Yoga, mathematics, and ganita

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Is yoga religious? Can it legally be taught in public schools? A judge at San Diego must decide this question raised by parents of Encinitas schools. The case has captured attention after the judge declared that he too was a practitioner of yoga, and Obama included yoga in his Easter party.

"It all depends", is an easy general answer[1]: for the words "yoga", "religion" and "Hindu" have multiple meanings. However, one expects the judicial system to rise above quibbles, and to examine the substantive issues involved, which are more complex than appears at first sight.

Many people today understand yoga (typically mispronounced "yogaa") as just a form of physical exercise, like aerobics. However, its Hindu roots are undeniable. The word yoga derives from the Sanskrit root *yuj*, similar to yoke. What is being yoked? Briefly, the *atman* ("soul") is being yoked to Brhman ("cosmic soul" within us, Nous), and this union (yoga) is believed to lead to *moksa*, or liberation from future lives, the ultimate objective of a Hindu, howsoever broadly defined (to include atheists, for example).

The religious understanding of yoga has been an issue earlier. Some five years ago, the Malaysian National Fatwa Council hence banned yoga for Muslims, but the prime minister stepped in to say Muslims could practice yoga minus the chanting.[2] Today, advertisements for yoga classes in Malaysia explicitly point out that there is no religious component to the yoga they teach.

That is clearly a widely acceptable solution: teach yoga minus its religious moorings, and solely for its practical value. The difficulty is that this cannot justly be applied to yoga in isolation: the same legal principles must prevail in other cases. As a trivial example, stating dates using AD and BC requires students to recite Christian religious beliefs in Jesus as saviour (Christ) and Lord (Domini). Such chanting during history lessons seems innocuous, but it may be offensive to some non-Christians, just as chanting during yoga lessons is offensive to some Christians. Ensuring religious neutrality in the present-day system of education is not easy because it copies the Western model, originally designed by the church to train missionaries.[3] Thus, consider mathematics. Like yoga, can we strip mathematics of its religious bias and teach it in public schools solely for its practical value?

How is mathematics religious? This is little known.[4] To begin with, the very word "mathematics" derives from mathesis which means "learning". According to Plato "all learning is recollection", meaning recollection of knowledge that the soul acquired in its previous lives. In the famous story of Socrates and the slave boy, in Plato's *Meno*, Socrates demonstrates the slave boy's innate knowledge of mathematics and claims he has thereby proved the existence of the soul and its past lives. In his *Republic*, Plato says mathematics should be taught, like music, for its spiritual value, and *not* for its practical value.

Both yogic meditation and mathematics (as Plato understood it) work similarly: they take away attention from the outside world and "compel the soul to turn its gaze inwards where there is the full perfection of being".[5] Proclus (5^{th} c.) explains that driving attention inwards reunites the soul with the Nous within, so that mathematics hence leads to the "blessed life". Exactly like yogic meditation, this

A yogic meditative pose to shut off external sensations and drive attention inwards. © C. K. Raju, Euclid and Jesus, 2012.

involves a spiritually moving experience: even atheists like Bertrand Russell indirectly admitted to it by ascribing to mathematics a "cold, austere beauty".

However, for most school students today, mathematics is a nightmare, not a spiritually moving experience. How did this sea change come about?

To begin with the church objected to the notion of soul underlying mathesis. That notion of soul was similar to the *atman* (underlying yoga)[6] and was also accepted by early Christians.[7] It involved belief in a series of past and future lives *across cosmic cycles*[8]. That belief became inconvenient to the church after it married the state, in the 4th c CE, and changed Christian doctrine into a tool of power for its priests. Church power derived from terrorising people with horrible tales of what would happen to them after death, for an eternity in hell, if they did not obey the priest. The belief in a long series of past and future lives worked against this terror ("why repent *now*, when there are many lives to come"). Hence, the church attacked the earlier notion of soul as an impediment to its power. The church now rejected reincarnation, and ordered people to believe in resurrection (life after death exactly once); it cursed the belief in past lives,[9] and drove out all philosophers (who did mathematics) from the Roman empire. That is, the church initially rejected mathematics because the underlying notion of soul was contrary to its worldly interests.

Centuries later, the church did accept back mathematics, but for a curious reason. The Crusades initially aimed to convert Muslims by force, the way pagan Europe had earlier been converted by force. But military force failed beyond Spain. What other way was there to convert Muslims who rejected also Christian scriptures? Since Muslims accepted reason, the church turned to reason as the only means to persuade Muslims to convert. But, first, reason had to be turned into a Christian weapon. To this end, Christian doctrine was again transformed (by Aquinas) to the Christian theology of reason.[10]

In line with this doctrine, mathematics was made soul-less, and reinterpreted as just a way to to teach reasoning to priests. That fulfilled the church need for a new technique of persuasion.

To justify the altered Christian doctrine, it was necessary to portray reason as a theologically correct Christian inheritance. This was achieved by concocting history: the origin of "mathematics as reasoning" was attributed to a mythical Greek, "Euclid". "NOTHING" is known about "Euclid" as a leading authority, the late David Fowler, publicly admitted,[11] when I first raised doubts over a decade ago. Euclid's name is not mentioned in Greek manuscripts of the book *Elements* attributed to him. But, till today, the myth of Euclid is taught to school-children, and propagated through Wikipedia. To demonstrate that people believe in Euclid without *any* evidence, I have offered a prize of USD 3300 for serious evidence about Euclid. The prize stands unclaimed.

Bad history leads to bad philosophy. Because the church needed reasoning only as a tool for persuasion, the West came to understand mathematics as only concerned with metaphysical (or deductive) proof, which it extolled over inductive or empirical proofs. That suits the Crusading theology of reason, but it does not fit even the very first proposition of the *Elements*, which uses an empirical proof! Such, however, is the power of church myths, that no one noticed this elementary difficulty for seven *centuries*. Let us understand its significance. If empirical proofs are permitted at one place, why not elsewhere? Why not prove the "Pythagorean theorem" empirically in one step (as Indian texts do)? Why go through 47 propositions, as in the *Elements*? I had this difficulty in school, but my teacher had no answer, for the difficulty cannot be resolved with the Christian reinterpretation of the *Elements.* To the ordinary mind, this fact alone destroys the myth that the *Elements* is about metaphysical proofs.

However, Bertrand Russell and David Hilbert found an ingenious way to "save the story": they rejected the book as incorrect, and rewrote it to fit the myth! An empirical proof is used also in the fourth proposition of the *Elements* (called side-angle-side theorem). This key proposition was changed into a *postulate*. That is how it is taught in schools since 1960's, as the side-angle-side *postulate*. Formalism is what ordinary folk called "new math", and identified with set theory etc. This was initiated by Russell and Hilbert, *after* they made the above correction to "Euclid". Formalism turns all mathematics into metaphysics, and that metaphysics suits the church. But is that metaphysics universal or even religiously neutral? (1322 words.)

Part 2

Most people today learn mathematics for its *practical* applications. And, contrary to Plato, it was for its practical applications that mathematics developed in most of the world. Historically, much of present-day school mathematics—arithmetic, algebra, trigonometry, calculus, probability[12]—originated in India for practical purposes, such as commerce, agriculture and navigation. Indian mathematics, called *ganita*, was useful for *calculation.* It was rigorous, but accepted empirical proofs.[13] Such empirical proofs do not affect the practical applications of mathematics, for any practical application always involves the empirical.

Europeans too initially sought Indian mathematics for its practical value: they imported "Arabic numerals" for their use in commerce, and calculus for its use in navigation. But the Christian theology of reason maintains that logic binds God who is free to create empirical facts of his choice. Hence, it regards logical proofs (which bind God) as stronger than empirical proofs. Hence, Europeans rejected empirical proofs as inferior, and demanded metaphysical proofs instead, as formalism does today. That metaphysics added nothing to the practical value of mathematics (in science and engineering), but,

because that metaphysics is *not* universal, it did inject a religious bias.[14]

Thus, *all* early systems of Indian philosophy accept empirical proofs (*pratyaksa*) as does Islam (*tajurba*) or science (experiments). Hence, teaching students to reject empirical proofs (or to regard them as inferior) tacitly teaches Hindus, Buddhists, Jains and Muslims that their philosophy is fundamentally unsound. This is a dangerous teaching, far more insidious than chanting during yoga. Students are being taught through mathematics that Christian metaphysics is more reliable than the evidence of their senses. By itself this is reason enough to change the way mathematics is taught in schools today.

Secondly, Christian theology declared reason as "universal", because reasoning rests on logic which binds God; but it never asked *which* logic? Hence, formalism, too, uncritically accepted binary logic as universal. But that is plain false: the Buddhist logic of *catuskoti*, the Jain logic of *syadavada*, quantum logic, or the logic of natural language, are all different from the binary logic, used in mathematical proofs.[15] By using a different logic, a different set of mathematical theorems can be proved from the same set of postulates, as was demonstrated long ago. This means mathematical theorems are mere relative truths which depend upon *both* postulates and logic. That goes beyond formalism which already admits that mathematical theorems are truths relative to postulates. For example, proofs by contradiction would fail with any of the above logics. Naturally, relative truths are of little value, so we would have to abandon theorem-proving, and revert to practical techniques of calculation as the source of value in mathematics. Conversely, if mathematics is used to teach that there is only one "right" kind of logic, that indoctrinates students into rejecting other logics: a clear religious bias, against Buddhists and Jains at least.

A third kind of religious bias arises through the postulates. Though, in principle, formalism allows one to choose the (metaphysical) postulates of a mathematical theory, that is not true in practice. The postulates underlying present-day school mathematics were all decided by a few influential Western mathematicians, and are treated as inviolable. The calculus involves infinite series, and postulates regarding infinity (as in set theory) bring in a subtler religious bias, through a culture-specific concept of infinity/eternity.[16] For example, college calculus uses limits, hence postulates the continuum, contrary to the belief in atomicity among Islamic thinkers like al Ashari, suggesting that Muslim thinkers were naïve. Students are never told that calculus historically originated with a similar belief in atomicity, in India, or that it could be done differently today with a different set of postulates, even within formalism,[17] as in calculations done on a computer which cannot handle infinity.

Turning mathematics into Western metaphysics has not only injected religious biases, it has also made mathematics difficult, especially through the metaphysics of infinity. Thus, the calculus was used in India to calculate precise trigonometric values (or the arc of a circle). Indians used infinite series in a sophisticated and simple way. That was then beyond the comprehension of of Europeans. Thus, Descartes amusingly declared in his *Geometry* that the length of a curved line was *beyond the human mind*. He thought that one *must* calculate it metaphysically using an infinity of straight line segments each of infinitesimal length. In fact, measuring a curved line is very easy, and any child can do it. Indian children were traditionally taught to do it using a flexible string.[18] However, that easy method is absent from present-day math teaching which blindly imitates the Western practice, and uses a compass box, no instrument in which can measure curved lines.

In Descartes' time, most educated Europeans were priests, so the metaphysics was piled on to mathematics in the West. Today calculus and differential equations are taught with heavy metaphysics such as the continuum (or formal real numbers). To reiterate, that metaphysics is irrelevant[19] to the

practical technique of calculation. Thus, the latest computer (or android phone) still calculates trigonometric values using the very same Indian infinite series; all that has changed is the metaphysical justification. (And the nomenclature; for, in line with the Western technique of falsifying history, those series today are all named after Westerners like Leibniz or Taylor.)

Likewise, to send a man to the moon, NASA must calculate the trajectory of the rocket. To do that it must still numerically solve differential equations. That was a technique of calculation initiated by Aryabhata in the 5th c. CE.[20] Of course, the calculation is today done on a computer, and uses a faster algorithm etc. But the point is that the metaphysics of the continuum is not relevant to the practical process of calculation. In fact, a computer cannot handle infinity. Hence, a computer calculation can *never* use the metaphysical continuum, and uses instead what are called floating point numbers. Thus, the actual practical process of calculation can be better justified with a *different* philosophy of mathematics, such as my realistic philosophy of zeroism.[21]

To summarise, Platonic and Neoplatonic mathematics was a spiritual practice remarkably similar to yoga. Though the church eliminated this spiritual aspect of mathematics, and reinterpreted it to fit its Crusading theology, that introduced another sort of religious bias. That religious bias persists today through formalism which, though not overtly religious, makes all mathematics metaphysical, but with a religiously-biased metaphysics. Eliminating that religious bias (and metaphysics) does not affect *any* practical uses of mathematics, such as sending a man to the moon. On the contrary, it makes mathematics easy even for non-math students. That *enhances* practical value, since students can now solve much harder problems, as my own children have demonstrated.[22] I have also demonstrated this in teaching experiments conducted in three countries.[23]

So, if Western metaphysics injected religious biases into mathematics, made mathematics difficult, added nothing to its practical applications, and mathematics can be done easily in another way, we ought to teach it that way. We ought to teach secular ganita.

Why, then, do we still teach Western mathematics in our schools? The reason is that Western metaphysics (and the related false history of Euclid, etc.) is a source of *power*. Formalists, like the church earlier, strike a pose of superiority: they grandly declare their own metaphysics as infallible, while declaring that other ways of doing mathematics (including all practical calculation on a computer) are erroneous and inferior. This pose of superiority, and related false history, was intensely cultivated by racist and colonial historians.[24] It was used to trick people into accepting the superiority of Western education and to globalise it during colonialism.[25] That led to the capture the colonised mind, which came to regard itself as inferior. That is how the West, which imported calculus from India, ironically gave it back to us in an inferior form which we today blindly imitate.

So, can we get over it? Can we today eliminate the religious bias in mathematics and teach it solely for its practical applications? That brings up a crucial difference between yoga and mathematics. While many people, whether or not informed about yoga, would readily offer an opinion about its religiosity, almost no one dares offer an opinion about the religiosity of mathematics. The first lesson they learnt in school is to trust authority in mathematics. Therefore, they would say, “We don't know, ask an expert”. However, all mathematics “experts”, today, have a vested interest in formalism. And their expertise itself is judged by marks of Western approval. That means there is a system in place to defend the soft power that the West derived form bad metaphysics and false history. Our legal system too imitates the West. Is it strong enough to overcome those defences and deliver justice? (1462 words.)

1 Philip Goldberg, “The Encinitas yoga case: Yoga is religious, only it is not”, http://www.huffingtonpost.com/philip-goldberg/the-encinitas-yoga_b_2892214.html.

2 http://www.reuters.com/article/2008/11/26/us-malaysia-islam-yoga-idUSTRE4AP2CA20081126.

3 For example, influential institutions in India controlled by the church are Loyola College Madras, St Stephen's College, Delhi, Loreto College, Kolkata, and St Xavier's College Mumbai. For more details of this church control, worldwide, see C. K. Raju, “Decolonising our universities: Time for a change”, GlobalHigherEd blog, http://globalhighered.wordpress.com/2011/09/11/decolonising-our-universities-time-for-change/

4 C. K. Raju, “The religious roots of mathematics”, *Theory, Culture & Society* **23**(1–2) Jan-March 2006, Spl. Issue ed. Mike Featherstone, Couze Venn, Ryan Bishop, and John Phillips, pp. 95–97. (http://www.ckraju.net/papers/Religious-roots-of-math-TCS.pdf.) For an account for the layperson, see C. K. Raju, *Euclid and Jesus: how and why the church changed mathematics and Christianity across two religious wars*, Multiversity, Penang, 2012. (http://ckraju.net/Euclid.)

5 Plato, *Republic*, VII.526. In: *The Dialogues of Plato*, trans. B. Jowett, Encyclopedia Britannica, Chicago, 1994, p. 394.

6 For an elaboration for the layperson, see C. K. Raju, “Geometry and the soul”, chp. 5 in *Euclid and Jesus,* cited above. For a more scholarly account of the traditional Indian notion of *atman*, see C. K. Raju,“*Atman*, Quasi-Recurrence and *paticca samuppada*”, in *Self, Science and Society, Theoretical and Historical Perspectives*, ed. D. P. Chattopadhyaya, and A. K. Sengupta, PHISPC, New Delhi, 2005, pp. 196-206. (http://www.ckraju.net/papers/Atman-quasi-recurrence-and-paticca-samuppada.pdf.)

7 Origen was a key representative of this pre-Nicene understanding of Christianity. This is part of the general similarities between what is today called Advaita-Vedant, “Neoplatonism”, and sufism. For more details see C. K. Raju, *The Eleven Pictures of Time*, Sage, 2003, and the further quotes from Origen's *De Princiipis*, in *Euclid and Jesus*, cited above.

8 Or a belief in quasi-cyclic time. See, C. K. Raju, “Life after death”, chp. 1 in *Eleven Pictures of Time*, cited above.

9 C. K. Raju, “The curse on 'cyclic' time”, chp. 2 in *The Eleven Pictures of Time*, Sage, 2003.

10 This adapted the earlier Islamic theology of reason (*aql-i-kalam*), to bring it in line with the earlier (post-Nicene) transformation of Christian doctrine. See, C. K. Raju, “Benedict's maledicts”, *Indian Journal of Secularism*, **10**(3) (2006) 79-90. http://www.zcommunications.org/benedicts-maledicts-by-c-k-raju.

11 http://mathforum.org/kb/message.jspa?messageID=1175734. However, the less well-informed Wikipedia account might insist hat “almost nothing” is known about Euclid. Typical of stories of grand achievements by Greeks, the “almost nothing” that is “known” comes from a passing remark attributed to a 5th c. author (in this case Proclus). In actual fact, that remark is sourced from another thousand years later, from a 15th c. Byzantine Greek manuscript on paper. In the case of “Euclid” that remark attributed to Proclus does not at all fit the spiritual (“Neoplatonic”) philosophy of mathematics he advocates in the rest of his preface, and shows knowledge of a 13th c. interpolation in a text attributed to Archimedes. Hence, that remark is a later interpolation, and is no evidence. Needless to say, Euclid, who supposedly lived in Alexandria, in Africa, is depicted in school texts as white skinned. Such history of mathematics, with a religious and racist bias, continues to be taught in school texts today.

12 For the origin of probability and sampling theory in India, see C. K. Raju, “Probability in Ancient India”, chp. 37 in *Handbook of the Philosophy of Science*, vol 7. *Philosophy of Statistics*, ed, Dov M. Gabbay, Paul Thagard and John Woods. Elsevier, 2011, pp. 1175-1196 (http://www.ckraju.net/papers/Probability-in-Ancient-India.pdf). For the Indian origin of calculus, see C. K. Raju, *Cultural Foundations of Mathematics: the nature of mathematical proof and the transmission of calculus from India to Europe in the 16th c. CE*, Pearson Longman, 2007. The origin of calculus is connected to the infinite series, and used to obtain precise “trigonometric” (rather circle-metric) values needed for the calendar, and navigation (http://www.ckraju.net/papers/GJH-book-review.pdf). Earlier transmitted to Europe through Arabs, these values were transmitted directly by Jesuit priests, afte rthe 16th c., in connection with the European navigational problem. The transmission of arithmetic algorithms (“Arabic numeral”) from India to Europe via al Khhwarizmi's *Hisab al Hind*, Pope Sylvester 2 (976 CE), and Florentine merchants, is well known. Algebra, deriving

from Brahmagupta, is also the title of another one of al Khwarizmi's books on algebraic equations: “Al jabr waa'l Muqabala”.

13 For a detailed examination of empirical proofs in Indian mathematics, see C. K. Raju, “Computers, mathematics education, and the alternative epistemology of the calculus in the Yuktibhasa”, *Philosophy East and West* **53**(2), 2001, pp. 325-62 (http://www.ckraju.net/papers/Hawaii.pdf).

14 For a more scholarly account, see C. K. Raju, “Teaching mathematics with a different philosophy. Part 1: Formal mathematics as biased metaphysics.” *Science and Culture* **77** (7-8) (2011) pp. 274–279. http://www.scienceandculture-isna.org/July-aug-2011/03%20C%20K%20Raju.pdf.

15 For Buddhist and Jain logic, see my article on “Logic”, *Springer Encyclopedia of Non-Western Science, Technology, and Medicine*, 2008. http://www.ckraju.net/papers/Nonwestern-logic.pdf. For quantum logic, see “Quantum mechanical time”, chp 6b in C. K. Raju, *Time: Towards a Consistent Theory*, Kluwer Academic, Dordrecht, 1994. http://arxiv.org/pdf/0808.1344v1. For the relation of the two, see the article on “Probability in Ancient India”, cited earlier.

16 Infinity is fertile ground for theology, and the first creationist controversy in the 6th c. between John Philoponus and Proclus actually concerned the eternity of the cosmos. Today, Stephen Hawking's claim that the cosmos was created (in a singularity) is based on precisely such manipulation of the notion of infinity. See, C. K. Raju*, The Eleven Pictures of Time*, Sage, 2003.

17 E.g. it can be done over a non-Archimedean field or with a discrete number system. See, C. K. Raju, “Retarded gravitation theory”, in: Waldyr Rodrigues Jr, Richard Kerner, Gentil O. Pires, and Carlos Pinheiro (ed.), *Sixth International School on Field Theory and Gravitation*, American Institute of Physics, New York, 2012, pp. 260-276. http://ckraju.net/papers/retarded_gravitation_theory-rio.pdf.

18 C. K. Raju, “Towards Equity in Math Education 2. The Indian Rope Trick” *Bharatiya Samajik Chintan* (New Series) **7** **(**4) (2009) pp. 265–269 (http://www.ckraju.net/papers/MathEducation2RopeTrick.pdf).

19 This irrelevance comes out most clearly in the manipulation of infinite integrals in quantum field theory (renormalization problem), and in junction conditions and the post-Newtonian approximation of general relativity, etc. See, appendix to *Cultural Foundations of Mathematics*, cited above. See, also, e.g. C. K. Raju, “Junction Conditions in General Relativity”, *J. Phys. A* **15** (1982) 1785–97, and “Distributional Matter Tensors in Relativity.” In: *Proc. 5th Marcel Grossman Conference,* D. Blair and M. J. Buckingham (ed), R. Ruffini (series ed.), World Scientific, Singapore, 1989, 421–23. arXiv:0804.1998.

20 Aryabhata's method of numerical solution is today known as “Euler's method”. For a detailed description and discussion of Aryabhata's method of numerically solving difference equations to derive precise sine and cosine values, see C. K. Raju, “Infinite series and π”, chp. 3 in *Cultural Foundations of Mathematics*, Pearson Longman, 2007.

21 For a quick account of zeroism and its advantages, see the paper on “Probability in Ancient India”, cited above.

22 Two hard mathematical problems which my children solved, while in school (K-12), were (a) the variation in the time period of the simple pendulum, and (b) the brachistochrone with resistance. For the pendulum, see, Suvrat Raju, http://ckraju.net/11picsoftime/pendulum.pdf. For the brachistochrone, see Archishman Raju, “A simple way to solve the brachistochrone problem with resistance”, *Physics Education* (India), **28** (3) (2012), http://www.physedu.in//uploads/publication/3/65/Archishman_Brachistochrone13July.pdf. For an anecdotal account, see C. K. Raju, “Time: what is it that it can be measured”, *Science and Education* **15** (2006) pp. 537-551. http://link.springer.com/article/10.1007%2Fs11191-005-5287-z#page-1 and http://ckraju.net/papers/ckr_pendu_1_paper.pdf.

23 C. K. Raju, “Teaching mathematics with a different philosophy. Part 2: Calculus without limits”, *Science and Culture* **77** (7-8) (2011) pp. 280–85. http://www.scienceandculture-isna.org/July-aug-2011/04%20C%20K%20Raju2.pdf.

24 E.g., the racist historian Rouse Ball, in his “classic” history of mathematics, declares that only the Greek race understood what “universal” mathematics is about, since the empirical proofs used in other cultures reduced geometry to “mere” land-surveying. Even a supposed liberal like Needham asks why didn't the Chinese imitate “Euclid”?

25 Macaulay used this argument in his infamous minute of 1835. See, e.g., C. K. Raju, *Ending Academic Imperialism,* Citizens International, Penang, 2010. http://multiworldindia.org/wp-content/uploads/2010/05/ckr-Tehran-talk-on-academic-imperialism.pdf .