Synoptic Contents

II The Calculus in India

The thousand-year background to infinite series in India and how they were derived

The underlying philosophy of *pramāṇa* and of number is brought out in the context of the derivation of the Indian infinite series. The full details, which are here presented for the first time, show that there was valid *pramāṇa* for the Indian infinite series (in contrast to Newton etc. who could not provide their contemporaries with any clear proof or derivation of the very same infinite series). Further, unlike the abrupt appearance of infinite series in Europe, starting in the 1630's, the Indian infinite series evolved over a thousand year period, as trigonometric precision was pushed from the first minute (Āryabhaṭa 5th c. CE) to the second minute (Vaṭeśvara 9th c. CE) to the third minute (attempted e.g. by Govindasvāmin, 9th c. CE, and achieved by Mādhava 14th–15th c. CE.). Āryabhaṭa used an elegant technique of finite differences and numerical quadrature, the numerical counterpart of the fundamental theorem of calculus. The use of second differences for quadratic interpolation was then extended to higher orders, using the fraction series expansion. "Limits" were handled using order counting, and a traditional philosophy of neglecting non-representables. In analogy with numerical series, continued fraction expansions were used to represent an infinite series of rational functions.

Why precise trigonometric values were needed in India for determination of time, latitude, longitude, and the size of the earth

The calculus developed in India to calculate precise trigonometric values needed in connection with the calendar—(still) a critical requirement for monsoon-driven agriculture which has long been (and remains to this day) the primary means of producing wealth in India. The similarity of cultural practices spread over a large area, India, led to a calendar standardized for the prime meridian of Ujjayinī, and recalibrated for the local place. Recalibration required determination of local latitude and longitude, early Indian techniques for which used the size of the globe as input. These techniques of determining latitude and longitude were needed also for celestial navigation for overseas trade, then the other important means of producing wealth in India.

Precise measurement of angles and the two-scale principle

The *kamāl* is a traditional navigational instrument used by the Indian navigator who navigated Vasco da Gama to India from Africa. Field work in the Lakshadweep islands led to the recovery of the instrument, used in traditional Indo-Arabic navigation, whose construction is here described. The *kamāl* primarily measures angles using a harmonic scale, marked by knots on a string. The novel feature is the use of the two-scale ("Vernier") principle for harmonic interpolation. This enabled very high accuracy in angle measurements, thus explaining also the instrumental basis of the precise early Indo-Arabic estimates of the size of the globe, and determination of local latitude and longitude.

III Transmission of the Calculus to Europe

General historiographic considerations and the nature and standards of evidence to decide transmission

We re-examine and reject the racist model that all (or most) scientific knowledge, especially of mathematics and astronomy, has a White origin either in post-renaissance Europe or in early Greece, from where others obtained it by transmission. Alexander obtained a huge booty of books from Persia and Egypt, some of which were translated into Greek. The conjectured scientific knowledge of early Greeks could not grow in Athens, but could grow only in Alexandria, on African soil, since it derived from transmission of knowledge from Black Egypt and other non-White sources. Since the actual evidence for the conjectured Greek knowledge in Alexandria comes almost wholly from very late Arabic sources, or even later Byzantine Greek sources, later-day world knowledge up to the 10th c. CE has also been anachronistically attributed to early Greeks, and is incompatible with the crudeness of Greek and Roman knowledge of mathematics and astronomy exhibited in non-textual sources. As an example, we consider the evidence that significant portions of the current *Almagest* text attributed to Ptolemy, derived by such transmission *from* India via Jundishapur and Baghdad. The cases of Copernicus and the rock edicts of Ashoka the Great are used to show how much and how systematically the standard of evidence varies with the direction of transmission. To avoid this racist double standard of evidence, often masked by an appeal to authority, we propose a new standard of evidence for transmission, involving opportunity and motivation, together with circumstantial, documentary, and epistemological evidence.

The European navigational problem and its solution available in Indian books easily accessible to Jesuits

At the beginning of the 16th c. CE, European navigators on the high seas could not determine any of the three "ells"—latitude, longitude and loxodromes—since their peculiar navigational technique was adapted to the Mediterranean. However, trade with India, China, and colonization of Americas was becoming the major source of wealth in Europe. This required good knowledge of navigation, to acquire which European governments took numerous big initiatives. Celestial navigation required accurate trigonometric values, and astronomical data, including an accurate calendar, all of which were then lacking in Europe. This provided huge motivation for transmission to Europe of precise Indian trigonometric values, and through them the infinite series and the calculus. Coincidentally, the first Roman Catholic mission in India was founded in Cochin, in 1500, and later turned into a college for the indigenous Syrian Christians, in the neighbourhood, who spoke Malayalam. The Raja of Cochin simultaneously patronized both Portuguese and the authors of key texts documenting expositions of the Indian infinite series used to derive accurate trigonometric values. This provided a splendid opportunity for the Jesuits, who systematically gathered knowledge by applying the Toledo model of mass translation to Cochin, soon after they took over the Cochin college in 1550 CE. Apart from the local languages, the Jesuits were soon trained also in practical mathematics and astronomy. Also, sailors and travellers returning from India routinely brought back books, as souvenirs or to be sold to collectors in Europe. From the mid-16th c. CE onwards, circumstantial evidence of the knowledge of Indian mathematical and astronomical works begins to appear in the works of Mercator, Clavius, Julius Scaliger, Tycho Brahe, de Nobili, Kepler, Cavalieri, Fermat, Pascal, etc. Indian sources were rarely directly acknowledged by these Europeans due to the terror of acknowledging "pagan" sources during the Inquisition, and the church doctrine of Christian Discovery, which preceded racism. (This is in striking contrast to the Arabs in the 9th c. CE who had enough religious freedom to acknowledge Indian sources.) The prolonged difficulties that Europeans had in understanding the epistemological basis of the calculus further characterizes the calculus as knowledge imported into Europe like the algorismus.

Berkeley's objections reflect the doubts about the nature of fluxions, infinitesimals etc., which neither Newton, nor Leibniz, nor their supporters could coherently explain to sceptical contemporaries. These doubts led eventually to the formalisation of "real" numbers using Dedekind cuts and set theory (itself formalised only in the 1930's), which finally gave a formulation of the calculus acceptable in the West. These prolonged European difficulties with the calculus arose because the Indian derivation of the infinite series used a philosophy of non-representables similar to *śūnyavāda*, and incompatible with Platonic idealism or formalism—thoughtlessly taken as the "universal" basis of mathematics in Europe. The central problem of representation was left unresolved by the formalisation of real numbers, which achieved nothing of any practical value. A similar problem had arisen earlier in Europe, in the dispute between abacus and algorismus, which involved zeroing of non-representables in a calculation. The *śūnyavāda* philosophy regards idealistic conceptualizations (as in Platonism or formalism) as empty and erroneous (e.g., in direct opposition to Platonism it regards an ideal geometrical point as an erroneous representation of a real dot). It is also better suited than Platonic idealism or formalism to numbers on a computer