(d) Calculate $\int \frac{1}{\sqrt{((1-x^2)(1-4x^2))}} dx$.

8. **Applications.** You learnt about Newton's laws of motion and the simple pendulum from class VIII to class XI.
(a) At approximately what angle should you throw a cricket ball so that it travels the furthest distance?
(b) Will the answer change if you use a tennis ball instead of a cricket ball?
(c) The formula for the time period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Therefore, the time period of a simple pendulum is independent of amplitude. Is this true or false?
(d) Did you ever experimentally verify any of your answers above?

Math Group: Calculus without Limits
Exam, Pre-test: A

Name: _____
Student Number: _____
Course: _____
Age: _____
Date: _____

- Please attach this question paper and return it along with your answer sheet.
- This is not a competitive test. The aim is to obtain feedback to decide what to teach and how.
- Since the group is heterogeneous, you may find some questions too easy, or some may be too difficult. Attempt as many questions are you are able to.

1. (a) Define a complete metric space.
   (b) The least upper bound property for $\mathbb{R}$ says that if $A \subset \mathbb{R}$ is non-empty and bounded above, then $\exists \alpha \in \mathbb{R}$ such that $a \leq \alpha$, $\forall a \in A$, and if $a \leq b$, $\forall a \in A$ then $\alpha \leq b$. Assume the least upper bound property and prove that $\mathbb{R}$ is a complete metric space.
2. (a) Define "infinite set", "countable set", "uncountable set".
   (b) Prove that $\mathbb{R}$ is uncountable.
   (c) If $\mathbb{N}$ is the set of natural numbers, and $P(\mathbb{N})$ is its power set, does there exist a bijective map $f : P(\mathbb{N}) \to \mathbb{R}$?
3. (a) Write down the binary representation of 41.
   (b) Write down the binary representation of 2.5
   (c) Rewrite your answer using a mantissa between 1 and 2.
4. Given $g(x) = \begin{cases} x^2 - C, & \text{if } x < 4 \\ -\sqrt{C}x + 20, & \text{if } x \geq 4 \end{cases}$
   (a) Find the value of C which makes g continuous on $(-\infty, \infty)$.
   (b) With the above value of C, is g differentiable? Explain your answer.
5. (a) Suppose f_n is a sequence of Riemann integrable functions which converges to the function f on $(0, \infty)$, convergence being uniform on compact subsets. Is it true that f is Riemann integrable and that $\int_0^\infty f_n \to \int_0^\infty f$?
   (b) Suppose f_n is a sequence of differentiable functions which converges uniformly to the function f on $(0, 1)$. Is it true that f is differentiable and that the sequence of derivatives ${f_n}' \to f'$?

6. (a) Give an example of a real-valued function f which is not Riemann integrable on $[0, 1]$. Is this Lebesgue integrable?

   (b) Does there exist a Riemann integrable function on $(0, \infty)$ which is not Lebesgue integrable?

7. The following ten numbers were drawn at random from $[0, 1]$ using a uniform probability distribution: 0.23, 0.74, 0.18, 0.79, 0.51, 0.34, 0.67, 0.44, 0.11, 0.44.

   (a) Find the average.

   (b) Explain why it is not 0.5.

   (c) If the average does equal 0.5 at some stage, can subsequent draws of further random numbers change that value?

   (d) An unbiased coin is tossed 100 times. The first toss is tails, and the subsequent 99 tosses are heads. At the 101st toss (i) is the probability of tails greater than that of heads or (ii) is the probability of heads greater than that of tails?

8. Suppose a monkey is typing at random on a typewriter which has 50 keys (z and Z having been dropped), and suppose that the monkey is equally likely to strike any key.

   (a) What is the probability that the first six letters the monkey types will spell the word "Hamlet".

   (b) Suppose we consider the letters typed by the monkey in consecutive blocks of six letters. What is the probability p_n that the first n blocks of six letters will have the word "Hamlet"?

   (c) Does $\lim_{n\to\infty} p_n$ exist? If so, what is it?

9. Differentiate the following with respect to x

   (a) $\sin^n x . \sin nx$

   (b) $\sec^{-1} \dfrac{\sqrt{x}+1}{\sqrt{x}-1} + \sin^{-1} \dfrac{\sqrt{x}-1}{\sqrt{x}+1}$

   (c) $x - \log\left(2e^x + 1 + \sqrt{e^{2x} + 4e^x + 1}\right)$

10. Evaluate the following indefinite integrals.

    (a) $\int \sqrt{3x+2}\, dx$

    (b) $\int \log x\, dx$

    (c) $\displaystyle\int \frac{dx}{\sqrt{\sin^3 x \cdot \sin(x+\alpha)}}$

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- within a few days they could manage hard problems
- and most found the post-test very easy.

Tutorial-1
Calculus without Limits

1. (a) Define an angle
   (b) Convert 32°into radians.
   (c) Convert 0.78 radians to degrees.

2. (a) Solve the ODE $y' = y$ with $y(0) = 1$.
   (b) Hence, calculate the value of e.
   (c) Define the exponential function e^x.

3. (a) Convert the second order ODE $y'' = -y$ to two first order ODEs.
   (b) Solve the system of two simultaneous ODEs with the initial data $y(0) = 0$, $y'(0) = 1$.
   (c) Calculate π correct to 4 decimal places.

4. (a) Define the function $\cos(x)$.
   (b) Calculate $\cos(42°)$.

5. (a) The equation for damped harmonic motion is often written as

$$\ddot{y} = -k^2 y - b\dot{y}$$

   . Convert this to a system of 2 ODEs, and solve with the initial data $y(0) = 0$, and $k = 1$, and $b = 0.1$.
   (b) How does the solution change if we use the initial data $y(0) = 1$?
   (c) Re-calculate the solution for $b = 0.2$, $b = 0.3$. Can you guess the solution for a general b?

6. The equation of motion for a simple pendulum is

$$y'^2 = (1 - y^2)(1 - k^2 y^2). \tag{1}$$

   The substitutions

$$\begin{aligned} y &= \mathrm{sn}(x) = y_1 && (2)\\ 1 - y^2 &= \mathrm{cn}^2(x) = y_2^2 && (3)\\ 1 - k^2 y^2 &= \mathrm{dn}^2(x) = y_3^2. && (4) \end{aligned}$$

converts this to 3-equations in Jacobi's form

$$y_1' = y_2 y_3, \tag{5}$$
$$y_2' = -y_3 y_1, \tag{6}$$
$$y_3' = -k^2 y_1 y_2, \tag{7}$$

Solve the above equations with the initial data $y_1(0) = 0$, $y_2(0) = 1$, $y_3(0) = 1$, and parameter k=0.4.
(b) Compare the Jacobian elliptic function $\mathrm{sn}(x)$ with $\sin(x)$.
(c) The time period of the simple pendulum is the first zero of $\mathrm{sn}(x)$. Calculate it.

7. (a) Van der Pol's equation is

$$y'' + \epsilon(y^2 - 1)y' + y = 0, \tag{8}$$

Convert this equation to two first order ODEs.
(b) Solve the resulting ODEs for $y(0) = 2$, and $y'(0) = 0$, and parameter value $\epsilon = 1$

8. (a) Solve the system of equations for the Lorenz model

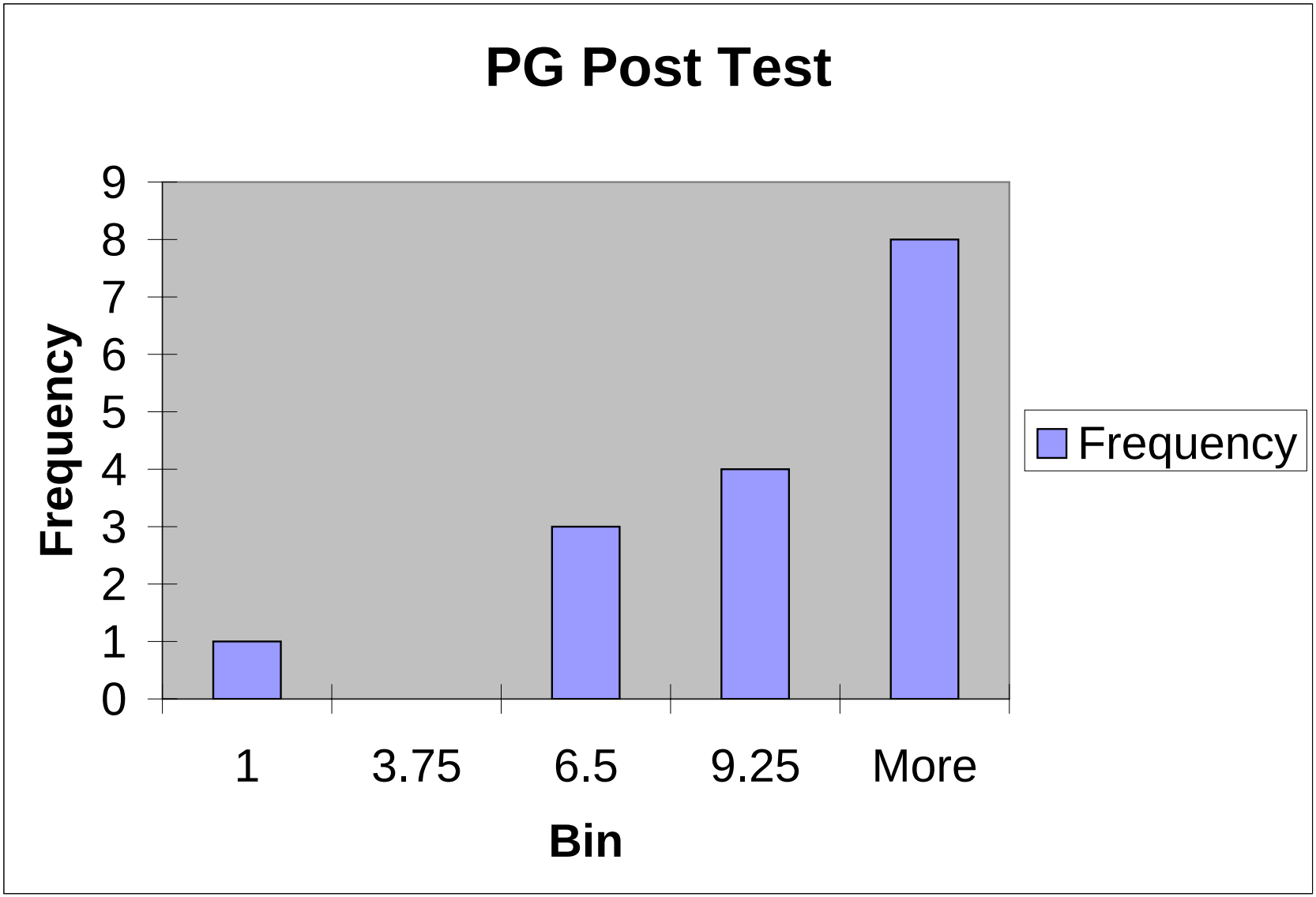
$$y_1' = -\sigma y_1 + \sigma y_2, \tag{9}$$
$$y_2' = -y_1 y_3 + r y_1 - y_2, \tag{10}$$
$$y_3' = y_1 y_2 - b y_3. \tag{11}$$

for the parameters $b = \frac{8}{3}$, $\sigma = 10$, $r = 28$, and initial data $y_1 = 8$, $y_2 = -8$, $y_3 = 27$, over the range [-2, 2].
(b) Draw the resulting phase plots.
(c) Switch to 3-d view, and animate.

9. A ball is thrown upwards at an angle θ from a height of 10 meters. Assuming a simple model of air resistance proportional to velocity, and assuming its coordinates at any instant are (y_1, y_2), the equations of motion are given by

$$y_1' = y_3, \tag{12}$$
$$y_2' = y_4, \tag{13}$$
$$y_3' = -\frac{b}{m} y_3, \tag{14}$$
$$y_4' = -g - \frac{b}{m} y4. \tag{15}$$

where b is the drag coefficient and m is the mass of the ball.
(a) The mass of a cricket ball is 155.9 gram and the mass of a tennis ball is 58.5 gram. Assume $b = 0.01$. Both balls are thrown with the same velocity 10 m/s, at an angle of 45°. Which ball will travel further? By how much?
(b) If the angle of throw is changed to 44° (for either ball) will it travel a larger or a smaller distance?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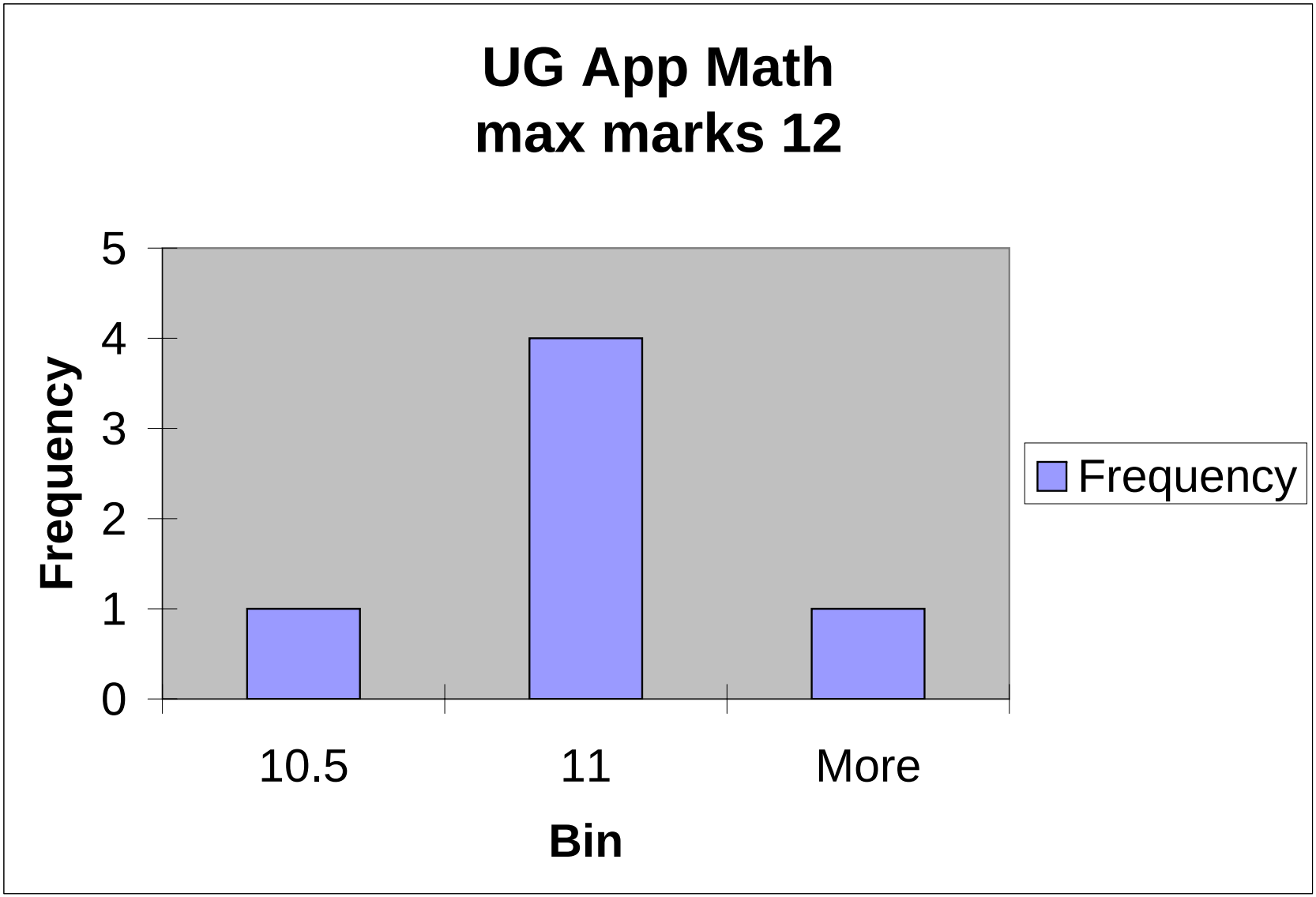
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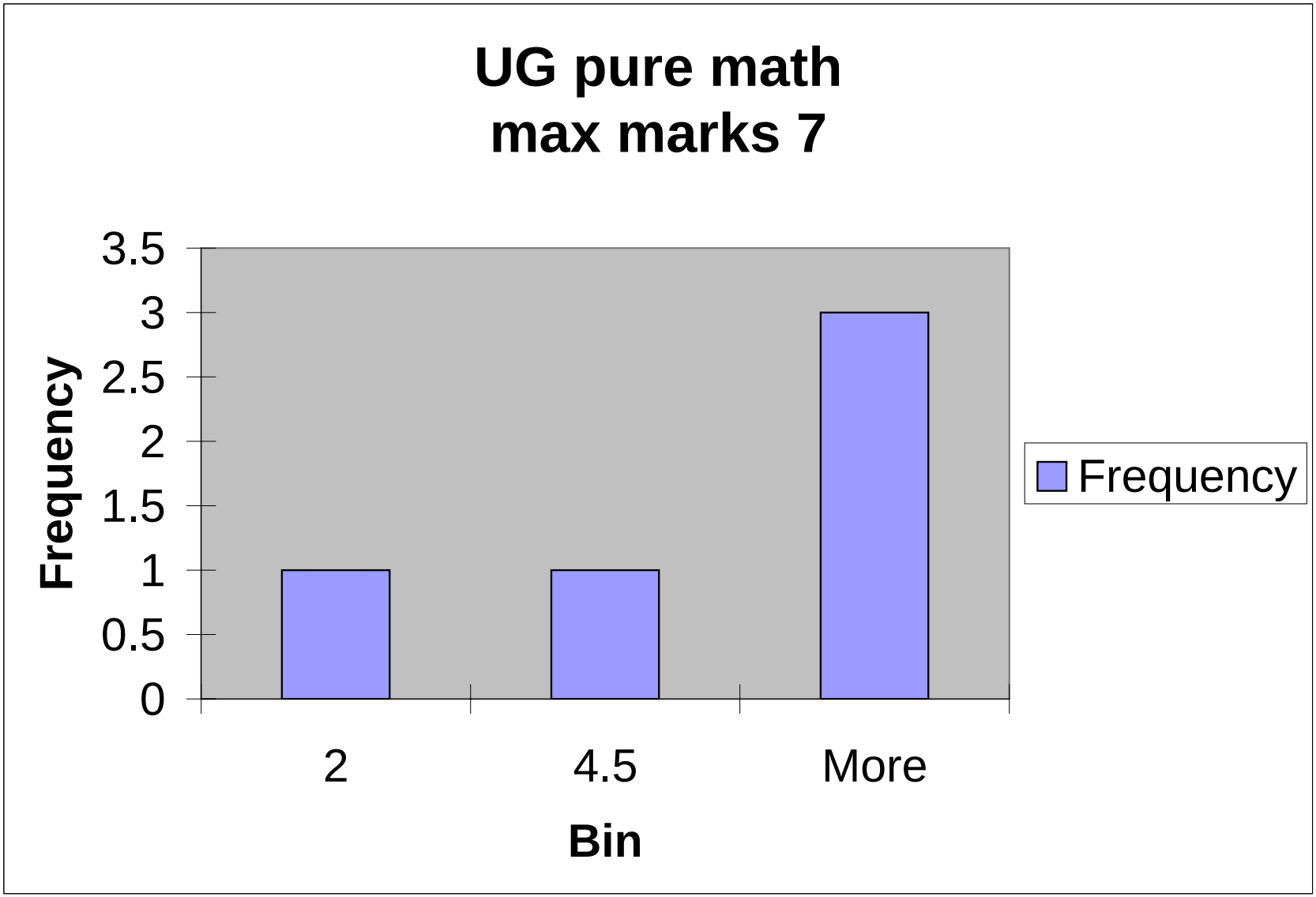
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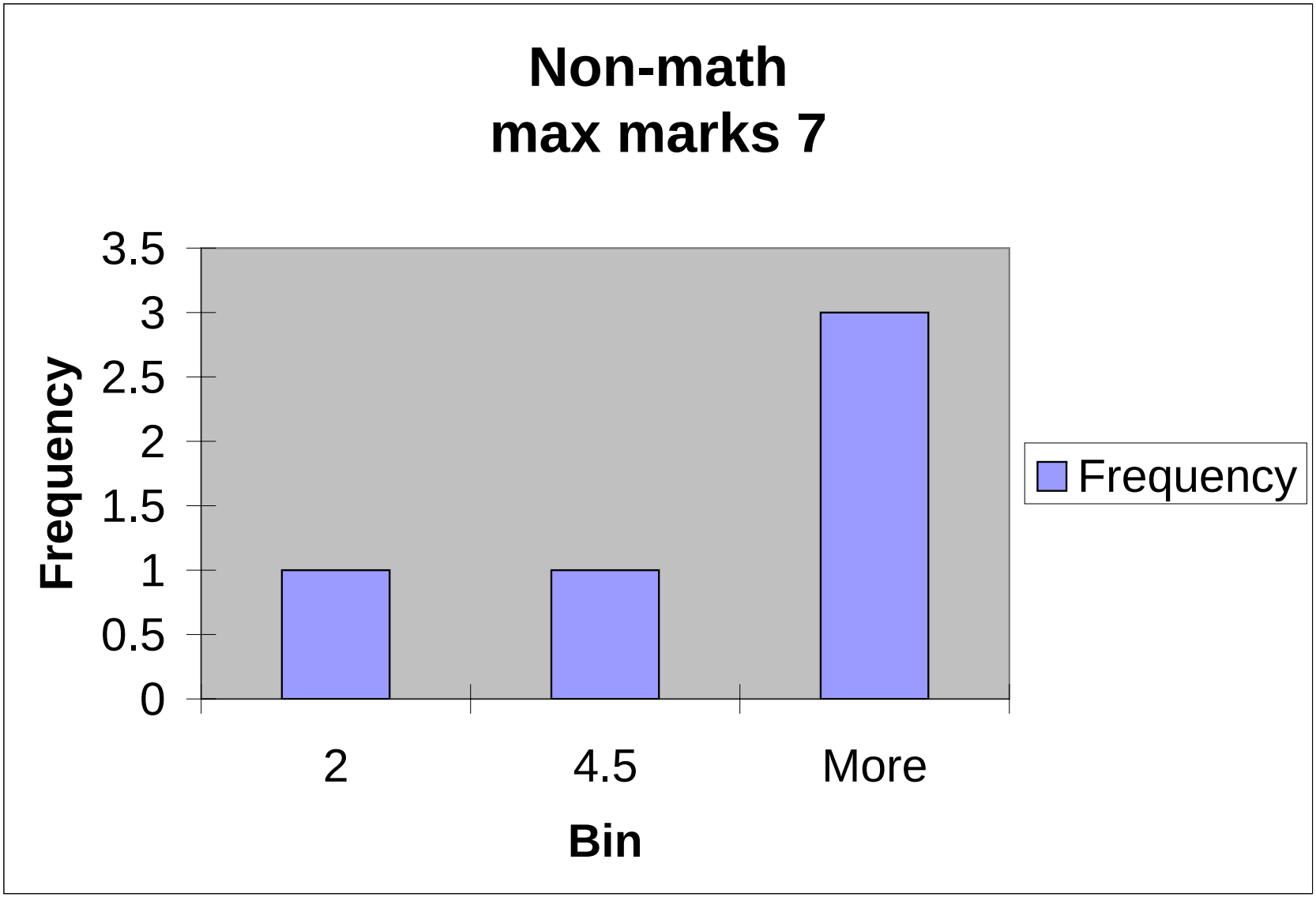
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CSISSC, Tehran

Ambedkar University, Delhi

SGT University, Delhi